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Sir Claude Inglis, Member, Chairman of the Division, in the Chair

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Author.

Hydraulics Paper No. 2

# "Measurement and Utilization of the Water Resources of the Nile Basin"

by

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## SYNOPSIS

The Paper deals with some of the results of the study of the Nile in the past 50 years.

At the Aswan Dam the discharge of the river is measured by means of the sluices, some of which have been calibrated by means of a large masonry tank, so that the method depends directly on the measurement of volume. The methods of deducing daily, monthly, and annual discharges from the individual measurements are described. Some information is given on the silt brought down from Ethiopia in flood time, and of the steps taken to keep a record of the diminution of the contents of the Aswan reservoir if there should be a deposition of silt.

A short account is given of the hydrology of the Nile including the wide variation of annual flow. The effects of these hydrological factors on the design of projects are indicated.

The Sudan Gezira Irrigation Scheme and recent projects are briefly described.

The Author's theory of over-year storage finds a relation between the capacity of the reservoir which would have given a steady discharge equal to the mean for the period, and the standard deviation and the number of years of observation. It also finds a relation between the capacity required to prevent the discharge falling below some definite quantity less than the mean, and the capacity required to maintain the mean discharge. These are statistical relations.

## Introduction

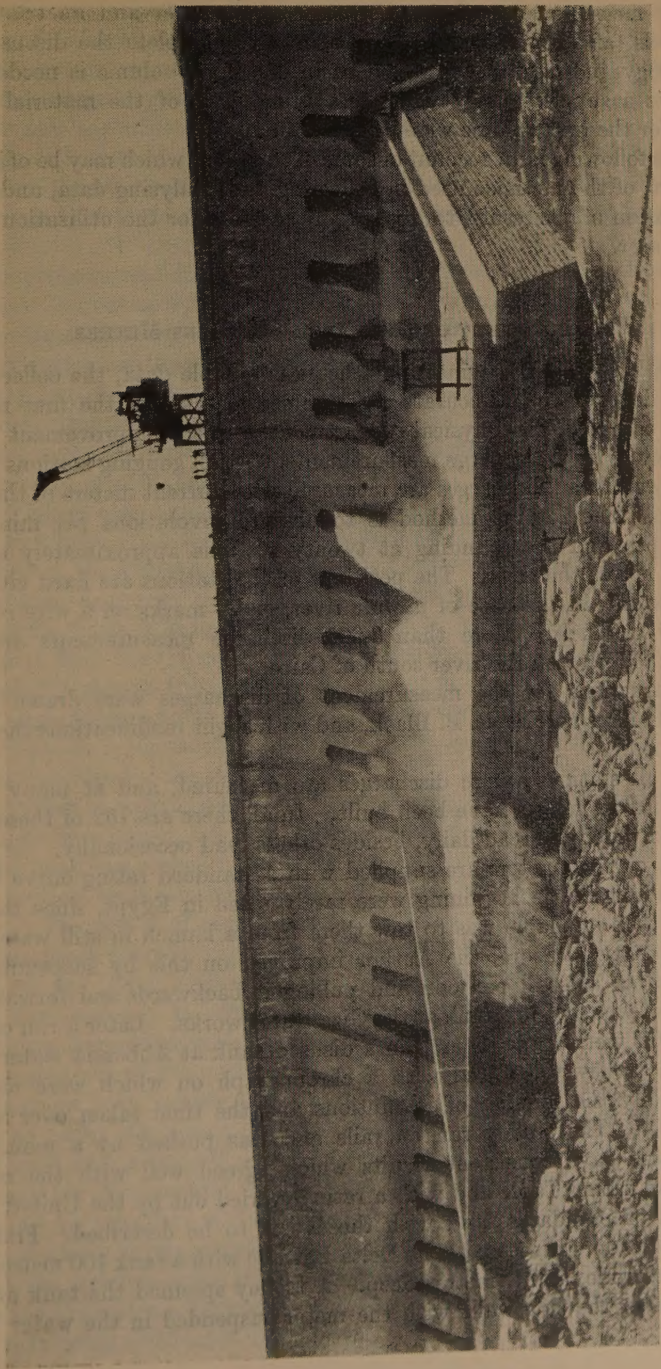
The subject of this Paper is of such wide scope that it is obviously only possible to deal with a very small selection of topics, some of which have been chosen to illustrate the scope of the subject, and others because they are important to those who have to study the regimes of other rivers.

Previous Papers relating to the measurement of the water flowing in the Nile have been presented to the Institution by Sir Murdoch MacDonald,[1] Mr D. A. F. Watt,[2,3] and the present Author, and Messrs R. F. Wileman[4] and H. W. Clark. The first and third Papers relate to the fundamental measurements of the discharge of the Nile before any of it has been used for irrigation in Egypt, and these are directly based on volumetric measurements by means of a large tank below the Aswan Dam (see *Fig. 1*). To the Author's knowledge, on no other large river is the discharge measured by means of a tank, and these Aswan measurements provide a standard for testing the accuracy of the ordinary method of river measurement by means of current meters. The fourth Paper gives a description of the ordinary current-meter method and the apparatus used, with the modifications needed to meet some of the varying conditions on the White Nile and its tributaries.

The scientific study of the hydrology of the Nile began after the re-opening of the Sudan and the completion of the Aswan Dam in 1902. The sluice measurements instituted by Sir Murdoch MacDonald (Adviser to the Egyptian Ministry of Public Works) were worked out and perfected, and current meters were introduced by Sir Henry Lyons, then Director-General of the Survey of Egypt, whose staff measured the discharge of the Blue and White Niles, and established for the first time their relative volumes. At the same time the reconnaissance journeys of Sir William Garstin (Adviser to the Ministry of Public Works) to the Great Lakes of East Africa, and of Mr C. E. Dupuis (later Adviser) to Lake Tana in Ethiopia, enabled the outlines of a scheme for the utilization of the Nile waters to be drawn up, and this has been the basis of most of the subsequent proposals. The Sudan Branch of the Egyptian Irrigation Service was formed in 1905 and started to collect hydrological data about the Nile on which projects could be based. This was interrupted to some extent by the 1914–18 war, and during this time the Physical Department was formed, whose work included the study of hydrology and the maintenance of instruments and records, though most of the routine measurement of discharges and reading of gauges remained in the hands of the Irrigation Department. It was clear that these needed to be multiplied and extended to the farthest parts of the Nile Basin, and that data had to be analysed systematically and the results published regularly. This led to the publication of "The Nile Basin,"[5] which now comprises eight volumes and ten supplements, containing the results of reconnaissances and studies, and

[1] The references are given on p. 25.

*Fig. 1.*



MEASUREMENT OF THE DISCHARGE OF A SLUICE AT ASWAN

all the measurements of the discharge of the river and its tributaries, its levels, and the rainfall on its basin. To complete the discussion of hydrology and description of the basin one more volume is needed, and perhaps a supplementary volume to bring some of the material up to date, for the first volume was published in 1931.

The following is an account of some of this work which may be of general interest, of the methods used in collecting and analysing data, and of the application of the results to drawing-up schemes for the utilization of the Nile water.

## Measurement of Discharge—Current Meters

The basis of any engineering scheme is reliable data, the collection of which depends upon accurate measurements. One of the first matters which occupied the Physical Department was the improvement of the accuracy of river discharge measurements. At all gauging stations except the Aswan Dam, discharges are measured with current meters of the Price pattern. The routine method is to observe revolutions per minute at half-depth and the sounding at twenty stations approximately equally spaced across the river. The positions of the stations are fixed either by box-sextant, in the case of a wide river, or by marks on a wire rope for smaller channels. More than 3,000 discharge measurements are now made each year on the river south of Cairo.

Instructions for the measurement of discharges were drawn up by Dr P. Phillips and Mr R. P. Black, and with slight modifications these still hold good.[6]

At all stations where discharges are measured, and at many others as well, river gauges have been built. In all there are 152 of these south of Cairo which are read daily, besides others read occasionally.

The current meters are supplied with a standard rating curve by the makers and at the beginning were rarely rated in Egypt, since the only means then available was to tow them from a launch in still water. As a temporary measure, the Author improved on this by suspending the meter from a small pontoon and pulling it backwards and forwards, by means of winches, in a tank at the Giza water works. Later a run of more than 100 metres was secured in a disused tank at Abbassia waterworks. Here a trolley was fitted with a chronograph on which were recorded electrically the number of revolutions and the time taken over a fixed distance.[7] The trolley ran on rails and was pushed by a man. This apparatus gave consistent results which agreed well with the makers' standard rating Table and with a rating carried out by the United States Bureau of Standards, and with those now to be described. Finally, a rating station was set up at the Delta Barrage with a tank 100 metres long, 2 metres wide, and 2 metres deep. A trolley spanned the tank and ran on rails on the side walls with the meter suspended in the water below.

An electric motor drives the trolley at any speed from 5 to 300 centimetres per second (Ward-Lamard speed control) and the number of revolutions and velocity are recorded on a chronograph. A similar rating station is being constructed by the Hydrological Survey of Uganda.

It is now a routine to rate a current meter after 100 hours of actual working, and about 300 ratings are carried out in a year. With careful treatment, the rating of a current meter does not vary very much, but lack of care can lead to damage to pivots or cups with consequent changes of rating.

The pontoon and hand-driven-trolley methods have been mentioned because it is understood that there is difficulty in getting current meters rated in Great Britain, and these improvisations can be used where a tank or a navigation canal 5 or 6 feet deep exists. The pontoon method, owing probably to turbulence set up by the pontoon, gave too many revolutions in a given distance, leading to discharges of the order of 2 per cent lower than are given by trolley ratings, but this effect could be reduced by a better design of pontoon.

Dr Phillips also made experiments on the effect of the proximity of the meter to a boat when the two are towed in the rating tank, and these are described in the Paper already mentioned. If the meter is near the boat its presence decreases the number of revolutions when the meter is in front of the bows, and increases them when it is suspended near the side. With a 16-foot boat there was no significant effect when the meter was suspended 2 metres from the side of the boat and 60 centimetres deep. When suspended in front of the bow at the same distances the maximum effect is about 1 per cent. The effect falls off rapidly with the depth of the current meter and of the stream.

The effect of rocking a discharge boat during towing was also examined. The meter was suspended from an arm 1 metre from the side of the boat and the boat was rocked as regularly as possible so as to give an up-and-down motion to the current meter, of an amplitude of about 15 centimetres. The period of an oscillation was about 1½ second. This amount of oscillation is not very excessive and may sometimes happen in discharge observations. Both a Gurley (cup type) and a Haskell (propeller type) meter were tested. At low and medium velocities of towing the effect of rocking was serious on both cup-and propeller-type meters. For example, at 0·5 metre per second, a very usual velocity on the Nile and its tributaries, the effect on both meters was about 11 per cent, but in the case of the cup-type meter the number of revolutions per second was increased and in the case of the propeller type it was decreased. The effect decreases with increase of horizontal velocity, but is not negligible in the case of the cup meter at velocities of 2 metres per second (Nile in flood). The effect on the propeller meter seems to be caused by the axis of the meter lagging behind the up-and-down motion and so being inclined to the relative motion of the water. Obviously, rough windy weather should be avoided when measuring discharges from a boat.

At this point it is convenient to mention the ingenious experiments performed by Dr Phillips to find the error of sounding with a suspended weight in deep and rapid water, as for example in the Blue or Main Nile in flood.[8] The sounding apparatus, consisting of a heavy conical lead weight suspended by a piano wire, was towed behind a launch in the still deep water of the Aswan reservoir. The length in the water, tension, and inclination of the portion of the wire outside the water were measured, and the speed of the launch by means of a current meter. The form taken up by the wire was computed theoretically. In the theory it was assumed that the force due to the water perpendicular to the wire was proportional to the square of the velocity component perpendicular to the wire. The experiments gave reasonably consistent results for the force on a 1-metre length of wire, 1·6 millimetre in diameter, held perpendicular to water flowing past it at a speed of 1 metre per second. This force was 0·18 kilogram (weight). The corresponding force along the wire was negligible compared with the tension in the wire resulting from a weight of 48 kilograms. From a number of experiments the corrections to soundings due to the water pressure carrying the wire and weight downstream were calculated. The results are summarized in Table 1.

It appears from Table 1 that it is only on deep swift rivers that any corrections to sounding are necessary with the above apparatus. On the Nile they are only required on the Blue and Main Niles and Atbara at the top of the flood. The effect increases with the thickness of the cable and will operate over a wider range of conditions if the cable supporting the current meter and sinker is used for sounding.

TABLE 1.—SOUNDING CORRECTIONS

Corrections (given in centimetres) to be subtracted from apparent soundings. A conical sinker was used, weighing 48 kilograms, and was suspended on steel piano-wire 1·6 millimetre in diameter.

| Apparent sounding : metres | | 5 | 10 | 15 |
|---|---|---|---|---|
| Mid-depth velocity : metres per second | 1·5 | 0 | 2 | 6 |
| | 2·0 | 0 | 5 | 16 |
| | 2·5 | 2 | 12 | 39 |
| | 3·0 | 4 | 24 | 74 |

## COMPARISON OF CURRENT METERS AND SLUICE MEASUREMENTS

In the early years of the twentieth century there was a great deal of discussion in Egypt as to whether the rating of a current meter in still water was applicable without serious error in flowing water, where the flow is necessarily more or less turbulent. In the Gurley meter, turbulent flow variable in magnitude and direction would tend to exaggerate the discharge. Several experiments were made by Mr B. H. Wade to detect

and measure this effect [9] and are referred to in the Paper indicated in reference 1. This question, which is discussed elsewhere,[1, 3] was finally settled by comparisons of the discharge measurements of the river at Aswan given by the sluices of the Dam, and by current meters at a good site 34 kilometres downstream. Some results showed [1, 3] that current meters and sluices agreed closely for low and medium velocities when referred to the ratings with the hand-driven trolley at Abbassia. The results obtained since ratings have been carried out by the motor-driven trolley at the Delta Barrage Rating Station are given in Table 2, which is based on the large mass of information now available.

TABLE 2.—COMPARISON OF CURRENT METERS AND SLUICE DISCHARGE MEASUREMENTS, ASWAN, 1923–1947

| Range of discharges: cumecs | Number of measurements | Norma Period | At upper limit of current meter discharges | | Mean discharge: cumecs | | Percentage difference (S—C) |
|---|---|---|---|---|---|---|---|
| | | | Velocity: metres per second | Depth: metres | Current meters | Sluices | |
| 0–2,000 | 294 | Nov. 25–July 20 | 0·7 | 5½ | 1,071 | 1,070 | –0·1 |
| 2,000–4,000 | 110 | July 20–Aug. 5<br>Oct. 25–Nov. 25 | 1·0 | 6½ | 2,976 | 2,991 | 0·5 |
| 4,000–6,000 | 121 | Aug. 5–15<br>Oct. 10–25 | 1·3 | 8 | 5,120 | 4,970 | –2·9 |
| 6,000–8,000 | 139 | Aug. 15–25<br>Sept. 15–Oct. 10 | 1·6 | 9 | 6,950 | 6,685 | –3·8 |
| 8,000–10,000 | 177 | Aug. 25–Sept. 15 | 1·7 | 10 | 8,890 | 8,510 | –4·3 |
| 10,000–11,500 | 65 | | 2·0 | 10 | 10,520 | 10,290 | –2·2 |

Table 2 shows that for mean river velocities of up to 1 metre per second, (2¼ miles per hour) current meters and sluices give practically identical results. This covers conditions on the Main Nile for three-quarters of the year, and on most of its tributaries under all conditions, and is the period for which sluice discharges are the most accurate and there is least difficulty in measuring with current meters. Moreover, during this period the discharge is kept steady for periods of 10 days or sometimes a month at a time. During the other quarter, the discharge is not usually regulated, and since the discharge site is about 20 miles downstream of the Dam, causing a lag of several hours, fluctuations of the river may cause small

differences owing to lack of exact correspondence between times of observation. During this time of flood the relation between the sluices and the tank is also not so direct [3] and the current meters give results from 2 to 4 per cent greater than the sluices. The mean difference for mean velocities from 1 to 2 metres per second is 3·3 per cent. This difference is too small to be of importance in most cases which arise in practice, and shows that the effects of turbulence on the Price pattern of current meter are negligible for velocities of up to 2 metres per second and that it is a very reliable instrument. This was also the conclusion reached by Mr Wade.[9] It should be noted that most of the current meter discharges depend on velocity observations at half-depth only, and consequently involve the conversion factor 0·96 to reduce half-depth velocity to mean velocity. This factor was obtained from about 500 observations on the Main Nile, where the velocity was measured at a number of points on each vertical including that at half-depth. Observations in America have given the same reduction factor. By more detailed analysis of the measurements than has been made it might be possible to trace the effects of the various causes of the difference.

Mr R. P. Black has designed a current meter of the cup type with a rotor like the Savonius S-rotor used on windmills. This rotor is robust and easily made, and moreover is unaffected by up-and-down motion caused by the rocking of the discharge boat, which may be part cause of the slightly higher results given by the Price meter than by the sluices in flood time. Black's meter under these conditions gave results about half-way between the Price meter and the sluices. Incidentally, small patterns with rotors 30 millimetres and 12 millimetres in diameter have been successfully used on hydraulic models to measure small velocities. Before leaving the Aswan sluice measurements, attention may be drawn to their use in establishing for the first time with precision on a large scale the relation between the discharges of a model and its prototype.[2] The scales of the models were 1 : 67 ; 1 : 50 ; and 1 : 33, and the discharges of the prototype varied from 1 to 100 cumecs, all measured in the tank. Models have since been used extensively in Egypt in the design of barrages, regulators, and other hydraulic works, and in the estimation of discharges passing through them. The accepted discharges through the Sennar Dam on the Blue Nile [10] are the results of model experiments, as are those through the Gebel Aulia Dam.[11]

## Measurements of Discharges and Their Computation

Since the formation of the Sudan Branch of the Egyptian Irrigation Service in 1905, the network of discharge measurements made with current meters has been extended southwards from Wadi Halfa, until now it includes all the tributaries of the Nile of any importance except those in Ethiopia. Some of these measurements have been started in connexion

with possible projects, some in order to be able to regulate the river for irrigation, and others with the object of studying its regime in detail. Many people—British, Egyptian, Sudanese, and latterly East African—have been engaged on the work and between them have built up a body of knowledge about the Nile waters which is not surpassed on any river of the world. One may say that the reconnaissance stage is now past, and many of the difficulties which confronted observers as late as 1930 are now very much lessened. Life in the Upper Nile Basin is much healthier and travel much quicker, provided that it does not involve the penetration of thick bush or swamp.

The computation of the results of the thousands of observations and and their tabulation and presentation in usable form involves a great deal of work, and employed a considerable staff in the Physical Department and now in Nile Control.

Some description of the methods of computation used in the tabulation of the observations will now be given. At most stations of importance for forecasting or regulation, discharges are measured every 4 or 5 days and in a few cases almost every day. In other cases where the river stage only changes slowly, or stations are less accessible and several have to be included in a tour, only one, two, or three observations a month may be made. From these observations, 10-day and monthly mean discharges are computed by methods which depend on the frequency of the measurements.[12]

The most usual method, when a considerable number of discharges have been observed at a site during the year, has been to plot a gauge-discharge curve for the year, distinguishing between rising and falling stages. From the curve the discharges corresponding to 5-day mean gauges are read off and the 10-day means, monthly means, and monthly totals are computed. In some cases, no discharges were observed during a particular year but the river gauges were read regularly; a general gauge-discharge curve is then constructed from all the discharges observed at the site and the discharges corresponding to the 10-day mean gauges read off. There are other cases in which discharges have been observed during the year but they are too few in number to construct a reliable gauge-discharge curve. In this case, a general gauge-discharge curve is constructed and is then modified to pass through the observed points as nearly as possible.

It is not easy to make definite statements about the accuracy of discharge observations, since so much depends upon the site, the observer, and the general conditions. For example, discharges of the Blue or Main Nile in flood are much more difficult to observe than those at low stage, or on the smaller and more placid tributaries. Under the best conditions, discharges have been carried out where the probable error was estimated at less than 2 per cent. A scrutiny of a very large number of discharge observations of all kinds which are susceptible of some outside check

has led to the conclusion that the probable error of a single discharge measurement is about 5 per cent.

Returning to the tabulation of observations, whilst much work is done by computing machinery, it is clear that anything which involves plotting and judgement in drawing curves introduces a great deal of work which can only be done by human labour. Moreover, years of training are required to produce reliable computers.

In the past, much time has been spent in trying to reduce river discharges to formulae, but without any great success as far as the Nile Basin is concerned.[13] If this were possible and permanent formulae could be found more of the work might be done mechanically. So far, in practice, the gauge-discharge curves which are drawn each year for all regular discharge sites are only a means of smoothing out experimental errors and the smaller natural irregularities which occur, and a means of interpolation and summing of the observations. At some stations, fairly regular curves are obtained, but a curve without any irregularities is rare. At most stations, the curves vary slightly from year to year, and also from the rising to the falling river, in the sense that for the same river level the discharge is higher when the river is rising than when it is falling, owing to the slight difference of slope between the rising and falling stages. There are very few sites in the Nile Basin, except where there are cataracts, where the gauge-discharge relation is permanent over long intervals of time. At Roseires on the Blue Nile, where the curves are regular in shape, there are differences from one year to another. For example, in 1928-29 a discharge of 500 cumecs occurred both rising and falling with a gauge of 12·85 metres, whilst in 1932–33 the same discharge on the river rising was at a gauge of 13·1 metres, and on the falling river of 13·25 metres. These effects are caused by changes of river bed and are greatest at low stage, and on those rivers where the low discharge is only a small fraction of the flood discharge and the low river winds about in a wide sandy or gravelly bed.

At gauge sites near the junctions with tributaries, back-water effects cause loops to occur on the gauge-discharge curves, with great variations from year to year. Loops also occur where there is spilling over the banks and a return later assisted by rain on the plains, as is the case on the Sobat.

Attempts were made to fit equations to gauge-discharge curves of smooth and regular shape chosen from the large number of curves which have been drawn. The form of the equation chosen was $Q = cwh^m$, where $Q$ denotes the discharge, $w$ the width, $h$ the mean depth (area/width), and $c$ and $m$ denote constants for the particular site. In the case of the Nile, where the section is usually wide compared with the depth, $h$ is nearly a linear function of the gauge reading and approximates to the hydraulic mean depth. The equation is fitted by taking logarithms, which produces an approximately linear relation between $\log Q$ and $\log h$.

which, however, is a little deceptive since it exaggerates the apparent goodness of fit.

Out of seventeen cases of fairly regular curves (rising or falling), five showed a bad or poor fit, and excluding these $m$ ranged from 1·3 to 3·6 with a mean of 2·0, and $c$ from 0·014 to 0·84 with a mean of 0·3. In the twelve cases of reasonable fit, the extreme deviations of observations from the computed curves averaged 11 per cent in discharge. To represent $Q$ completely an equation is needed which takes account of slope. However, the variation of $S$ is often small and dependent on $h$, so that in these cases $Q$ can be represented as a function of $h$ for the rising curve and as another function for the falling curve.

There is difficulty in measuring the actual slope at the gauge site since slope varies from point to point, and although a fair value may be derived for the mean slope over a long reach containing the site, this may not be the slope at the site itself. The conclusions from the investigation were that for natural streams with irregular beds the relation between discharge and depth can sometimes be represented by a formula of the type $Q = cwh^m$, but the values of $m$ and $c$ vary from site to site, even on the same stream and also from year to year. Where there is a correlation between $h$ and $S$, as is often the case on natural water-courses, a formula of the form $Q = ah^pS^q$ can sometimes be found, but it is not unique as to $p$ and $q$ because of the correlation between $h$ and $S$. General formulae derived from experiments on regular channels such as canals may not represent the facts even approximately on natural channels.

## Other Measurements

River gauges have been mentioned. These consist of metric scales on marble set in masonry, and near the gauge there are always one or more bench marks. In the Sudan, the topsoil is often a grey clay called cotton soil, which shrinks and cracks in the dry season and swells and rises in the rains. Consequently, several screw-piles are sunk into the permanent subsoil and bench marks are set on these, so that it is easily possible with a level to see if there has been vertical motion of the gauge or its component sections. If a movement of more than 5 centimetres is established at the annual inspection, the scale is re-set. First-order levelling extends up the White Nile from Alexandria to Lake Victoria and up the Blue Nile to Roseires. The probable error of this is of the order $\pm$ 0·8 millimetre per kilometre, so that the level of the water surface along the main branches is known with a high degree of precision. It was carried out by officials seconded from the Survey of Egypt, and has been extended in recent years by the Sudan Survey Department. Many surveys have been made for projects, and some of these have added considerably to the available knowledge of remote parts of the Basin. In connexion with schemes for the prevention of water losses in the Sudd Region of the Bahr el Jebel,

the swamp country and its edges as well as the country around Lake Albert were completely mapped by the Air Survey Company, London. The maps on a scale of 1 : 50,000 which were produced added very considerably to the available knowledge of these regions, as well as giving a record of what the country was like between 1930 and 1932.

Other important work has been the measurement of the quantity and quality of the silt carried by the Main Nile in flood time. This is brought down by the Blue Nile and Atbara from the Ethiopian plateau. The measurement was started in connexion with the questions of how early it was possible to begin filling the Aswan reservoir after the crest of the flood and the maximum quantity of suspended silt had passed, and how much deposition was caused when the reservoir was partially filled during high floods as a measure of protection for Egypt. The results of the work have been published,[14] but a few facts may here be given about Nile silt.

From January to nearly the end of July there is very little suspended solid matter in the Nile in Egypt and the concentration is usually less than 100 parts per million by weight. However, the rising flood, which is halfway to its maximum at Wadi Halfa by the end of July, is then carrying something like half its maximum concentration of solids, the maximum being reached towards the end of August before the maximum discharge is reached. From then it falls away until by the middle of November the concentration is unimportant for most practical affairs. In a good flood, the concentration may average 4,000 parts per million for a few days and for the whole flood about 1,500 parts per million. This is much less than that carried by many rivers, for example, the Colorado, Missouri, Indus, and Orange rivers. For the whole flood, the proportions of constituents are :—

| | |
|---|---|
| Coarse sand—none or traces | |
| Fine sand | 30 per cent |
| Silt | 40 ,, |
| Clay | 30 ,, |

Coarse sand consists of particles which do not pass through a sieve of 0·2-millimetre mesh. After this, fine sand is defined as the suspended solid which falls more than 10 centimetres in still water at 20° Centigrade in less than 4 minutes 48 seconds. Following on, silt falls 10 centimetres in less than eight hours, and clay takes longer than this to fall 10 centimetres.

Careful surveys have been carried out on the Aswan reservoir to see if its occasional use to reduce the crest of the flood has diminished its capacity. These surveys have usually been carried out by sounding during the period when the reservoir is full or nearly full and its level changes but slowly. Sections have been fixed at intervals of 1 kilometre and more sparsely towards the upper end of the reservoir. The terminals of these are marked by concrete blocks in which iron pipes are set so that ranging rods with flags can be used during the work. Bench marks are

also fixed near-by and connected with a line of first-order levelling up the Nile. The maximum level of the reservoir, which is indicated by a water mark on the rocks, is also a useful standard to which to relate the actual water surface at the time of observation. The terminals of the sections have been fixed by triangulation, and in fact every precaution has been taken so that any section can be re-measured in the future without any uncertainty as to its actual position. The work can be extended if necessary to find the capacity which would be added by raising the top level of the reservoir. So far there has been no silt deposition of any importance. In addition to the hydrological observations already described one may mention the existence of rainfall and meteorological stations scattered over the Nile Basin, in many areas rather sparsely.

With regard to meteorology, the vast area of the Nile Basin and the lack of funds and population to maintain stations in remote parts have so far prevented the collection of detailed information uniformly over the Basin. This is particularly the case in Ethiopia, the Southern Sudan and the arid parts of East Africa. It is the case, however, and probably always will be, that the rain-water finding its way into the Nile is far better determined from the measured discharges than from any deductions based on rainfall. Nevertheless, rainfall observations have great local importance in connexion with agricultural development, and the same is equally true of meteorology.

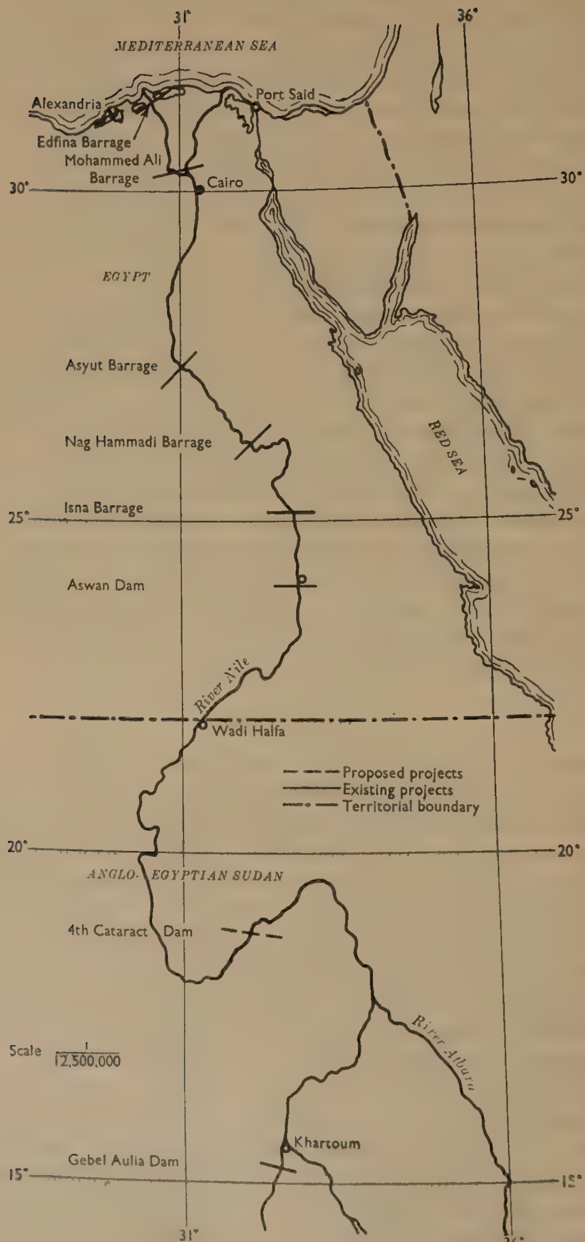
In concluding the remarks on measurements, emphasis should be placed on the importance of long series made under the same conditions owing to the variability of natural phenomena.

## Hydrology

The utilization of the Nile water depends on a knowledge of the hydrology of the river, which has been acquired by the system of measurement of which some features have been described. A very short sketch of the hydrology will now be given, the main points of which are illustrated by the maps in *Figs 2* and *3*, and in *Fig 4*. The principal tributaries are the Blue Nile and the White Nile, which join at Khartoum to form the Main Nile which from there to the sea—1,900 miles as the river flows—only receives water from its other large tributary, the Atbara. The contributions of these are roughly in the proportions, Blue Nile 4, White Nile 2, Atbara 1, where the unit is 12,000,000,000 cubic metres a year. This unit spread over Yorkshire would cover it $2\frac{1}{2}$ feet deep.

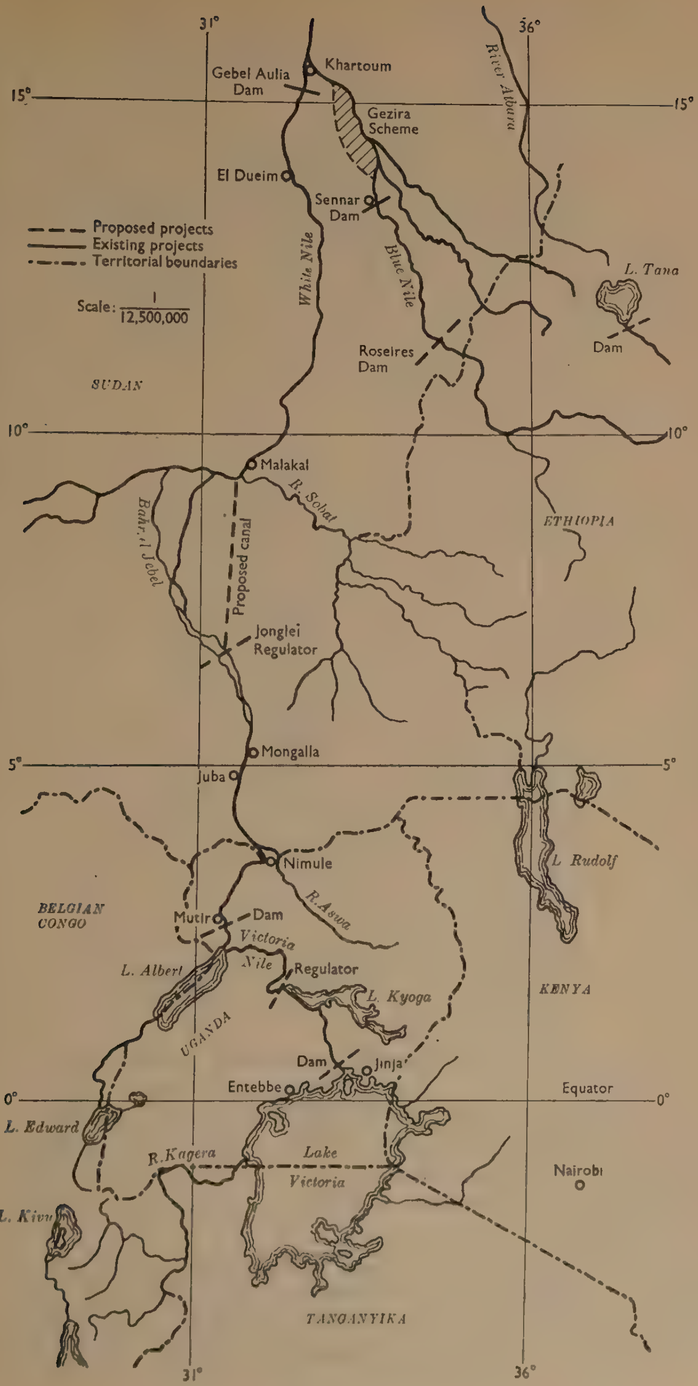
The White Nile is the longer stream and its most distant source is in high country near Lake Tanganyika, 4,160 miles from the sea as the river flows. It receives its supply from rainfall on the Lake Plateau in Central Africa and from the Sobat, a large tributary whose water comes for the most part from Ethiopia. The Blue Nile receives practically all its water from Ethiopia from the many torrential streams which have cut deep valleys in the plateau, and from the Dinder and Rahad which rise on

*Fig. 2*



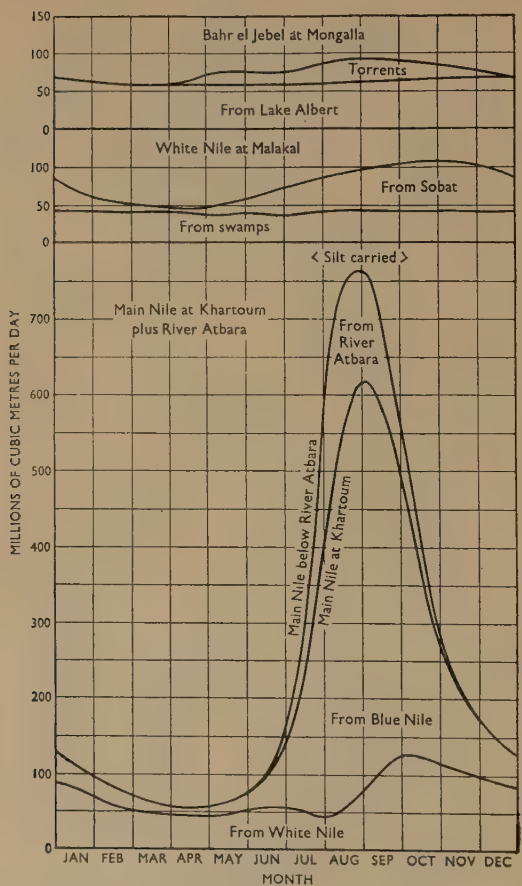

THE NILE BASIN—NORTHERN PART

*Fig. 3*




THE NILE BASIN—SOUTHERN PART

*Fig. 4.*




Average Discharges of the Nile and its Main Tributaries, 1912–1936

the edge of the plateau and join the Blue Nile in the Sudan. Although there is a good rainfall on the plains of the Southern Sudan, owing to their flatness there is little run-off and most of it disappears by evaporation or transpiration by vegetation. The Atbara receives nearly all its supply from Ethiopia.

Notable features in the hydrology of the White Nile are Lakes Victoria, Albert, and the smaller Lake Edward. They equalize the flow in the Bahr el Jebel, as the White Nile below Lake Albert is called, which would otherwise have a big seasonal variation like the other tributaries of the

Nile. Also, the fact that evaporation is very largely compensated by rainfall and their large area make the lakes eminently suitable for storage reservoirs, and as such they have formed a part of every comprehensive scheme for the conservation of Nile water. *Fig 4* shows the average discharge through the year of various tributaries for the 25 years previous to the working of the Gebel Aulia Dam on the White Nile, which exercises some control on the flow of the Main Nile. It begins with the water passing Mongalla on the Sudan Plain 270 miles below Lake Albert. This consists of a major portion from Lake Albert, which is fairly steady through the year, and a portion of about one-sixth of the whole from torrential streams entering between the lake and Mongalla. Most of these are dry for 3 months of the year.

The next point on the diagram is Malakal below the junctions of the Bahr el Ghazal and the Sobat, and below the swamps of the Sudd Region which border the river for 450 miles and cause the loss of about half the water passing Mongalla. The existence of these swamps has been known at least since the time of Nero's centurions, but it was only when the discharges above and below them were measured with current meters during Sir William Garstin's reconnaissances of 1900–1903 that the large losses of water were discovered. Since that date, much study has been devoted to the water losses in the Sudd Region,[15] which will be mentioned again later. Besides the loss of water, *Fig 4* shows the effect of the swamps in producing an almost steady discharge throughout the year. Another feature not shown by a diagram of averages is the increasing loss as the discharge at the head of the swamps increases. Thus, whilst a discharge of 40 million cubic metres per day at Mongalla produces a little more than 30 million cubic metres at the tail of the swamps, 160 million cubic metres per day produces only about 58.

The Sobat delivers about the same amount of water as flows out of the swamps, but has a considerable variation from flood to low stage. The contribution of the Bahr el Ghazal to the White Nile is almost negligible, although its tributaries coming mainly from the Nile-Congo divide drain a vast area. Most of the tributaries terminate in a large swampy area, and on one only, the Jur, is there a continuous waterway to the junction of the Ghazal and Jebel. It is estimated that about the same amount of water is lost in the swamps of the Ghazal as on the Jebel. Their hydrology, however, has not yet been as intensively studied. From the Sobat mouth to its junction with the Blue Nile, the White Nile receives no important contribution, though various Khors (water-courses) may bring some water in years of heavy rain. The lower part of *Fig 4* shows the contributions of the three main tributaries—the Blue Nile and Atbara with heavy discharges in flood which are reduced to a small quantity and nothing at the low time of the year from January to June, and the White Nile with a much more uniform distribution. A comparison between the White Nile at Malakal and the White Nile at Khartoum brings to light

an interesting phenomenon, namely, the effect of the rapid rise of the Blue Nile in holding back the discharge of the White Nile, which begins again to increase when the rate of rise of the Blue Nile slackens. From Khartoum to Aswan, losses are small and there is no tributary except the Atbara, so that the Main Nile below the Atbara is practically the natural supply entering Egypt. It is modified by the Gebel Aulia reservoir and the Aswan reservoir which store water from the flood period, and together add about 7 milliard cubic metres to the low period between February and July, thus increasing the average low-stage supply by about 50 per cent.

An important feature of the Nile discharge is its great range of variation, from 42 milliard cubic metres in the lowest year about which anything is known (1913–14, August to July), to 151 milliards in the highest (1878–79, August to July). The exact figure for 1878–79 is uncertain and the above is based on a gauge-discharge curve, but all records confirm the large and unusual quantity of water which came down. By great economy and good distribution of water, the year 1913–14 was passed without disaster, but cultivation has expanded since then so that the present crop requirements of Egypt and the Sudan are now greater than the whole discharge of the river in that year. In such a year the storage reservoirs, expanded since 1913, could be only partially filled, thus increasing the difficulties of the low stage.

### The Effect of Hydrology on the Utilization of Nile Water

Two factors which have had a dominant effect on projects for the storage of water are the regular annual flood of the river followed by the much smaller discharge of the low stage, and the silt carried when the river is in flood. These determined the design of the Aswan Dam, with its under-sluices constructed to pass the whole flood without any obstruction which would cause harmful deposition of silt, and its programme of filling which begins when the silt in the flood-waters has sufficiently abated. The principle that no risk must be taken of serious reduction of the capacity of the Aswan reservoir by deposition of silt has remained until the present time, though it has been relaxed a little to allow for storage of some water in high floods to reduce the danger of breaching the river banks in Lower Egypt. The site of the Gebel Aulia Dam was chosen because the water of the White Nile was practically free of silt, and the rise of the Blue Nile already created a natural reservoir where water was ponded up and released on the fall of the Blue Nile. The dam enabled the ponded water to be increased and held until it was required to supplement the low river for irrigation.

Before the 1914–1918 war, a project had been drawn up for a dam on the Blue Nile near Sennar, to store water and raise the level of the river to such a height as would enable the irrigation of an area of the Sudan Gezira, the land between the two Niles. This work, including the canalization of 300,000 acres, was finished by 1926, and the area has been gradually

extended to make the fullest use of the water available without infringing the rights of Egypt, and now comprises about 1,000,000 acres. A small increase of storage has recently been agreed upon by Egypt and the Sudan, which will allow some expansion. A project for a dam and reservoir upstream of Roseires to increase further the stored water is under consideration.

The Gezira Scheme is a remarkable example of co-operation on a large scale between a government, foreign concession companies, and native African tenants. The concessions have now lapsed and only government and tenants are concerned. It is also remarkable because of the very accurate measuring and accounting for the water in gross and in detail. A full account of the scheme is given in "Agriculture in the Sudan,"[16] in the chapter on irrigation by W. N. Allan and R. J. Smith, from which the following is taken.

The manner of partition of the Blue Nile water between Egypt and the Sudan is specified in the Nile Waters Agreement of 1929 with its attached Working Arrangements. These state that the Sudan may fill the Sennar reservoir between the 15th July and the 30th November in a certain manner, and that the natural flow of the river and its tributaries is reserved for Egypt from the 19th January to the 15th July (Sennar dates). This last provision is controlled by a water account, which starts on the 1st January with a credit which is the amount allowed from the 1st to 18th January, plus the gross content of the Sennar Reservoir. Against this is debited water taken into the Gezira canal for irrigation (January to April) and for drinking water when the irrigation season is finished (May to July), evaporation losses from the reservoir, and a small amount of water to compensate for pump irrigation to land in excess of an agreed area. The fundamental basis and the keeping of this account depend on water measurement by means of the sluices of the Sennar Dam and the Gezira Canal Head Regulator, as already mentioned on p. 10. The same careful measuring is continued on the main canal and its branches, through the major down to the minor distributary canals which feed the field water-courses, each of which waters nine plots of 10 acres each, and daily discharges are recorded at all control points. Cropping is carried out on a uniform system throughout, which leads to economy of water and avoidance of over-cropping. The Gezira Scheme is probably unique in its uniformity, detailed measurement, and control of water, and its device for avoiding watering by night.

Sir William Garstin[17] had suggested the reduction of losses in the Sudd Region and the possible use of the Great Lakes as reservoirs, where he thought the annual rise and fall would provide more than enough water for the needs of Egypt and the Sudan. The very low year 1913–14 and another low one in 1915–16 led Sir Murdoch MacDonald to propose the use of Lakes Albert and Victoria to hold a reserve above the needs of annual regulation[18] big enough to meet a succession like 1913–16.

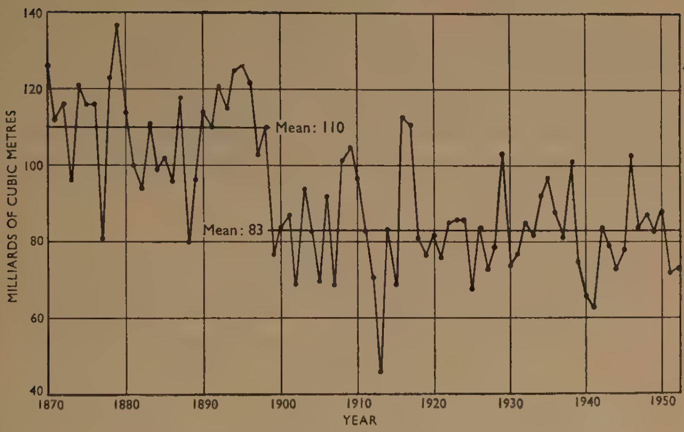
During the period between the wars the Sudd Region and the Great Lakes were studied and various detailed proposals for dealing with them were put forward.[19] After the 1939–45 war the Minister of Public Works, H. E. Abdel Qawi Ahmed, asked the Author's department to review the whole question of projects in the light of all the information collected, and suggested that a connected account of present ideas should be written, showing the interdependence of Nile projects. The result of this examination was published in 1946 as volume VII of "The Nile Basin,"[20] and contained a scheme of projects for the extended utilization of Nile water.[21] After discussion by a committee composed of three former Ministers of Public Works, the scheme was accepted as the policy of the Egyptian Government.

This scheme contained a selection of projects from among the many which had been proposed at one time or another. The chief components were a main over-year storage reservoir in Lake Victoria and a smaller one in Albert to act also as a balancing reservoir; the Jonglei Diversion Canal to enable loss in the swamps of the Bahr el Jebel to be reduced to a reasonable amount; a reservoir in Lake Tana on the Blue Nile and another one on the main Nile to be used for flood protection, and for annual storage and regulation of the discharges coming down from the Upper Nile reservoirs. These are interdependent and include, as a combining principle, over-year storage on a large scale. Some previous projects for reservoirs in the Equatorial Lakes had contemplated storage over periods of years, but until the theory was worked out it was not known how over-year storage would work or what it could do in the way of guaranteeing a minimum discharge.[22] This theory is of general importance wherever there are storage problems, and a short account of it will be given here. Since the scheme of southern projects was accepted by the Egyptian Government proposals have been made for a high dam to impound a very large quantity of water and produce a great deal of power at a site a few miles south of Aswan. This is a considerable departure from previous ideas which would have far-reaching effects and is at present being studied.

*The Theory of Over-Year Storage*

The problem at its simplest can be expressed as follows: given a variable source of supply, such as a stream on which a reservoir is constructed, what is the relation between the capacity of the reservoir and the minimum annual discharge which it can guarantee over a long period? In the solution of this problem, annual total discharges are used and variations within the year are left for consideration later; it can be illustrated from the Main Nile discharge at Aswan, which is shown in *Fig. 5*, where annual totals are plotted. Given these totals, it can easily be computed what amount of storage would have been required (*a*) to enable year by year a constant discharge equal to the average to have been given (excess losses by evaporation from the reservoir being ignored); or (*b*) to guarantee

*Fig. 5.*




ASWAN DISCHARGE. ANNUAL TOTALS

that the discharge would not have fallen below a stated quantity less than the mean of the period. Both of these can be found by the following general method. The annual total discharges are written in order and their mean is obtained. A convenient number near the mean is chosen as a base to lessen the amount of computation, and the departures of the discharges from the base are written down. In the next column the squares of the departures are written and in the final column the continued sums of the departures. The portion of the calculation shown in Table 3 illustrates the work.

TABLE 3.—ASWAN DISCHARGE—CONTINUED SUMS OF DEPARTURES

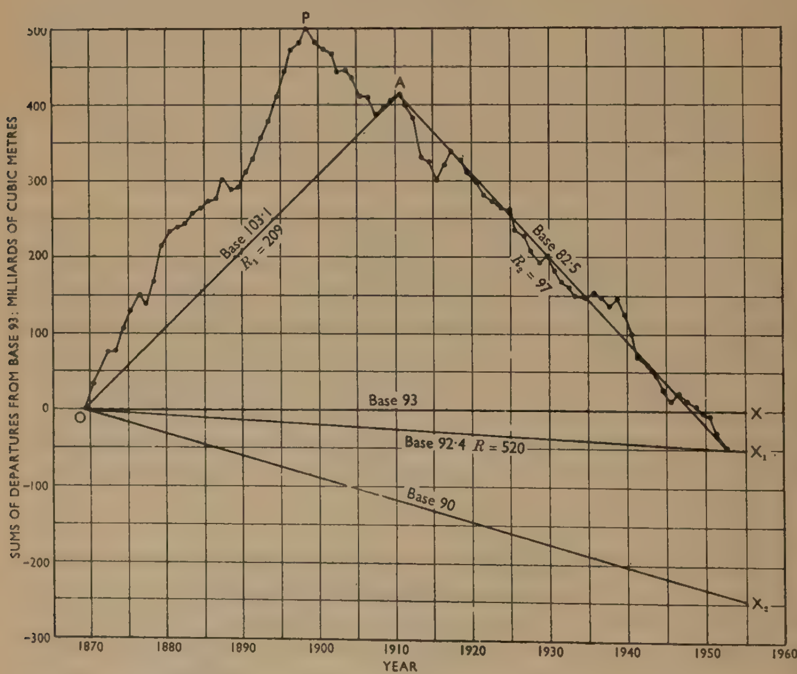
Quantities are in milliards of cubic metres

| Year | Discharge | Departure from base 93 | | Square of departure | Continued sums |
|---|---|---|---|---|---|
| | | + | − | | 0 |
| 1870 | 126 | 33 | | 1,090 | 33 |
| 1871 | 112 | 19 | | 360 | 52 |
| 1872 | 116 | 23 | | 530 | 75 |
| 1950 | 88 | | 5 | 20 | −7 |
| 1951 | 72 | | 21 | 440 | −28 |
| 1952 | 73 | | 20 | 400 | −48 |
| Sums | 7,671 | 624 | 672 | 27,940 | −48 |
| $N = 83$ | | | 48 | | |
| Mean | 92·4 | | 0·6 | 337 | −0·6 |
| | | | | $\sigma = 18{\cdot}4$ | |

It will be seen that except for the squares the computation is self-checking. It is important in order to lessen labour not to use figures beyond what are significant. The computation gives the mean, standard deviation, and the continued sums, all of which are required. The continued sums are plotted as in *Fig. 6*, which gives the answers to (*a*) and (*b*). Diagrams like this are the basis of research on the problem.

*Fig. 5* shows how considerable is the variation of the annual discharge, a fact that has been previously mentioned. It also shows that the mean

*Fig. 6.*




Aswan Discharge. Annual Totals. Continued Sums of Departures

discharge from 1870 to 1898 was 110 milliards, whilst from 1899 to 1952 it was only 83 milliards, the mean for the whole period being 92·4. The curve of continued sums of departures from base 93 (*Fig. 6*) shows that if a discharge of 93 had been given continuously and excesses had been stored, the amount stored would have been 503 milliards by the end of 1898 and for most of the time from then onward the stored water would have been decreasing until by 1949 it would have been exhausted, and by the end of 1952 there would be a deficit of 48 milliards on the amount required to maintain 93 milliards each year. It is obvious that continued

sums of departures from the average 92·4 are obtained by taking the axis of the curve as $OX_1$ instead of $OX$. Consequently, the storage which would have maintained the average over the whole period is 520, the total range of the curve from the axis $OX_1$. This range shall be called $R$. Now if it is wished to find the storage which would just have maintained a discharge of 90, the axis $OX_2$ is drawn below the base at a slope of 3 milliards per annum. In this case the curve of continued sums with relation to a base of 90 rises to 588 and falls to 193 by the end of 1952. If the reservoir were full by the end of 1898 (peak P of the curve), a draft of 90 would produce a maximum deficit of 395 during the remainder of the period. The maximum deficit ($S$) is the capacity required to guarantee a mimimum draft of 90. It is to be noted that the maximum deficit is not the range of the curve when the base (or draft) is less than the mean. It is clear from this that the curve of continued sums of departures is a very useful tool in connexion with storage problems, but it is not in general use, though the clumsier mass curve of discharges is well known. Other interesting results can be obtained from the Figure. For example, if the period is divided into one of 41 years and a second of 42, then the mean discharges for these periods are given by the slope of $OA$ and $AX_1$, that is, $93 + \frac{416}{41}$ and $93 - \frac{464}{42}$ and the capacities $R_1$ and $R_2$ required to guarantee these discharges are 209 and 97. Not only, therefore, is there a considerable variation between the means for the whole period and the two halves, but there are also considerable variations of $R$. These calculations are possible when dealing with the past and all the information is available, but what about the future ?

It is clear that an idea of the possible variations of means and ranges in the future can be formed only by analysing a number of past records of river discharges covering as long terms as possible.

Unfortunately, not many such records exist, so rainfall statistics, which are more numerous and cover longer periods, have been included. Following these, the work was extended to temperatures and pressures, and to much longer records of the thickness of tree rings and the thickness of varves. Seventy-five different phenomena were used and 729 values of $R$ were computed for periods of lengths ranging from 30 to 2,000 years.

It was found that the average relation between the variables could be represented by the equation :

$$\log \frac{R}{\sigma} = K \log \frac{N}{2}$$

where $\sigma$ denotes the standard deviation of the phenomenon ; $N$ denotes the number of years in the period considered ; and $K$ denotes a coefficient which has a mean value of about 0·72, with a standard deviation of 0·09. $K$ does not vary much from one group of phenomena to another : for

example, for river statistics it had a mean value of 0·75, for rainfall 0·70, and for temperature and pressure 0·70. For the less precise phenonema, for which long periods are available, tree rings (up to 900 years) gave 0·80; varves at Lake Saki (up to 4,000 years) gave 0·69, and in Canada and Norway (up to 1,200 years), 0·77.

The equation is a statistical one and due consideration has to be given to the variations of $R$ and $K$ in its application. Using the mean value of $K$, the average obtained is:

$$\frac{R}{\sigma} = 0{\cdot}61 N^{0{\cdot}72}$$

Example:—If $N = 100$ years, $R = 16{\cdot}7\sigma$.

If the discharge from a reservoir over a period is kept steady at the mean, the range of the curve of continued sums of departures from the mean is taken as the storage required to maintain the constant discharge, and the final sum of the departures is zero. If the departures are taken from a base less than the mean, this corresponds to the case where the discharge out of the reservoir is not allowed to fall below the base, and the maximum accumulated deficit ($S$) is the storage required to prevent this. The results of a number of computations of $S$ for actual cases concerning rivers, rainfall, and temperatures which have occurred gave a relation between $S$ and $B$ (the draft to be guaranteed) which can be represented by either of the following equations, both of which fit the observations equally well within their limits:

$$\log \frac{S}{R} = -0{\cdot}08 - 1{\cdot}00\left(\frac{M - \mathrm{B}}{\sigma}\right)$$

$$\frac{S}{R} = 0{\cdot}97 - 0{\cdot}95\sqrt{\frac{(M - B)}{\sigma}}$$

where $M$ denotes the mean of the observations.

The feature of this relation is that a small reduction of the draft below the mean makes a much greater proportional reduction in the storage. For example, if $M - B = 0{\cdot}1\sigma$, then $S = 0{\cdot}65\,R$.

One fact which is clearly shown by the investigation is the tendency with natural phenomena for high or low values to occur grouped together more often than would be the case with random events. This leads to greater variability of means and standard deviations as exemplified by the Aswan discharge, and the possibility of such variations must be taken into account in applying the previous equations to calculate storage. A discussion on this will be published later, including means of using the storage.

The above is a very bare account of the investigations, and the reader is advised to consult reference [22] for a full account of the theory. It has been applied to Lakes Victoria and Albert as long-term storage reservoirs.

## Acknowledgements

The work of which an outline has been given covers fifty years and is the result of the labours of very many people—British, Egyptian, Sudanese, and other nationalities. A great deal has been done by the Egyptian Irrigation Service in the Sudan, who have always had the co-operation of the Sudan Government and its Irrigation Department, and of the Governments of East Africa in carrying out work on the Upper Nile. In recent years, the Uganda Hydrological Survey has taken over the collection of information on the Lake Plateau. A large amount of experimental work, and analysis and publication of Nile data has been done by the Physical Department and is continued by the Nile Control Inspectorate-General of the Irrigation Service.

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The Paper is accompanied by one photograph and five sheets of diagrams from which the Figures in the text have been prepared.

## Discussion

**Mr B. D. Richards** observed that the importance of hydrological records, taken over a long period to form the essential basis of schemes for the control and utilization of a river, had evidently been appreciated at an early stage by those responsible for the Nile Basin, with the result that a remarkable collection of data had now been built up. In contrast, the information on some of the other large rivers of Africa was somewhat meagre. In East and Central Africa the importance of hydrological records had been recognized only in the past few years and, whilst there were now Water Development Departments in all the territories concerned, it must from the nature of the work take many years to make up the leeway.

The proper utilization of the water supplies in Africa was of paramount importance, and the need for hydrological records could not be overemphasized. They were of the same order of importance as topographical maps and geological surveys; but, whilst the two last-named could, with the use of modern appliances and methods, be speeded up, the establishment of hydrological records by their very nature must be a very slow and

costly process. The expenditure required, whilst very great, did not bring an immediate return, but on a long-term view it was both essential and remunerative.

It appeared that the standard practice in making river discharge measurements on the Nile was to base the rating curve on current-meter observations taken at mid-depth on a series of verticals across the river. He had no doubt that that method had been determined from long experience as being the most suitable for the Nile, but he would suggest that in a deep and fast-flowing river it might be that a method of taking the readings at a lesser depth would be easier and would not detract very much from the accuracy. Was the echo sounder now used?

Mr Richards was in full agreement with the Author's statement on p. 13 that the establishment of the yield of a catchment by discharge measurements was generally preferable to computation from the rainfall. In East Africa the coefficient of annual run-off was only from 7 to 8 per cent, so that an error of only 1 per cent in assessing that coefficient would mean an error of more than 12 per cent in the annual discharge. In certain cases, however, of small steep impermeable catchments, with a consequent very high coefficient of run-off, he thought that the rainfall method might be as good as or even better than the discharge method, because the discharge method in such a case, with rivers being very "flashy," would involve a continuous record by an automatic recorder of the river levels.

Mr Richards did not quite appreciate the advantages of the Author's over-year storage method over the mass curve. Table 3 showed the calculation of storage required for regulation of the annual variations, but in many cases where the storage was limited the seasonal variations would be of equal or possibly of more importance than the annual variations. A mass curve based on monthly flows lent itself very readily to the calculation of the storage required for any degree of regulation, and from it a storage-to-utilization curve could be prepared. The critical years in the series would be at once evident from the mass curve, and those years could be re-plotted to a larger scale.

On p. 10 the Author had pointed out the necessity of taking rating measurements both on the rising and on the falling river, and had noted that on most of the Nile stations a given height on the gauge connoted a higher discharge on the rising than on the falling flood. Mr Richards had recently had occasion to examine the discharge measurements on a large river in Central Africa. There existed a record of the river level for nearly 50 years, but the rating diagrams had been made only in the past 3 or 4 years, and at a station many miles downstream of the gauge. At the same time a second gauge had been put in at the lower station. He found that the discharge measurements, plotted against the levels of the lower gauge, showed very little difference between the rising and the falling river, but to make use of the 50-year record it had been necessary to try to establish a correlation between the upper and lower gauges. Plotting the discharge

measurements against synchronized readings of the upper gauge, there had been found to be a very considerable difference between the rising and falling river. A given level on the upper gauge gave a considerably lower discharge on the rising than on the falling river, and that difference between the two tended to increase as the peak flood increased.

He thought that the explanation was that a short way below the upper gauge the river narrowed, and the effect of that constriction was that on a rapidly-rising flood the river became gorged, and the levels on the upper gauge rose more quickly than those on the lower gauge. On the falling flood the reverse process happened. The floods on that river were very steady and entirely seasonal—single-peak floods as a rule—and it might be said that a flood which was going to have a high peak was a rapidly-rising flood, and one which had a low peak was more slowly rising, which would account for the reduced width of the loop formed by the curves of rising and falling flood, as the peak decreased.

**Mr W. N. Allan**, who had been concerned with Nile waters on behalf of the Sudan for many years, paid a warm tribute to the value of the work which had been done in collecting and observing the data and in analysing and publishing the results which the Author had described. A notable feature of those results, he observed, was the series of volumes known as "The Nile Basin." Without them, studies for future projects would not be possible, and Mr Allan made a plea for a continuance of that work in the future in as efficient a way as it had been done in the past, and for its extension in those parts of the Nile Basin where as yet information was somewhat deficient, notably in Ethiopia on the upper part of the Blue Nile and on the Atbara and Sobat.

He said it was clear that future development on the Nile would call for over-year storage, and one of the more important practical implications of that was the question of evaporation losses. At a site such as Aswan (see *Fig. 6*) the evaporation rate was heavy. *Fig. 6* indicated that during the years 1870 to 1952 the total storage necessary to maintain the average flow would be about 520 milliards of cubic metres, but in actual fact evaporation would occur at the surface of any reservoir as it filled up and the consequent decrease in the amount of water stored would accordingly reduce the range required. The important figure was the net effective mean flow which could be maintained each year after the evaporation had been allowed for, and it seemed to Mr Allan that the curve of accumulated departures from the mean, or the mass curve, whichever was used for any site, would have to be adapted to take account of that if possible. Where evaporation, as at Aswan, occurred on a considerable scale, it might be possible to plot a curve of the net mean available flow against the storage, and that, he thought, would rise to a maximum point, indicating the amount of storage which would be hydraulically the most efficient.

In the case of the Equatorial Lakes, evaporation was of much less importance. Over the year, as the Author had shown, evaporation was, in

general, made good by rainfall, so that it was possible to discuss the flows sufficiently nearly without allowing for the evaporation. That, in one way, made it very much easier to consider the problems of over-year storage in that part of the basin.

There were, however, other factors peculiar to that region, which affected the control and operation of over-year storage. For instance, storage in the Equatorial Lakes, Victoria and Albert, and the project for the Jonglei diversion canal would all have to be operated as a single scheme, and the primary object would be to maintain at Mongalla an annual flow as near the mean as was reasonably possible. But on the stretch of the river from Mongalla down to a point about mid-way between Malakal and El Dueim the local population were pastoral, and they depended for their living on the present annual flow and reflux of the river. In the flood season they moved back from the river with their animals, and in the dry season they moved down to the regions uncovered by the falling flood and were able to find fresh green grazing, which was their only dependable source of grazing during that period.

When the project was developed, the conditions in that region would change in two main respects. First of all, the annual spill or flood would be very greatly diminished. The whole object of the Jonglei diversion canal was to reduce losses in the swamps. Secondly, for a great many years at a time the flow would be stabilized and would continue year after year on a very similar scale ; there would not be the variations between individual years which took place at the present time. In those circumstances, that population, whose livelihood must be maintained, would have to be re-settled in the regions which were near the river, where they could still get at water in the dry season, and where grazing would have to be made available for them. When that was done, it would no longer be possible to allow them to be exposed to the danger of a high flood, as they were at present.

The effect of that would be that the maximum discharge which could be safely passed at Mongalla was limited ; it was as if the spillway capacity of the Equatorial reservoirs had to be limited. That factor must be taken into account in considering the range of storage which would be required and the method of operation, because over-year operation was obviously going to be rather different from annual operation. It was not possible merely to let the reservoirs fill up and then spill, because that would cause the floods which it was desired to avoid. The whole operation of the system of reservoirs had to be thought out so that the tendency towards excessive discharges was avoided, and the flows which had to be passed, so far as information went, were limited to those which were safe.

What those figures would be it was not yet possible to say. As the Author had pointed out, the averages of the past were known, but no one knew what the future conditions would be, nor whether the incidence of high and low years would be similar to those of which records were

available. That provided all the greater reason for care and for flexibility in the working arrangements.

**Mr R. F. Wileman** referred to the method of discharge-taking which involved the measurement at half-depth only in several verticals, and asked whether there had been time and opportunity in the past few years for testing the validity of the conversion factor of 0·96 in channels other than the Main Nile. He believed that there were some channels in the Upper Nile region which showed very different cross-sections from the Main Nile. During Mr Wileman's period of service in the Nile Basin, only the half-depth method had been used, but perhaps it had been possible to do more detailed discharge observations since he had left, many years ago.

He had not himself been able to study the latest American findings in that connexion, but the Author might have had more opportunity to do so. It would be interesting if the Author could give his opinion, therefore, on the limitations of the safe use of that coefficient of 0·96 in channels of various cross-sections. Mr Wileman felt that if it were possible in the United Kingdom, where there was a shortage of money which would probably continue, to simplify methods safely, it would help towards carrying out gauging as economically as possible.

**Professor Herbert Addison** suggested that the importance of the work described in the Paper could be gauged, if there were any doubt about it, by comparing the present Paper with Hydraulics Paper No. 1.[23] That Paper had dealt with a very small river which had caused a great deal of damage, while the present Paper dealt with a very large river which was so well regulated that it never seemed to get out of control. In his own experience, during 30 years in Cairo, never had the river Nile flood caused loss of life or serious damage, at least in Cairo and Lower Egypt, so that he had the best of reasons for appreciating the work which had been done.

One aspect of the Paper which might be worthy of further study was that of the storage of flood-water. In the scheme of over-year storage described by the Author, presumably no silt-laden water needed to be stored ; but other schemes which were just mentioned, such as, for instance, a project involving a very much heightened Aswan Dam and a very much enlarged Aswan reservoir, would require, presumably, the impounding of silt-laden water. What would happen in that case ?

The evidence so far about the deposition of silt in the Aswan reservoir was based on only a few occasions when the reservoir had been partly filled. Only once, Professor Addison thought, had the reservoir been completely filled with flood-water, and that had been in 1946, as an emergency measure during the exceptional flood of that year. He believed that the water had been quickly released and he did not think that it stayed in the reservoir for more than a few weeks before the normal filling began. In America

[23] C. H. Dobbie and P. O. Wolf, "The Lynmouth Flood of August 1952." *Proc. Instn Civ. Engrs*, Part III, vol. 2, p. 522 (Dec. 1953).

the experience of storing silty water was rather ominous. It was believed there that some of the largest reservoirs might be out of use within the space of 200 years and would be silted up completely. It was true, as the Author had pointed out, that those rivers might have a much higher silt content than the Nile, which perhaps were reassuring.

In order to examine completely, however, the question of storage of silt-laden water, it might be advantageous to go to the basis of silt suspension, about which much more was known now than formerly. In a normal flood river, without artificial obstructions, the silt was kept in suspension by the turbulence of the flowing water. When the water was ponded up in a reservoir by a dam, the energy was not lost but part of it was available ; it could, if necessary, be dissipated by extreme turbulence downstream of the dam when the water was released through sluices. On the other hand, the water could be released through a hydro-electric plant and could then provide useful energy more or less at the end of a wire.

The question arose, therefore, whether it would be possible to feed back this electrical energy into the reservoir in order to keep the silt in suspension by some sort of stirring process. It might be profitable to find out whether any artificial means of creating turbulence would suffice to keep even a small amount of the silt in suspension which would otherwise help to choke the reservoir. If that were possible, another question arose : what would happen when the water was released from the reservoir ? It would be silt-laden water which, during the low stage in February, March, and April would be flowing down the river with a much smaller velocity than the normal flood velocity of the river. Was it not likely that silt would then be deposited in the river bed, and especially in the ponds upstream of the barrages such as the Asyut Barrage ?

Those, then, were the two points upon which it would be very interesting to hear the Author's view. First of all, did he think that there was any possibility of the artificial sustenance of silt in reservoirs, and secondly, did he think that it would be possible to pass silty water down the Nile at the low stage without excessive deposition in rivers and canals ?

**Brigadier C. G. Hawes** made further reference to the question of measuring velocities at 0·5 of the depth. He had had a long experience in India on the Indus, where discharges had been measured at 0·6 of the depth with no coefficient for modification of the velocity measured. The decision to use that depth had been based on more than 1,000 experiments which had been made by taking the velocities at one-tenth of the depth all the way down, plotting them, and finding the point where the mean velocity occurred. Those experiments had been carried out in canals, large and small, and in the Indus, and the conclusion had been drawn that the mean velocity occurred at 0·6 depth, as near as it could be ascertained for practical purposes. Had that system ever been considered in Egypt ?

Brigadier Hawes had started the Hydrological Survey in Uganda in 1947. Previous to that, the local administration had not bothered about

measuring any discharges of their rivers. They had been asked by Egypt to undertake certain measurements on the Nile and the Kagera River at the instance of the Author, in 1938, but the Public Works Department had not been interested and would not produce a man to do the work, and so Egypt had made its own measurements. The Uganda Hydrological Survey had taken over that work, which was now being done, as it should be, by the Colonial Hydrological Survey.

The metric system was used for all gauge and discharge records and in the standard Egyptian practice for discharge gaugings, velocity measurements were taken at 0·5 depth.

Good meteorological records were available for East Africa.

The main preoccupation in hydrological work in Uganda, in addition to finding out where the water came from in the territory and what happened to it, was to collect data about Lake Victoria. A certain amount of work had already been done. The Meteorological Service in Uganda had made one small investigation on the correlation of rainfall in the area of Uganda in the Lake Victoria catchment itself and the movement of the lake surface, and had obtained an equation relating the rainfall of one month with the change in lake level between the mean level of that month and the mean level of the succeeding month. The correlation coefficient was 0·68 and, although significant,was considered to be too low for the regression equation to be of value.

That kind of investigation would go on since, in East Africa, it was necessary to arrive at some way of forecasting movements of the Lake surface from rainfall data in the Kagera catchment, in Ruanda Urundi— and in the Congo, and for stations on the Lake shore. Such forecasting would simplify the operation of the working arrangements for the release of water from Lake Victoria, particularly at high and low lake levels.

The working arrangements had been drawn up only in draft and had still to be discussed with Egypt and the Sudan. Provision was made, however, for maximum release to start when the lake reached a certain point, the object being to stop the lake running up over the new maximum level. Similarly, at the low levels some provision would have to be made to limit Egypt's right to take water below a certain figure, which was the requirement for the power station at the Owen Falls, when the lake reached a low gauge, in order to prevent the lake level dropping below the recorded minimum and so creating difficulty for shipping at one or two of the ports round the lake. The work being done would be continued, so that in 15 to 20 years' time, when the Nile projects would have to be operated, the authorities should be in a much better position to know what would happen and what could be done to exercise control.

The Author had mentioned that a rating tank was being erected in Uganda. That tank was now in operation and was being used by the Central and East African territories for rating their meters. It was

provided with a trolley with speed control giving fixed speeds, and automatic recording of meter rotation, time, and so on.

The Uganda Hydrological Survey authorities were very interested in the rainfall on Lake Victoria itself. They felt that the evaporation from the lake was as a rule balanced by the rainfall which fell direct on to the lake surface, but they had no information about the rainfall at the centre of the lake itself. It was hoped that it would soon be practicable to install an automatic recording rain gauge on Godzeba Island, which was well out from the Southern shore. That would help to determine the true position of the 50-inch isohyet. Automatic recording gauges were being installed at 5 or 6 points round the Lake shore. The records from those gauges would all have a common time base and would make it possible to study whether there were any movements on the Lake in the nature of seiches and so on.

The Author had stated on p. 10 that the probable error of a single discharge measurement was about 5 per cent. Did that mean that discharge measurements varied from +5 to —5 per cent of the true value ?

In Sind it was considered that the accuracy of discharge measurements was from +2 to —2 per cent of the true value.

**Mr R. L. Fitt** joined with Mr Richards and Mr Allan in emphasizing the tremendous importance of meteorological observations and river discharges. He added that any funds made available for the development of underdeveloped territories should first be devoted to the accumulation of data in respect of climate, rainfall, and run-off.

In the case of the upper reaches of the White Nile there were other difficulties, apart from lack of funds and lack of trained observers, in the establishment of some of that information. Brigadier Hawes had already referred to the rainfall over Lake Victoria. The size of that problem could be visualized when it was realized that Lake Victoria was the size of Ireland and that it represented one-quarter of the total catchment area above the Ripon Falls. The conclusion that evaporation over Lake Victoria was largely compensated by rainfall had been reached, Mr Fitt believed, by comparing the volume of water flowing into the lake with the measured discharges at the Ripon Falls. Since those quantities were comparable, the estimation of rainfall and evaporation over the area of the lake had not, in the past, been of great consequence, because if the water flowing into the lake and that flowing out of it roughly balanced, it did not matter very much what the rainfall and evaporation figures were. On the other hand, evaporation losses played a very important part, in the development of water resources for irrigation, for domestic requirements, and for hydroelectric power ; in countries where little or no statistical information was available on those matters, it was a question of how such losses could possibly be estimated.

He referred to the extremely valuable work recently carried out by Dr Olivier. It was to be hoped that the results of that work would shortly

become available. The significance of the evaporation and transpiration losses in Africa was shown by the fact that, to use the Author's figures, only 7·6 per cent of the rainfall falling on the upper catchment of the White Nile passed down the river at the outlet from Lake Victoria.

On p. 18 the Author had referred to the 1913–14 discharge of the Nile of 42 milliard cubic metres, and had stated that that would have been inadequate to meet the demands for present-day irrigation in Egypt and the Sudan. As would be seen from *Fig. 5*, the Author had selected a year in which the discharge had been about 50 per cent lower than in the next lowest year over the whole period from 1870 to the present day. It would be interesting to have the Author's opinion on whether such abnormal conditions should be taken as a controlling factor in the development of the enormous agricultural potential of the Nile Valley, because if so there must be a very low limit set to such development.

In the next lowest year, 1941–42, there had been an additional 20 milliard cubic metres of water available in the rivers concerned, the White Nile and the Blue Nile combined and the Atbara, and, given adequate reservoir facilities to store the water in the flood river, when the flow of the river was in excess of the day-to-day irrigation demands of Egypt and the Sudan, it would be possible to irrigate further a very large area; moreover by the establishment of grain reserves and some form of equalization fund, the cultivators would not suffer disaster if in one year out of hundreds the crops could not be brought to maturity owing to lack of water in the critical growing period. Even in 1913–14 there had been an excess of river flow over present-day irrigation requirements of about 7·6 milliard cubic metres during the peak of the flood, and that would have gone a long way towards filling the three reservoirs at Aswan, Gebel Aulia, and Sennar.

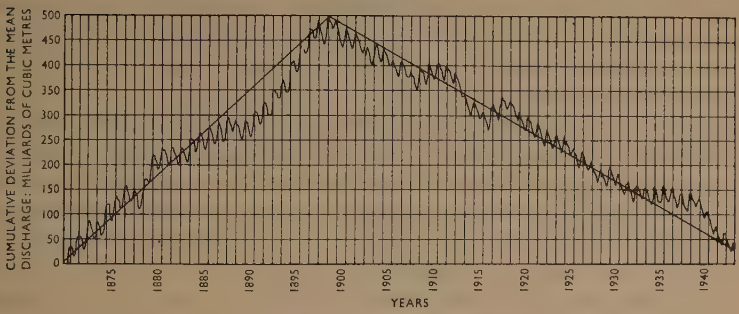
In *Fig. 5*, there seemed to be a strange jump in the Aswan discharge curve in 1898–99, and that was reflected in the deviation from the mean curve which was shown in *Fig. 6*. The first impression received from those two curves was that there might have been something wrong with the discharge calculations prior to 1899, but the Author had emphasized the tendency of natural phenomena to group themselves, and Mr Fitt had therefore examined discharge curves for the Thames, for which records were available over a long period of years. In *Figs 7* the Thames discharges were plotted in a similar manner to *Fig. 6*, and it would be seen that there had been the same tendency, but in the reverse direction, in the case of the Thames, as the Author had shown in the case of the Nile. Although *Figs 5* and *6* looked a little peculiar it would appear that there was a somewhat similar phenomenon in a river in Britain, where variations of discharge were not nearly so wide.

The Author and his colleagues in the past had always made their calculations of Nile discharges in calendar years. Mr Fitt wondered whether the Author thought that it would be more appropriate, when considering river discharges in relation to irrigation development, to base the statistics

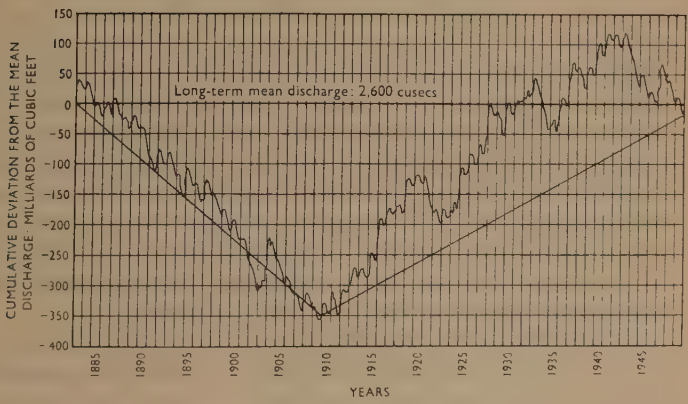
on the agricultural year whenever that was possible. It would vary, of course, according to the river being studied.

Professor Addison had mentioned the possibility of mechanically maintained turbulence in reservoirs to prevent siltation. Mr Fitt felt that that would be a very expensive proposition, and probably the answer would be

*Figs 7*




MASS CURVE FOR THE NILE AT ASWAN




MASS CURVE FOR THE THAMES

to devote the same sum of money to intensive soil conservation and afforestation in the upper catchment areas of the rivers which produced most of the silt.

**Dr W. L. Lowe-Brown** observed that the very exceptional Paper which had been presented summarized the most important part of the Author's life-work, connecting it with a whole library of other volumes bearing his name and those of other members of his organization. Without question, the knowledge available today of the Nile Basin was attributable entirely to him.

The Paper carried Dr Lowe-Brown back more than 50 years to a time towards the end of the nineteenth century when, as a young assistant engineer, he had been called upon to accompany one of his seniors in measuring the discharge of the Nile a few miles downstream of Aswan. Their equipment had consisted of a boat, a sounding lead, a few empty bottles to use as floats, a sextant, a level, and a pocket watch. Their first day's work had been entirely thrown away because the peg which they had established as a bench-mark disappeared in the night. The following morning the village headman had triumphantly brought it to them, saying "Here is your peg. I knew it belonged to the Government, and I did not want it lost, and so I took it to my house to prevent it being stolen." (The Author had used screw piles.)

Subsequently, Dr Lowe-Brown himself had levelled, by ordinary levelling methods, all the way from Aswan to Wadi Halfa and back, fixing bench-marks on the rock from which the surface level, reservoir full, could be measured. Dr Lowe-Brown was glad to say that when those levels were used, although the slope was only about 1 in 1,000,000, all the slopes of the surface were found to be positive!

To him, the most important sentence in the Paper related to the Aswan reservoir and stated that "So far there has been no silt deposition of any importance." That was confirmation, after 50 years of use, that the original design of the Aswan Dam with 180 under-sluices had been absolutely sound.

**Mr G. R. Hoffman** referred to the unsuccessful attempts which had been made to fit equations to the gauge-discharge curves on the Nile. Such equations were often the most reliable way of deciding the height to which extreme floods would rise, and they could be extrapolated beyond observed ranges of level with more confidence than the simple extension of the gauge-discharge curve itself; the resulting estimated levels were, however, only approximate, mainly because the effect of the shape of the cross-section was not normally taken into account, and the river might overflow into its flood plain above the range of observed levels.

Mr Hoffman then outlined a method of deriving gauge-discharge equations which included the effect of the shape of the cross-section of the river and which might possibly account for some of the discrepancies found on the Nile.

B. A. Bakhmeteff[24] had found that, for most natural river cross-sections, an equation of the following form held good :—

$$AR^{\frac{2}{3}} = kH^{a}$$

where $A$ denoted the cross-sectional area, $R$ the hydraulic mean depth, $H$ the depth to the lowest point on the cross-section, and $a$ and $k$ were constants. In that equation, $H$ could be replaced by $(G + b)$, where $G$ denoted the gauge reading and $b$ was a constant, giving :—

[24] B. A. Bakhmeteff, "The Hydraulics of Open Channels." McGraw-Hill, 1932, p. 84

$$AR^{\frac{2}{3}} = k(G+b)^a$$

Putting that value for $AR^{\frac{2}{3}}$ into Manning's equation (with the usual notation) :—

$$Q = \frac{1{\cdot}486}{n} AR^{\frac{2}{3}} S^{\frac{1}{2}}$$

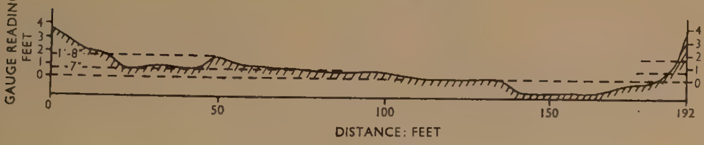
gave the required gauge-discharge equation :—

$$Q = \left(\frac{1{\cdot}486}{n} S^{\frac{1}{2}} k\right)(G+b)^a$$
$$= c(G+b)^a$$

Mr Hoffman then gave an example showing the derivation of gauge-discharge equations at a point on the River Shin, in Sutherland.

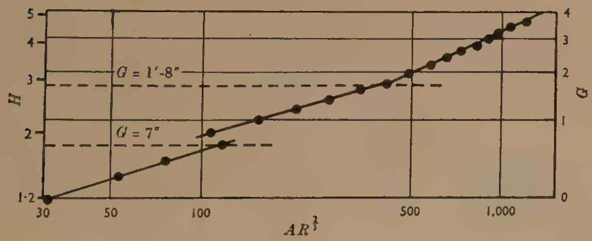
*Fig. 8* was the cross-section of the river. Normal water levels ranged

*Fig. 8*




River Cross-Section

*Fig. 9*




from about 3 inches on the gauge to 3 feet, and in that range there were two major irregularities in the cross-section, the first at 7 inches, where the river overflowed into a small side channel, and the second at 1 foot 8 inches. Those irregularities were more clearly shown in *Fig. 9*, in which log $H$ was plotted against log $AR^{\frac{2}{3}}$. It would be seen that Bakhmeteff's relation held good only over the intervals between irregularities in the cross-section where straight lines could be drawn with a reasonable approximation through the plotted points. The discontinuity at 7 inches on the gauge was caused by the sudden increase in perimeter at that level, and similar discontinuities would occur where rivers overflowed into their flood plains

The slopes of the straight lines in *Fig. 9* gave values for $a$ in the gauge-discharge equation ; values for $b$ and $c$ were found by plotting $Q^{\frac{1}{a}}$ against the gauge readings, $G$, for observed discharges, as shown in *Fig. 10*. In that Figure, $b$ was given by the intercept at $G = 0$, and $c$ by the gradient of the

*Fig. 10*

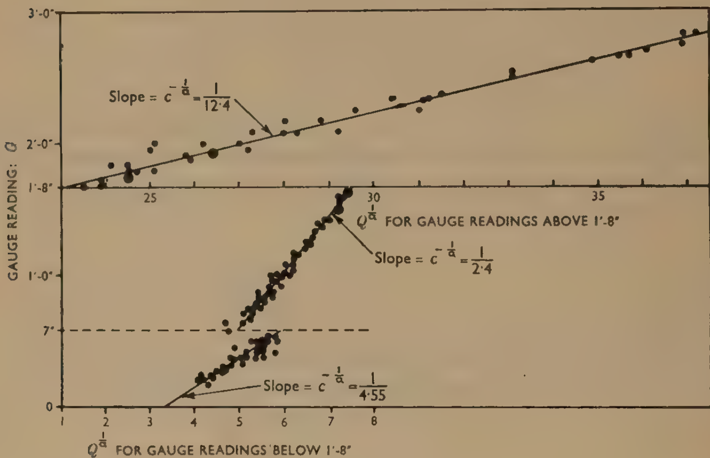
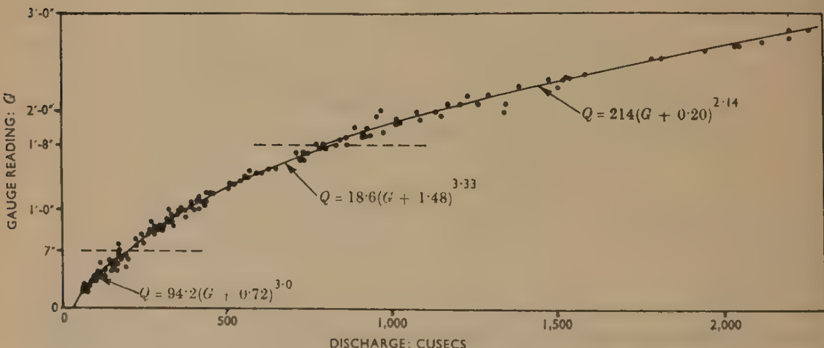



*Fig. 11*




Gauge Discharge Curve

*Equations :—*

(1) Bakhmeteff: $AR^{\frac{2}{3}} = kH^a = k(G + b)^a$

(2) Manning: $Q = \frac{1\cdot486}{N} A R^{\frac{2}{3}} S^{\frac{1}{2}}$

(3) Gauge discharge equation : $Q = \left(\frac{1\cdot486}{N} S^{\frac{1}{2}} k\right)(G + b)^a$

$$= C(G + b)^a$$

curve raised to the power $-a$. The straight lines in *Fig. 10* showed that $c$ was approximately constant for each range of levels; that would be expected, since variations in $S^{\frac{1}{2}}$ are not normally large enough to affect $c$.

The values of $a$, $b$, and $c$ found from *Figs 9* and *10* gave the gauge-discharge equations shown in *Fig. 11*, with their corresponding curves.

The example given was one of the two cases where the method had so far been tested. The maximum flood level under existing conditions was required at the cross-section shown, which was not at the gauging site on the river. The 4-foot range of levels and the 180-foot width were small when compared with the river Nile, but there did not appear to be any reason why the method should not apply equally to large rivers. The cross-section chosen when applying the method should be in a reach of the river where the flow was uniform and Manning's equation applied. Under those conditions it seemed possible that the different curves for the rising and falling stages on the Nile might be reproduced by putting appropriate values for $S^{\frac{1}{2}}$ into the gauge-discharge equations.

**Mr G. W. Grabham** said he felt sure that the Author would agree on the value of Sir Henry Lyons's work in the early days. Sir Henry's book on the Nile Basin, published in 1907, had been issued at a time when very few data were available, and even now that book could be consulted with considerable profit.

Mr Grabham also wished to draw attention to the topography of the Sudan. The discussion had been concerned with the Nile, but there had been references to the production of grain and other food. The vast plains of the Sudan produced crops which were collected and brought to market, and that great production did not depend on the variations in the Nile, or on irrigation, but simply on the rainfall. One remarkable fact was that it was possible to have quite a good total rainfall but a poor crop, because the rain had not been evenly distributed throughout the season.

In prehistoric times, storage tanks ("napirs") dug by hand and filled by rain, had provided water for cultivators and their animals at the beginning of the dry season, while their crops matured and were harvested. That system, which continued in use until modern times, had been greatly developed since the 1939–45 war by the use of bulldozers to dig tanks, usually with capacities of at least 10,000 cubic metres. The heavy clay soil was almost impermeable, so that the loss from seepage was negligible, whilst depths of more than 20 feet allowed for the rapid evaporation; water could thus be conserved for more than a year unless the tanks were heavily drawn upon. Some of those tanks held more than 100,000 cubic metres.

The problem which had been mentioned in the discussion of cattle grazing alongside the Nile could be partly solved by means of such tanks, because the cattle could be watered and could feed on the straw from the crops round about.

*** **Mr H. F. Wilmot** referred to the Author's statement, on p. 8, that the conversion factor of 0·96 was used to convert half-depth velocity to mean velocity. Did that hold for all depths and all velocities? Was it not possible that the value 0·96 was valid at lower velocities but diminished at greater velocities? Admitting that, it could be shown that the departure for the 6,000–8,000 group of meter readings would disappear if the coefficient were 0·92.

The method employed for mass curves in *Fig. 6* had been used for a considerable time, and had been described by Lang.[25] It had also been employed for the variation in level of Lake Victoria in the Report on the Owen Falls project, published in January 1948—the relationship there between levels and volumes was to all intents and purposes linear. The graph produced for that Report was entitled "Effect of Regulation on Lake Victoria" and showed the deviations from mean storage level plotted against time for the period 1899 to 1946. It illustrated Dr Hurst's contention with regard to the groupings of natural phenomena into high and low groups, showing nine well-formed groups in a 50-year period.

During a study of the Thames, two curves had been drawn for the period 1883 to 1949, one showing the mass discharge plotted against time, the other showing the deviation from the mean discharge plotted against time. The contrast between those two graphs afforded striking evidence of the superiority of the "deviation" method over the "mass-summation" method, enabling the vertical scale used for the former to be at least, say, four times that required for the latter.

Whilst for considering long-term storage by means of the mass-deviation curve only the mean annual value was required as used by Dr Hurst, thus greatly simplifying the arithmetical work, nevertheless the annual storage needs permitted no escape from the mass of detailed computation.

Mass-summation curves could be very useful for supplying the data when plotting storage regulation curves (showing storage as a percentage of the run-off against percentage of time in years); such curves, however, could be misleading, since great variation was possible when the graphs were based on a relatively short period of time. Thus for the River Thames, to ensure a supply of 80-per-cent run-off as the mean flow, the storage required, expressed as a percentage of the mean annual flow, for 1883–1949 (67 years) was 104 per cent, for 1883–1928 (45 years) 110 per cent, and for 1928–1949 (22 years) 62 per cent.

Mr Wilmot felt that the Author's ambitious general formula for storage capacity expressed in terms of standard deviation and period, giving $K$ the constant value of 0·72, should be accepted only with reserve, in spite of the many cases of natural phenomena, ranging up to 2,000-year periods, upon

*** This and the following contributions were submitted in writing upon the closure of the oral discussion.—SEC. I.C.E.

[25] J. D. Lang, "On a Determination of Storage Reservoir Capacity," *J. Instn Engrs Aust.*, vol. 22, p. 85 (April and May 1950).

which it was based. It was tempting to believe that both it and the equation given by the Author connecting the mean of observations with the draft had absolute validity. The line of approach was a valuable contribution to that difficult subject.*

**Mr W. N. McClean** observed that the unit of 12 milliards of cubic metres per year referred to on p. 13 was equivalent to about 380 cumecs, or 13,400 cusecs. It was of the same order of magnitude as a high Thames flood, for which the run-off had been recorded at 0·13 inch per day over a catchment area of 3,812 square miles. Would the Author state: (*a*) the area of the Atbara catchment; (*b*) the annual run-off in inches; and (*c*) the daily run-off during the flood period?

The Author's method of assuming mean velocity to be 0·96 times the velocity at half-depth was very convenient, and was indeed ideal for a standard assessment of discharge where there was neither acceleration nor retardation of flow. In the United States of America there was a preference for using the velocities measured at 0·2 and 0·8 of the depth—a method also used in the United Kingdom. Mr McClean himself preferred, for accurate gaugings on British rivers, to assume that mean velocity $= C \times$ maximum velocity, where the coefficient $C$ varied to some extent with the depth and was derived from actual depth-velocity measurements and not from hydrological deductions.

Mr McClean had devised a gauging apparatus which spanned 200 feet or more. In 8 hours' work he could obtain about 160 velocity measurements, all observed on the same verticals across the section; greater sounding-depths of water, at the higher velocities, did not give rise to any appreciable difficulty.

Referring to the Aswan discharge shown in *Fig. 6*, would the Author state if the discharge below the dam included irrigation supply? It would appear that the supply of irrigation water might be the cause of the lower discharges observed since 1898. It was, perhaps, an effect similar to that of the sudd, where some of the loss might go to lower levels than the transpiration limit of about 30 feet.

Mr McClean would be interested to know the exact manner in which the measurement, by tank, of the sluice discharge was carried out. *Fig. 1* showed turbulence and overflow.

Lake Victoria was known to have an area as large as Ireland, and a maximum depth of 250 feet. Would an increased storage level result in an increase in the evaporation during the longer duration of storage? Lake levels might be considerably affected by local conditions.

**Mr Frederic Newhouse** remarked that the Author had given the conversion factor 0·96 to reduce half-depth velocity to mean velocity when taking current meter velocities on a vertical in a river. It had been found as a mean of more than 300 discharge measurements of rivers in the

* Mr Wilmot's contribution was accompanied by three sheets of diagrams illustrating the points he raised.—Sec. I.C.E.

Highlands that that factor was 0·95. The error in that coefficient was about 4 per cent, and it would be interesting to know how that compared with the standard deviation in the Author's 500 observations on the Main Nile. It should be added that the 338 measurements of discharge of rivers, from which the coefficient 0·95 had been deduced, involved at least ten times as many velocity measurements by current meter. A discharge observation required velocity measurements on two or three verticals in the smaller streams and up to eight or nine in the larger ones, whilst the number of observations on each vertical varied from three to about fifteen. The latter number were taken in some cases where it was decided to measure the velocity at every 6 inches of depth down each vertical. Those Highland streams had steeper slopes than the Main Nile and were sometimes almost of a torrential nature, whilst the bed was seldom sand or mud but usually gravel and pebbles of a fairly large size. The close correspondence between rivers so different as the Main Nile and Highland streams was of great interest.

It was to be hoped that the Author's conclusion that the probable error of a single discharge measurement was about 5 per cent—a statement with which Mr Newhouse heartily concurred—would receive careful attention by the numerous enthusiasts now dealing with river measurements. Many were inclined to give the results of measurements correct to five significant figures, whilst one case was known to Mr Newhouse in which a foreign Government presented lists of discharges of a major river correct to seven significant figures. A great deal of time would be saved if all concerned would recognize that river measurements were correct to only two significant figures, and that any figure derived by multiplication of a river discharge was therefore also correct to only two significant figures. It would indeed be a blessing, for instance, if those writers who quoted costs of hydro-electric power, estimated from river discharges, at 0·7965*d*. per unit, or some such, would say 0·8*d*., and thus avoid giving an entirely unwarranted appearance of accuracy to arithmetical statements.

The Author had drawn attention to the variability of the gauge-discharge relation at any station on a river where there was a sandy bed, and would, no doubt, recall the case where a 30-centimetre erosion in the bed of the Nile had caused such confusion and so many accusations of inaccuracy and even fraud, that the matter could only be settled by an International Commission and a Public Inquiry.

Turning to the Author's description of the proposed projects for the development of the Nile, Mr Newhouse believed that decisions had been taken, or were said to have been taken, before the whole subject had been thrashed out by an independent Commission of Engineers. Departmental reports had been made by officials who had studied the question for years and had reached certain conclusions; contrary conclusions had been reached by other officials, also after years of study. Those various projects had been considered *in camera* by a Committee of three ex-Ministers of

Public Works to the Egyptian Government, who were all engineers but obviously had had no time in the course of their great and varied duties to devote the requisite attention to such an intricate question.

It was not generally appreciated in Egypt, or elsewhere, that it would not be possible for Egypt to benefit to any appreciable extent by works on Lake Victoria and Lake Albert until after the completion of the Sudd Channel. An average of half the discharge entering the swamps of the Upper Nile was lost annually, and in years of high discharges the loss increased enormously; but, on the average, it seemed that 14 milliards of cubic metres of water were lost in those swamps every year and it was not known what happened to about 6 to 8 milliards of that loss. That was the approximate quantity of the loss that could not be accounted for by evaporation or absorption. There was no spillway over the watershed of those swamps into a neighbouring watershed. It would require more study to decide whether the fact that that great loss could not be accounted for was of importance for the future development of the Upper Nile. No case had been made out for spending millions of pounds at Lake Victoria and Lake Albert to increase Egypt's water supply, before many millions had been spent on abolishing those swamp losses.

One of the first facts to be settled in considering the development of Egypt was the area that could be irrigated in that country. The Survey of Egypt gave the area as 7,300,000 feddans (1 feddan was approximately 1·06 acre), of which at one time it had been proposed to reserve 200,000 feddans in the lakes of the Delta for fisheries. In addition, there were believed to be a few high-lying plateaux, such as the one at Komombo, which could economically be irrigated by pumps, and it was optimistically believed that, by not having any reservation for fisheries, and by including some cultivable areas alleged to be available east of the Suez Canal, the total cultivable area of Egypt might be 7,600,000 feddans. Various estimates of water requirements had been put forward, based on a cultivable area in Egypt of 8,500,000 feddans or even 10,000,000 feddans, but no map had ever been published showing where those areas were. There could be no reality in the discussion of Egypt's needs for water until a decision was reached on the cultivable area and the average requirement of water per feddan per annum. Such figures were the basis of the calculations which had been made more than 30 years ago to support the Nile Control Projects then put forward by the Egyptian Government, advised by Sir Murdoch MacDonald, and accepted by an International Commission. The changes in those basic figures had never, to Mr Newhouse's knowledge, been supported by any evidence.

**The Author,** in reply, said that an echo sounder had been obtained with the idea that it would be used on the survey of the Aswan reservoir, but when that work was being done the echo sounder had not been available because it had been borrowed to do surveying on the coast of the Red Sea. It was an early model, probably made before the 1939–45 war, and was

not very suitable for their purpose because it had not a wide enough scale. He believed another one had been secured later, but that had been since he had had anything to do with the administration, so that he did not know what became of it.

When referring to a mass curve he had been thinking of the curve where the actual discharges were measured every year and added up, producing a sort of ladder, whereas his own practice had been to take the departures from the mean and add those up, thus producing the sort of curves which had been shown, which went up in some cases and down in others, and which were more manageable and gave a better idea of what the phenomenon really was.

In most of the work which he had done he had neglected the annual variation, because for his purpose it had been important to deal with the main phenomenon first so as to have an idea of the variation from year to year. With very large reservoirs it was possible to smooth out a great deal of the effect of the period within the year; in fact, it counted only at the highest and lowest levels.

With regard to the half-depth factor, the experiments which were mentioned were experiments on the main river, where many measurements were made at numbers of points on the vertical, and then curves of variation of velocity with depth were drawn. He thought that there had been something like 500 complete discharge observations to go upon, with twenty or more vertical sections in each. An examination had been made of the various methods in use or proposed in other places for obtaining the mean velocity on a vertical, for example, 0·6 depth as being practically the mean, or two different depths (0·8 and 0·2) which gave approximately the mean, and so on, but at that time his organization had inherited the practice of measuring at half-depth, and that came out at the factor 0·96, so that they continued to use that. Most of the cross-sections that they had to deal with on the Nile were wide and relatively shallow, so that although the streams varied a good deal in size the type of section did not vary very much. He thought that the evidence obtained from measurements where two streams joined together to form another stream and the fact that there was such agreement between the sum of the two tributaries and the main stream showed that there could not be a great deal of error with the factor of 0·96. He did not know what the most recent American results were, but those that he had seen at the time that the work concerned was being done agreed with the results obtained.

The recent proposal for a very large reservoir on the Nile took account of the silt, inasmuch as it allowed a volume which was not likely to be filled in any short time, perhaps 500 years, as a sort of pit into which the silt would be dropped, whereas the main part of the reservoir would be up above that.

The Author agreed entirely with what Brigadier Hawes had said about the value of rainfall observations and other meteorological observations

which were being made all round the Lake Victoria basin. The more observations that could be made, the more satisfactory would be the position of the man who had to deal with the problem. The Author could not recollect within his own experience any case where there were so many observations that one did not know what to do with them.

The very low year which he had quoted, 1913–14, had never been taken as a basis for design. But matters had been arranged, in the various projects dealt with, that if a year like that of 1913–14 did occur it would be possible to get through it without disaster. The basis for design had been something very much more. He ought to add, however, that in the year which came next to that one, namely 1899–1900, the total water supply, if it could all be used, would have been only just about as much as the estimated requirements of Egypt and the Sudan, and the requirements which had been estimated for the Sudan in those days were probably a good deal less than would be estimated after intensive work done in the past few years.

There was a little difficulty in using the hydrological and agricultural year inasmuch as it varied when one went down the river. The flood at Roseires, towards the Abyssinian frontier, had a maximum 10 days or a fortnight before the maximum at Aswan, and 3 weeks before the time at Cairo. On the whole, he thought that for their purposes on the Nile it was more convenient to stick to the calendar year, the end of which occurred during the time of closure of irrigation canals. The statistics were put out in 10-day means, and there was no need to worry about going very much finer than that.

He would like to study Mr Hoffman's curve at leisure, but the point that was made in the Paper was that on the Nile there were very few places where one had a permanent bed, and the fact that the bed shifted from year to year nearly everywhere and became eroded or silted upset the possibility of having any absolutely fixed gauge discharge curve.

The work of Sir Henry Lyons, as Mr Grabham had said, had been of inestimable value, because that was the first time that the whole of the data which could be found in the reports of travellers and others who had anything to do with the river had been put together in one volume and in a form which enabled use to be made of them. Until fairly recent times, that had been the only existing book giving a detailed scientific account of the Nile, and even at the present time it was still possible to go back to it to obtain information about some of the things which happened before 1900.

With regard to Mr Allan's question about evaporation, it would be possible to deal with that in the case of a reservoir where the relation between water level, surface area, and content were known. For instance, the behaviour of a reservoir could be calculated if the known data included a set of incoming annual discharges, an initial content, a corresponding set of outgoing discharges based on some scheme or regulation, and a relation between surface area and content. If $C_0$ denoted the initial

content, $a_0$ the corresponding surface area, $Q_1$ the entering discharge in the first year, $D_1$ the draft, and $e$ an annual average depth of evaporation, then $C_1$ (the content at the end of the year) was given approximately by :

$$C_1 \doteq C_0 + Q_1 - D_1 - ea_0$$

If necessary a correction could be applied to $C_1$ by replacing $a_0$ by $\frac{1}{2}(a_0 + a_1)$ where $a_1$ denoted the area corresponding to $C_1$. Repetition of that operation year by year gave the successive contents under the regulation. The operation could be done graphically by the use of the curve of accumulated departures. In many cases it would be near enough to use evaporation losses which referred to particular ranges of content.

The operation of over-year storage reservoirs would be discussed in a forthcoming Paper.

Some further information on Mr Wileman's question about the variation of the coefficient of 0·96 used to reduce velocities at half-depth to the mean velocity over the vertical, would be given by Mr R. P. Black in the Correspondence on the Paper.

In the scheme for a very large reservoir at Aswan, mentioned by Professor Addison, silt would certainly be deposited ; that had been allowed for by the provision of plenty of capacity below working levels to form a silt container. It was estimated that that capacity would be enough for 500 years.

Nile silt contained very little sand and two-thirds of the suspended material consisted of fine particles, so that a fair proportion would pass through the reservoir. The scheme would develop a high-power output but, having regard to the depth of the reservoir, it seemed doubtful whether it would be feasible to use some of that power to keep matter in suspension.

Further to Brigadier Hawes's remarks on vertical velocity-distribution on the Indus and on canals, the distance 0·6 depth below the surface, which had been found by the Brigadier for the mean velocity, had also been the position of the mean velocity found in about 500 vertical velocity curves, for depths ranging from 4 to 12 metres, which had been observed on the Nile below Aswan. The reasons for preferring the half-depth position were that the observer was less likely to make a mistake in fixing the depth of his current-meter, and any displacement from the correct depth which might occur owing to the current meter being carried a little downstream, had less effect at half-depth than at 0·6 depth below the surface. The use of the maximum velocity as proposed by Mr McClean might have some advantages in a deep and rapid river, as suggested by Mr Richards, since it occurred at about 0·12 of the depth below the surface. Also, the velocity varied very little between 0·07 and 0·17 of the depth. It suffered, however, from the drawback, compared with the half-depth method, that a correction of 13 per cent had to be applied to obtain the mean velocity instead of 4 per cent. It might be noted that in very small

shallow streams the velocity would not be uniform over the rotor of the current meter and the presence of the instrument would disturb the velocity distribution, so that very accurate results could not be expected.

With regard to Mr Hoffman's remarks on fitting formulae to gauge-discharge relations, when he said that the cross-section chosen when applying the method should be in a reach of the river where the flow was uniform and Manning's equation applied, the Author found that the difficulty on the Nile was that so far he had not found an equation of Manning's form or any other form which applied generally.

Fitting formulae to observations could be very misleading. Unless a formula had a sound physical basis it was merely a convenient way of representing a set of observations, and more than one representation could usually be devised. The danger was that, outside the set from which the formula had been constructed, it might have no relation at all to the phenomenon. The following example illustrated that.

*Fig. 12* showed a set of observations of the discharge of an Aswan sluice (represented by circles) over the range of heads 2 to 15 metres, and through those a curve had been drawn by hand to fit as closely as possible. The formula $Q = CA\sqrt{2g(H - F)}$, or $Q^2 = aH + b$ had been fitted and the results were shown by crosses. Another formula, $Q = c + eH + fH^2$, shown by squares, had also been fitted. It would be seen that both those equations were a good fit for the observations. Also observations on the sluice gave $H = 20{\cdot}74$ and $Q = 27{\cdot}45$. Calculating $Q$ from equation (1), the result was $Q = 27{\cdot}3$, and from equation (2), $Q = 24{\cdot}7$, a value which was 10 per cent too low. The first formula had a theoretical basis and had been established by very many experiments on different sluices under a wide range of conditions, whilst the second had no theoretical basis and applied only to that particular sluice over the range of heads from 2 to 14 metres. That example showed the danger of extrapolation on an empirical formula.

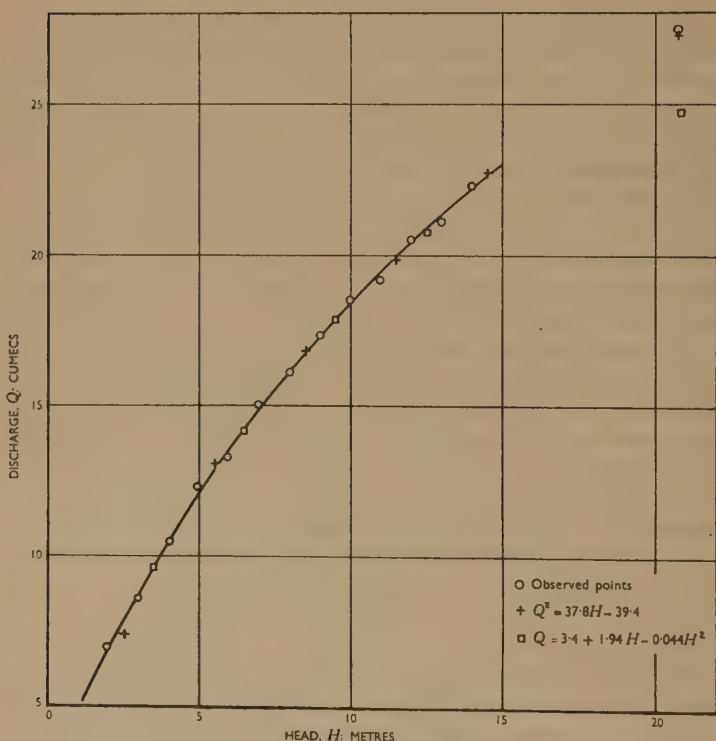
In reply to Mr Wilmot, the difference between sluice and current-meter discharges above 6,000 cumecs probably resulted from errors in sounding. Considerable skill and experience were required to reduce errors to the minimum in flood time, and it was not always possible to provide the best observers during the height of the summer. The Author's method of using accumulated deviations had been first given in "The Nile Basin," vol. V, published in 1938. In the formula for storage capacity given on p. 23 and mentioned by Mr Wilmot, as explained in the Paper, $K$ was not a constant but a quantity which had a Gaussian frequency distribution and a mean of $0{\cdot}72 \pm 0{\cdot}09$, and, like any other mean value, that should be used with due regard to its probable error.

In reply to Mr McClean, (1) the area of the effective part of the Atbara catchment, above the junction of the Setit and Atbara, was about 100,000 square kilometres; (2) the average annual total of the Atbara discharge was $11{\cdot}6 \times 10^9$ cubic metres, which was equivalent to 4·6 inches spread

evenly over the effective catchment; and (3) the highest normal 10-day mean discharge of the Atbara was $193 \times 10^6$ cubic metres a day, that was, 0·08 inches per day over the catchment.

The discharge below the Aswan Dam was not affected by irrigation except by the small amount of water taken by the Sudan; that did not begin until 1925. There was no question that the discharge of the Nile

*Fig. 12*




Discharge of a Sluice

had decreased considerably in the present century from what it had been in the latter part of the 19th century.

The method of measurement of sluice discharges by the tank was described in the Paper, reference 1. The splashing over shown in *Fig. 1* was trivial compared with the volume of the tank, and turbulence did not enter the question, since the content of the tank was not measured for a matter of 2 hours after the finish of the discharge, by which time all movement had ceased and the water surface was level.

In regard to Lake Victoria, the slight increase of area from higher levels would have little effect, since evaporation and rainfall were approximately equal and compensated each other.

Turning to Nile Projects and the questions relating to them which were raised by Mr Newhouse, the projects had been drawn up by the Author and his colleagues after a review of the many previous projects which had been proposed. They were described in "The Nile Basin," vol. VII, "The Future Conservation of the Nile" (reference 20), in which all the basic data were given. That was published in 1946, approved by a committee of three ex-Ministers of Public Works, and then by the Council of Ministers. It had formed the basis of discussions between Egypt, the Sudan, and Uganda and, so far as was known to the Author, had met no adverse criticism. Contrary to Mr Newhouse's opinion, the dependence of reservoirs in the Great Lakes on the reduction of losses in the swamps of the Upper Nile had been explained in all Egyptian publications on the subject, and was well known to all in Egypt who took an interest in development projects. Mr Newhouse had referred to the loss of water in the swamps, where there was a discrepancy between actual loss measured by discharges and loss calculated from estimated swamp areas by means of measurements made in a tank in which papyrus was grown and which was placed in the midst of swamp vegetation. The question had been discussed at some length in vol. V of "The Nile Basin" (1938) (reference 5). The conclusion had been that "The most probable explanation of the water losses both from the Bahr el Jebel and the Bahr el Ghazal is that:

(1) The transpiration losses from the swamp vegetation are of the order of 30 per cent greater than those measured in the tank.

(2) We must add to the area shown on the maps as permanent swamp an area which is inundated in the flood and dries out in the dry season. In the case of the Bahr el Jebel this area is greater than the area of permanent swamp."

In support of (1), it might be said that, up to the time when it was written, the vegetation in the tank had never been as luxuriant as in the swamp outside. Also there was collateral evidence to show that freely growing vegetation with an ample water supply in a hot climate transpired considerably more water than was evaporated from open water, whereas the papyrus in the tank had disposed of very little more. It was hoped that more recent figures for the papyrus tank would soon be available.

The cultivable area of Egypt, had been discussed in vol VII of "The Nile Basin" where the possibilities were examined. Work was going on at the present time to see whether the estimate of 7·5 million feddans there given can be extended. The water requirements were also given and were based on quantities of water which had been actually used, in the same way as had been done by the Author in his estimate made for

the projects in Nile Control mentioned by Mr Newhouse. It was well known that the limit to cultivation both in Egypt and the Sudan was the Nile water-supply, and not the amount of land.

Correspondence on the foregoing Paper is now closed and no contribution, other than those already received at the Institution, will now be accepted.—Sec. I.C.E.

## STRUCTURAL AND BUILDING ENGINEERING DIVISION MEETING

8 December, 1953

Professor A. J. S. Pippard, Member, Chairman of the Division, in the Chair

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Author.

Structural Paper No. 36

# "A Moment Distribution Method for the Analysis and Design of Structures by the Plastic Theory"

**by**

**Michael Rex Horne, M.A., Ph.D., A.M.I.C.E.**

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SYNOPSIS

A fundamental principle in the plastic theory of structures states that if any *arbitrary* state of stress in equilibrium with a given set of loads acting on a structure can be found (the yield stress being nowhere exceeded), then the structure would support the given loads. The term *arbitrary* is here meant to imply that no account need be taken of the requirements of strain compatibility, and hence the postulated state of stress will not usually be one which can occur in practice. This principle is the basis of the method described in the Paper for determining the maximum loads which could just be supported by a ductile framed structure. The method bears some resemblance to the moment distribution method for the analysis of continuous structures in the elastic range. It is particularly advantageous when used as a method of design, that is, as a means of deriving suitable sizes of members in a structure required to support given loads. Examples of the various applications of the method are given.

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## INTRODUCTION

THE derivation of the plastic collapse load of a structure involves finding a bending-moment distribution which gives a state of equilibrium with the external loads, the full plastic moment of the structure being reached at a sufficient number of sections for either the whole structure or part of it to become a mechanism. At the same time, the full plastic moment must not be exceeded at any section. These principles have already been sufficiently expounded elsewhere [1, 4] and will not be elaborated in this Paper.

In the case of structures such as continuous beams or portal frames it is

[1] The references are given on p. 76.

possible to derive collapse loads either by inspection or by simple graphical methods.[1] More complicated frames can also be analysed in this way, but only after considerable experience. A formal method of solution, employing sets of linear inequalities, and theoretically applicable to frames of any degree of complexity, has been presented,[6] but, unfortunately, this method becomes excessively laborious except for the simplest frames.

The above methods all aim to satisfy, in a single operation, the requirements of a bending-moment distribution representing a state of collapse. These requirements may be referred to as :—

(1) The equilibrium condition (that is, the bending-moment distribution must be in equilibrium with the applied loads).
(2) The collapse mechanism condition (that is, there must be sufficient plastic hinges for either the whole structure or part of it to become a mechanism).
(3) The yield condition (that is, the full plastic moment must nowhere be exceeded).

Whilst it may be difficult to satisfy all three requirements simultaneously, they are much more readily satisfied in pairs. Thus, in a method of analysis recently presented by Neal and Symonds,[7] emphasis is placed on the first two conditions, solutions satisfying these conditions being progressively modified so that ultimately the third condition is also satisfied. This gives a very rapid method of analysis, based on the principle [2, 3, 4] that the plastic collapse load of a structure is the least load for which a bending-moment distribution satisfying conditions (1) and (2) can be found. Hence, in this method of analysis, the collapse load is approached from above, and at any intermediate stage of the working, an estimate on the high side is obtained.

An alternative procedure in developing methods of analysis is to obtain bending-moment distributions satisfying the equilibrium and yield conditions (1) and (3), and to modify these distributions systematically until there are sufficient plastic hinges for collapse to occur (condition (2)). Use may then be made of the fact that the plastic collapse load of a structure is the highest load at which a bending-moment distribution satisfying conditions (1) and (3) can be found.[3, 4] Hence, in this case, intermediate estimates of the collapse load will be on the safe side. This Paper is concerned with such a method of analysis, in which some of the conceptions employed are similar to those used in the moment distribution method of analysis for elastic structures. This has led to the adoption in the title of the term "Moment Distribution Method," but it is not to be imagined that the processes are in any way identical. A knowledge of the moment distribution method for elastic structures is not necessary for the understanding of the present Paper.

Methods of structural analysis are concerned with the determination of structural behaviour, the cross-sections of the members being already

determined. The place of analysis in structural design is thus confined to checking the adequacy of structures already proportioned by some other means. The plastic moment distribution method described here is an exception to this general rule, since it may be used directly to assign suitable full plastic moments to the members of a structure subjected to given loads. In this application of the moment distribution process, the moments which are distributed are regarded not only as equilibrium moments, but also as contributions to the required plastic moments of resistance of the members. This use of plastic moment distribution is indeed simpler than its application to the analysis of given structures, and will therefore be given first. Plastic moment distribution also provides a direct design process for cases where a structure may be subjected to two or more different loading combinations, so long as these do not occur in a cyclical manner so many times that complications arise owing to shake-down effects.[5, 8]

It should be noted that the word " design " is used here in the restricted sense of assigning suitable values to the full plastic moments of resistance of the members in a structure of given form. It is assumed that instability does not prevent these moments of resistance from being fully effective in resisting the external loads.

The plastic moment distribution process and its application to the design of continuous beams is discussed in the section on " Design of Continuous Beams." The design of plane frames under one or more combinations of loads is dealt with in the section entitled " The Design of Plane Frames "; whilst the final section deals with the method of deriving the collapse loads of structures in which the full plastic moments of resistance are already known.

## The Design of Continuous Beams

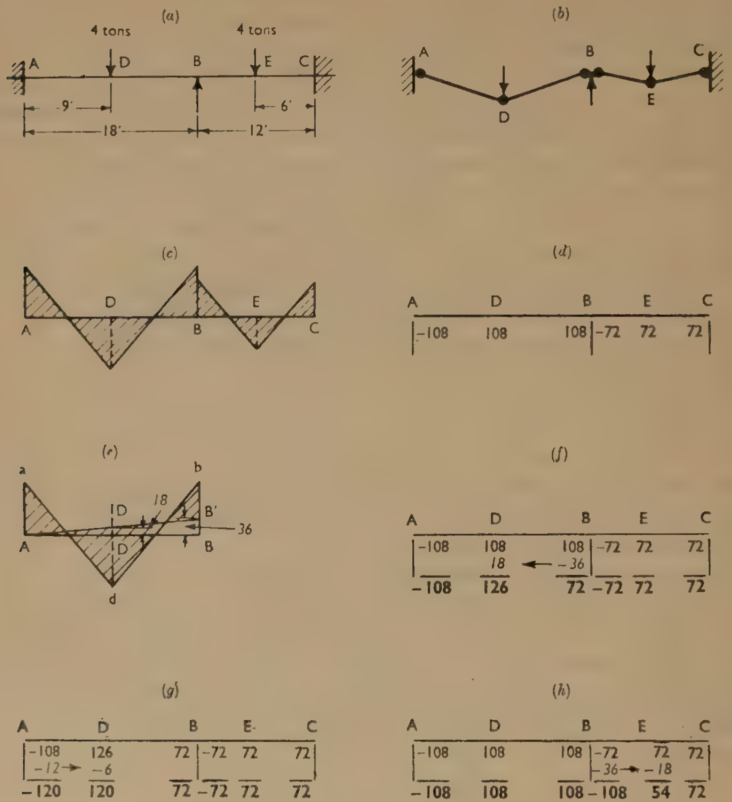
The method of designing continuous beams will be described by reference to a numerical example. It is not claimed that the following is the best means of designing the beam chosen. The method of plastic moment distribution becomes advantageous only for more complicated structures, but the basic conceptions are more easily introduced by considering a simple example.

*Example 1*

The beam ABC (*Fig. 1* (*a*)), 30 feet long, is fixed at both ends and simply supported at B which is 18 feet distant from A. It is required to design the beam so that it would just collapse when subjected to a central concentrated load of 4 tons on each span.

The beam is first designed by the plastic theory as if there were a rigid restraint at B preventing rotation of the beam at this section. Both spans may then be designed separately, giving plastic hinges as shown in *Fig. 1* (*b*).

*Figs 1*




The span AB then has a moment of resistance of $\frac{4 \times 18 \times 12}{8} = 108$ ton-inches and span BC has a moment of resistance of $\frac{4 \times 12 \times 12}{8} = 72$ ton-inches. The bending-moment distribution is shown graphically in *Fig. 1* (*c*), hogging moments being plotted upwards. The numerical values of the bending moments at the ends and centre of each span are shown in *Fig. 1* (*d*), where the following sign convention is adopted: clockwise moments acting on the end of a member are positive, whilst within the length of a member, sagging bending moments are positive.

The above plastic moments of resistance are insufficient to support the loads when the restraint is removed, since the joint at B cannot then remain in equilibrium. Equilibrium can only be achieved at B by reducing the bending moment in AB, increasing the moment in BC, or both.

The effect is then considered of reducing the moment at B in span AB by

36 ton-inches, so that at and near this section the moment of resistance of the beam could, if desired, be reduced from 108 to 72 ton-inches. Since there are moments elsewhere in the beam greater than 72 ton-inches this would actually involve using a beam of varying section. Whilst it would usually be intended to have beams of uniform section only, it is useful during the design process to use temporarily the conception of a non-uniform beam.

Returning to the beam AB in *Fig. 1* (*a*), the plastic moment of resistance at A is considered to remain at 108 ton-inches, so that the change of equilibrium moments occasioned by the reduction of the moment at B by 36 ton-inches will be represented by the movement of the bending-moment baseline from AB to AB′ (see *Fig. 1* (*e*)). The shaded area in *Fig. 1* (*e*) then represents the resultant bending-moment distribution. The required full plastic moments of resistance of the hypothetical beam of varying section are thus 108 ton-inches at A; $108 + \frac{1}{2} \times 36 = 126$ ton-inches at D; and $108 - 36 = 72$ ton-inches at B. The above changes in the required plastic moments of resistance at D and B are in italics in *Fig. 1* (*f*), and the net moments of resistance are shown in bold type. Since instead of a beam of varying section, one of constant section will actually be used, spans AB and BC will require full plastic moments of resistance of 126 and 72 ton-inches respectively. The "balancing" of the joint at B, as in *Fig. 1* (*f*), thus immediately provides a safe design, although it will be shown later that the design may be improved.

It is convenient to think of the above process as one of "plastic moment distribution." Each span is first treated as though it were rigidly fixed at the ends, and is designed so that it just collapses under the applied load. The joints are then "balanced," the moments introduced to achieve balance being essentially changes in the required minimum plastic moment of resistance at the appropriate sections. The balancing moments must then be "distributed." *Fig. 1* (*f*) demonstrates the fact that, if the moment is altered at the right-hand end of a beam and remains constant at the left-hand end, there is a change of moment at the centre (according to the accepted sign convention) of $-\frac{1}{2}$ times the change of moment at the end. The factor $-\frac{1}{2}$ may thus be regarded as the "carry-over factor" appropriate to the case described. All the cases which it is necessary to consider are shown in *Figs 2*, and the appropriate carry-over factors are listed in Table 1. Thus, when the right-hand end moment remains constant (*Fig. 2* (*a*)), the carry-over factor from the left-hand end to the centre is $+\frac{1}{2}$ (see row (*a*) of Table 1). When the left hand end moment is constant (*Fig. 2* (*b*)), the carry-over factor is $-\frac{1}{2}$. If the central moment is constant (*Fig. 2* (*c*)), the carry-over factor is $+1$. These carry-over factors must be memorized in order to apply the plastic moment distribution method. It should also be noted that a change in the bending-moment distribution may also be instituted by modifying the central moment and keeping one of the end moments constant. In this case, there is a carry-over factor of

*Figs 2*

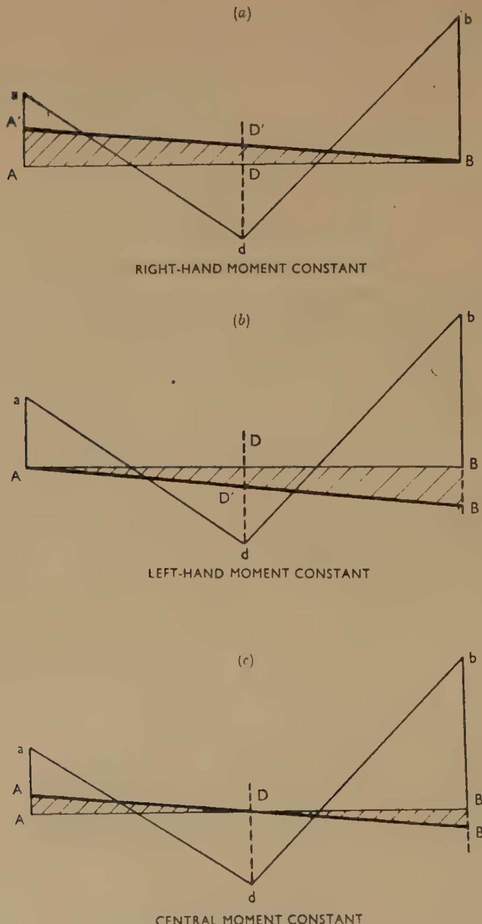



TABLE 1

| | Left-hand end moment | Central moment | Right-hand end moment |
|---|---|---|---|
| *a* | 1 | $\frac{1}{2}$ | 0 |
| *b* | 0 | $-\frac{1}{2}$ | 1 |
| *c* | 1 | 0 | 1 |

$+ 2$ to the left-hand end (obtained by doubling row (*a*) of Table 1) or of $- 2$ to the right-hand end (obtained from row (*b*)). The plastic moment distribution process may therefore be regarded as a reversible one.

The design already obtained for the beam shown in *Fig. 1* (*a*) may now be improved. With the bending moments deduced in *Fig. 1* (*f*), the span AB will not collapse if it is of uniform section, since there will not then be a plastic hinge at A. The first row in Table 1 shows that a change of $+ x$ in the moment of resistance at A is accompanied by one of $+ \frac{x}{2}$ at D. Since the moment at D controls the plastic moment of resistance of the span AB, it is advantageous to reduce this moment until it is numerically equal to the plastic moment required at A. It can therefore be written :

$$- (- 108 + x) = 126 + \frac{x}{2}$$

Hence $$x = - 12$$

Thus the further distribution of moments given in italics in *Fig. 1* (*g*) can be made, showing that the beam AC will be satisfactory if the spans AB and BC have plastic moments of resistance of 120 and 72 ton-inches respectively.

The balancing process described above was started by distributing the out-of-balance moment of 36 ton-inches at B (*Fig. 1* (*d*)) into the span AB. This out-of-balance moment could equally well have been distributed to span BC, giving the total moments shown in *Fig. 1* (*h*). If the beam were designed according to this set of moments, it would have a uniform plastic moment of resistance of 108 ton-inches, collapse occurring in span AB but not in span BC. No advantage is to be gained by modifying the minimum moments of resistance demanded at E and C, since the strength of the span BC is controlled by the moment at B. Hence, this provides an alternative rational design to that previously obtained. It should be noted that the actual moments at E and C are in this case indeterminate. Row (*b*) in Table 1 may be applied to vary these moments without modifying the design, and it is in fact found that the moment at E may vary between $+ 108$ and $+ 36$ ton-inches whilst that at C varies between $-36$ and $+ 108$ ton-inches. This indeterminacy arises since span BC does not collapse, and the calculation of the true moments would involve a lengthy elastic-plastic analysis. Such an analysis is not, however, necessary to establish the design, and the indeterminacy of the moments at E and C may be ignored.

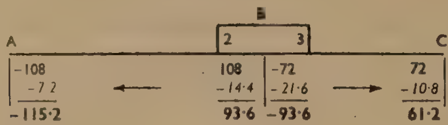
It may be observed that by distributing the initial out-of-balance moment at B between AB and BC in any desired ratio, a series of designs intermediate between those already derived are obtained.

The plastic moment distribution process is readily applied to the design of beams continuous over a number of supports, and it is unnecessary to give further examples. The above treatment is confined to central

concentrated loads, but distributed loads are discussed in the following section of the Paper.

It is instructive to compare the plastic moment distribution method with elastic moment distribution as applied to continuous beams. The elastic analysis of the beam shown in *Fig. 1* (*a*) is given in *Fig. 3* on the

*Fig. 3*




assumption that the beam is of uniform section throughout. The carry-over factors employed in elastic moment distribution are shown in Table 2. The following comparisons between the elastic and plastic moment distribution processes may be made :—

(1) In elastic moment distribution, the out-of-balance moments are distributed in proportion to the stiffnesses of the members meeting at a joint.

TABLE 2

| | Left-hand end moment | Central moment | Right-hand end moment |
|---|---|---|---|
| *a* | 1 | →(½)→ | ½ |
| *b* | ½ | ←(−½)← | 1 |

In plastic moment distribution, the out-of-balance moments may be distributed between the members meeting at a joint in any desired ratio.

(2) In elastic moment distribution, there is a carry-over factor (for prismatic members) of $+\frac{1}{2}$ to the other end of the member, this process being irreversible. The central moment is not recorded.

In plastic moment distribution, it is possible to choose between several alternative " distribution patterns "—that is, there may be a carry-over factor of $\pm\frac{1}{2}$ to the centre of the member or a carry-over factor of $+1$ to the remote end of that member. These processes are reversible, so that if a change of moment is instituted at the centre of a member, there is a carry-over factor of $\pm 2$ to one end.

It will be seen that plastic moment distribution is a more flexible process than that of elastic moment distribution. Such flexibility must exist in a plastic design process, since it is usually possible to postulate an infinite number of structures which will just collapse under a given set of loads.

## The Design of Plane Frames

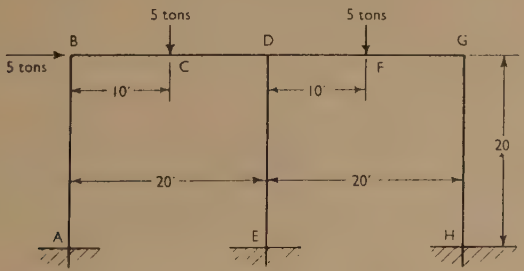
### (a) *Design for a Single Set of Loads*

It is again convenient to introduce the method by reference to a numerical example.

*Example 2*

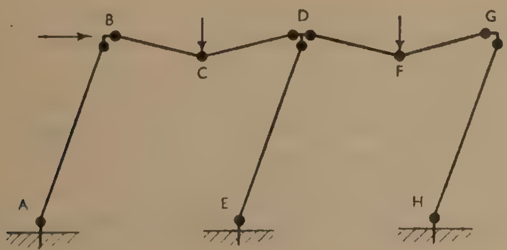
The two-bay portal frame shown in *Fig. 4* is to be designed so that it just collapses under the loads indicated. The beams BD and DG are to be of identical section, and the stanchions AB, ED, and HG are also identical (although not necessarily of the same section as the beams).

*Fig. 4*




The first step is to design the frame on the assumption that external restraints prevent rotation at any joint, but not necessarily displacement. The structure could then be made to collapse as shown in *Fig. 5*. The plastic moments of resistance at the ends and centres of the beams and the

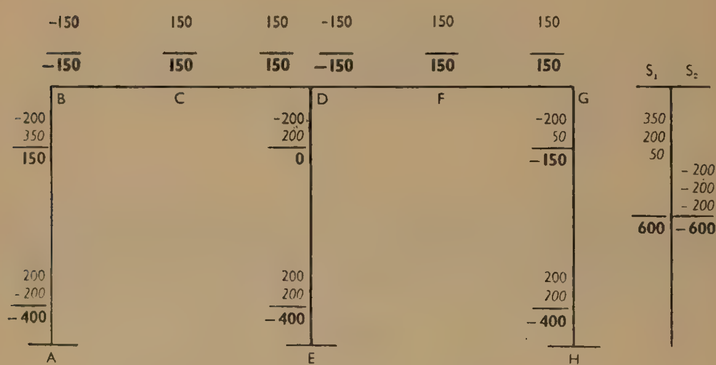
*Fig. 5*



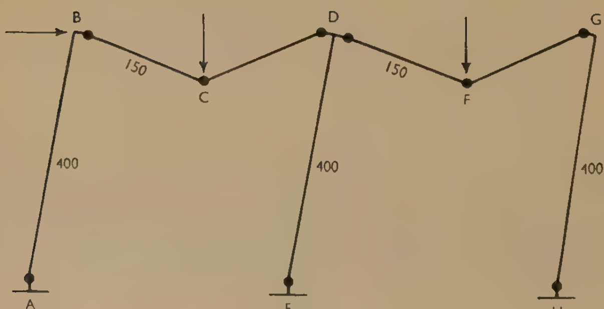
ends of the stanchions would then be as shown in plain type in *Fig. 6 (a)*. It will be noted that the horizontal shear load of 5 tons (giving a storey shear moment of $5 \times 240 = 1{,}200$ ton-inches) has been equally divided between the stanchions, since these are to be of the same cross-section.

*Figs 6*

(a)




(b)



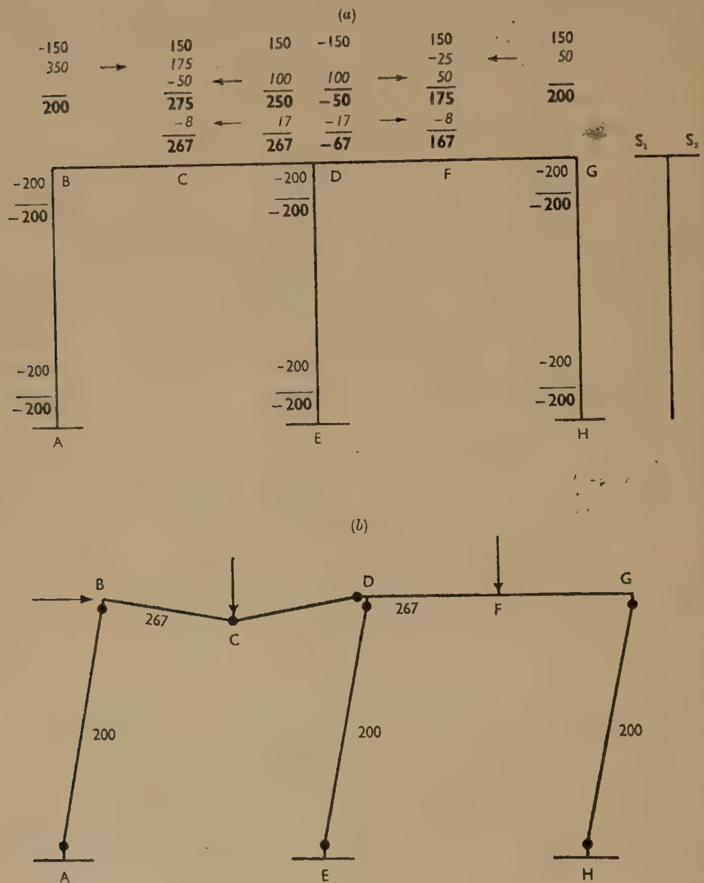

The second step is to balance the joints. It has already been seen, in the case of a continuous beam, that the method adopted for balancing the joints affects the final design. The actual choice of a suitable section will also depend, however, on the sections available, and it will often be useful to determine the upper and lower limits within which the final plastic moments of any particular member in the frame must lie. The lower limit for any member is given by the moment of resistance obtained in the first step, since no member can be lighter than one in which both ends are restrained against rotation. If, in the present example, the beams

retain their minimum section, all out-of-balance moments must be distributed to the stanchions, thus adding moments of + 350, + 200, and + 50 ton-inches to the tops of stanchions AB, ED, and HG respectively (see numbers in italics, *Fig. 6* (*a*)). This process destroys the sway equilibrium of the frame. These balancing moments are therefore entered in a "sway-balancing" column $S_1$ on the right-hand side of *Fig. 6* (*a*), giving a total out-of-balance moment of 600 ton-inches. This moment must be balanced by distributing a moment of — 600 ton-inches to the feet of the stanchions. Since the stanchions are to remain of equal section, this is distributed equally, the distributed moments being entered both at the joints in question and in the second sway-balancing column $S_2$. Hence, provided the totals of the moments shown in columns $S_1$ and $S_2$ are numerically equal but opposite in sign, the frame remains in equilibrium against horizontal forces. The total moments of resistance (given in bold figures in *Fig. 6* (*a*)) show that the maximum necessary moment of resistance of the stanchions has the value 400 ton-inches. The mode of collapse is shown in *Fig. 6* (*b*).

The result of distributing the initial out-of-balance moments to the beams is shown in *Fig. 7*(*a*). The changes of moment are carried over in each case to the centres of the beams, the maximum resultant moments being 275 ton-inches at C, and 250 ton-inches at D in beam BD. Since the bending moment at B is determined by the moment of resistance of the stanchions, it cannot be increased if the stanchions are to retain their minimum section. It is therefore impossible to use either of the distribution patterns in rows (*a*) and (*c*) of Table 1 to decrease the moments at C and D. The only step that can be taken is to use the distribution pattern of row (*b*) to equalize these two moments. Inspection shows that the addition of 17 ton-inches at D in beam BD, accompanied by the addition of $-\frac{1}{2} \times 17 = -8$ ton-inches at C, results in two equal moments of 267 ton-inches. Finally, the joint D is balanced by distributing — 17 ton-inches to the beam DG. The resultant moments then show that the maximum necessary moment of resistance of the beams is 267 ton-inches, and the corresponding mode of collapse is shown in *Fig. 7* (*b*).

It has thus been found that the beams should have a plastic moment of resistance of between 150 and 267 ton-inches, whilst that of the stanchions should be between 200 and 400 ton-inches. It is then supposed that a convenient practical beam section has a moment of resistance of 210 ton-inches, and that it is required to find the corresponding suitable stanchion section. It is apparent from the two designs already obtained that hinges will certainly occur at C and D in beam BD (see *Figs 6* (*b*) and *7* (*b*)), and the first step is therefore to amend the moments at these sections to the value 210 ton-inches (see *Fig. 8* (*a*)). The amending moments at C and D are carried over to B. The joints B, D, and G are then balanced by taking the out-of-balance moments of 170, 140, and 50 ton-inches respectively into the stanchions. Part of the resulting — 360 ton-inches out-of-balance

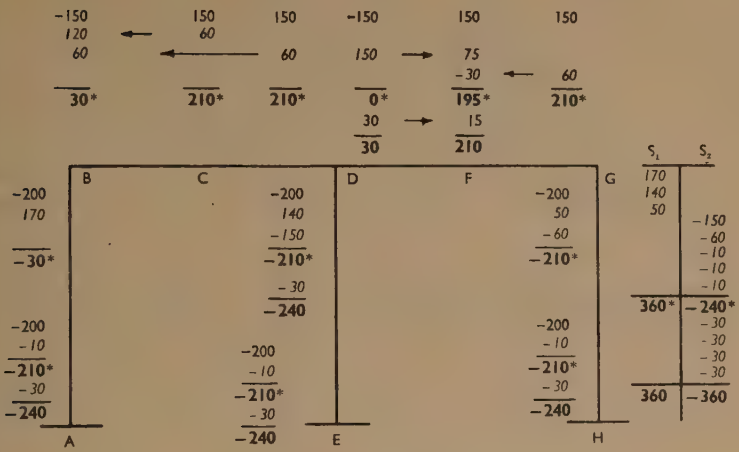
*Figs 7*



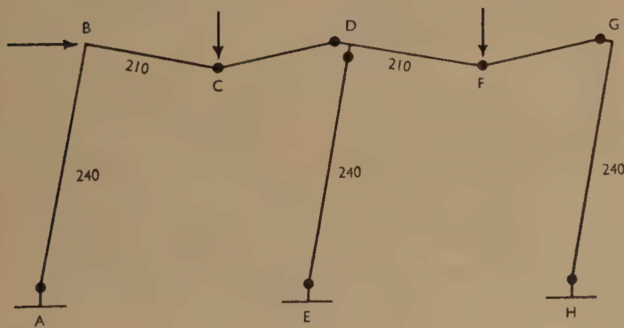

sway moment can be absorbed by increasing the stanchion moments at D, G, A, E, and H to — 210 ton-inches. The moment at B cannot be increased, since this would in turn increase the moment at C or D in the beam BD. The resultant moments at this stage are indicated in *Fig. 8* (*a*) by asterisks, leaving — 120 ton-inches out-of-balance sway moment still to be distributed. Since the moment at G cannot be further increased, this remaining out-of-balance moment is distributed equally to the four stanchions at A, D, E, and H, giving final maximum stanchion moments of 240 ton-inches. This represents the minimum allowable plastic moment in the stanchions when that in the beams is fixed at 210 ton-inches. The mode of collapse is shown in *Fig. 8* (*b*).

*Figs 8*

(a)




(b)




(b) *Design for More than One Set of Loads*

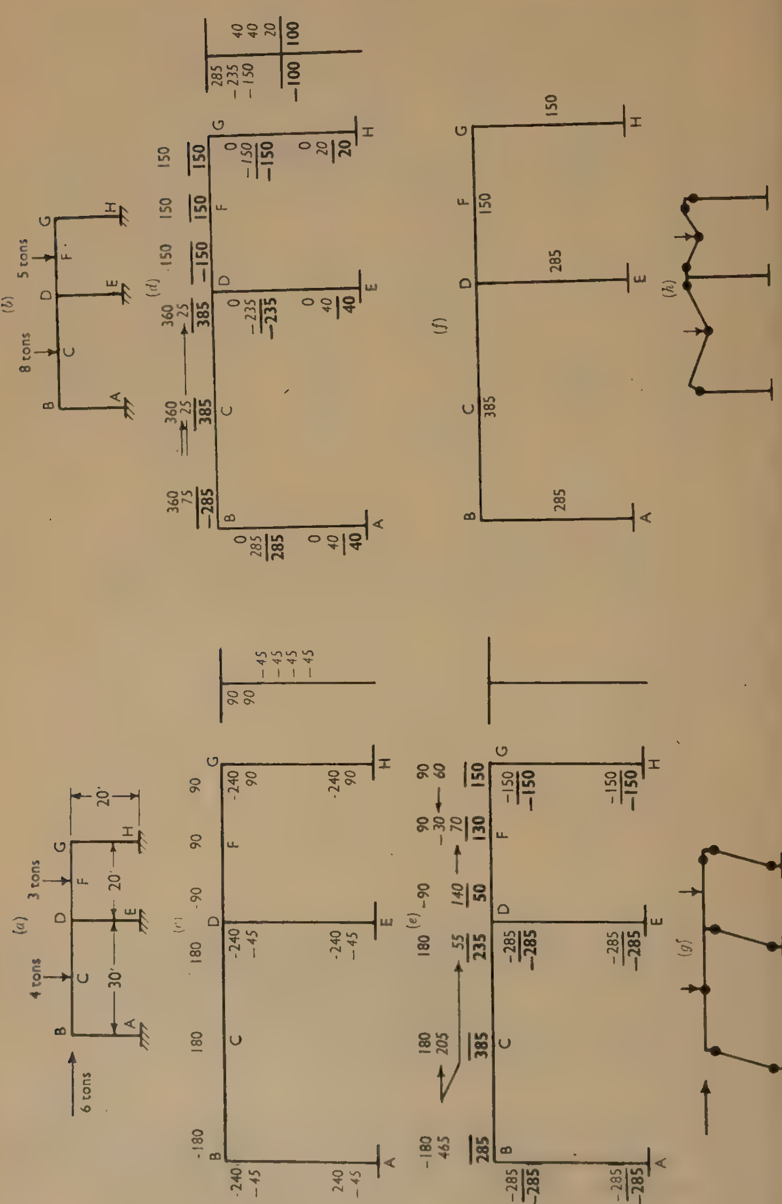
It was mentioned in the Introduction that plastic moment distribution enabled a structure to be designed in a single operation to withstand several different combinations of loads. This application will be described by considering the following example.

*Example 3*

Suitable full plastic moment values are to be assigned to the members of the two-bay portal frame in *Fig. 9* (*a*) so that it will support either of the load distributions shown in *Figs 9* (*a*) and *9* (*b*).

The plastic fixed-end moments for the two load distributions of *Figs 9* (*a*) and (*b*) are shown in plain type in *Figs 9* (*c*) and (*d*) respectively. It

*Figs 9*



immediately appears that the mean stanchion full plastic moment cannot be less than 240 ton-inches (*Fig. 9* (*c*)), whilst the beams BD and DG have minimum full plastic moments of 360 and 150 ton-inches respectively (*Fig. 9* (*d*)). The stanchion GH carries less axial load than stanchions AB and DE, and may therefore be given a lighter section. Since a moment of resistance at G of 150 ton-inches is sufficient for the maximum beam load in DG, it seems reasonable to restrict the full plastic moment of resistance of stanchion GH to this value. Hence, in *Fig. 9* (*c*) the stanchion moments at G and H are reduced by 90 ton-inches, and the resulting out-of-balance sway moment is distributed equally to the stanchions AB and DE.

At this stage attention may be directed to *Fig. 9* (*d*). The joint B is first balanced, partly by raising the stanchion moment to that deduced in *Fig. 9* (*c*) (that is, 285 ton-inches) and partly by reducing the moment at the end of the beam. The 75 ton-inches balancing moment at B in the beam BD is distributed to C and D in such a way that the moments at these sections remain equal. This course is adopted since it leads to the smallest possible increase in the required moment of resistance of the beam. The joints D and G can be balanced by taking all the out-of-balance moments into the stanchions. Finally, the resulting out-of-balance sway moment is distributed to the feet of the stanchions. The beam BD has a minimum full plastic moment of 385 ton-inches, whilst a value of 150 ton-inches remains satisfactory for beam DG.

It is now necessary to return to the loading shown in *Fig. 9* (*a*). This further working could have been carried out on *Fig. 9* (*c*), but for the sake of clarity it is shown in *Fig. 9* (*e*). Starting with the moments derived in *Fig. 9* (*c*), the joint B is first balanced, the out-of-balance moment of 465 ton-inches being taken into the beam. Of this moment, 410 ton-inches is distributed to C, thus bringing the central beam moment up to the value of 385 ton-inches already derived as a suitable moment of resistance (*Fig. 9* (*d*)). The remaining 55 ton-inches is distributed to the end D of the beam. The joints D and G are balanced by taking out-of-balance moments into the beam DG and distributing to the centre. The frame is then fully balanced, and it is seen that full plastic moments of 285 ton-inches for the stanchions AB and DE and of 150 ton-inches for stanchion GH are satisfactory.

Suitable full plastic moments of resistance have thus been assigned to all members, and are shown in *Fig. 9* (*f*). The resulting frame would just collapse as shown in *Fig. 9* (*g*) under the loads given in *Fig. 9* (*a*), and as shown in *Fig. 9* (*h*) for the loads of *Fig. 9* (*b*).

The general method of designing against more than one load distribution should be clear from the above example. The plastic fixed-end moments for each load distribution give lower limiting values for the full plastic moment of resistance of each member. Some members are designed on the basis of these minimum values, and the joints are then balanced for each

of the load distributions. This enables suitable full plastic moments to be assigned to the rest of the members.

*(c) Distributed Loads*

The maximum sagging bending moment in a beam usually occurs at the centre of the span when the load itself is concentrated at the centre but occurs, in general, away from the centre under any other load distribution. It is therefore generally necessary, when using the plastic moment distribution method, to make special calculations of the maximum sagging bending moments. This need not be done until the balancing process is complete, when the plastic moment of resistance of the member chosen must not be less than the value of this maximum moment.

During the distribution process, it may be desired to maintain an equality between the maximum sagging and hogging bending moments in any beam. The beam BD was so treated in *Fig. 9 (d)*. This is not so easy to achieve for distributed loads as for central concentrated loads. Fortunately, in most cases, the maximum sagging moment is very little greater than the central moment, and it is usually sufficient to assume that the maximum sagging moment occurs at the centre until the final sections are chosen. If desired, the central moment may be made somewhat smaller than the maximum end moment—a rough calculation will be sufficient to indicate the approximate magnitude of the correction which should be made.

When the loads are uniformly distributed, the chart given in *Fig. 10* may be of use in calculating the maximum sagging bending moment. The ratios of the two end moments ($M_A$ and $M_B$) to the central moment $M_C$ in the beam (see inset on *Fig. 10*) are plotted along the two axes. The contours give the ratio $\frac{M_S}{M_C}$, where $M_S$ denotes the maximum sagging moment.

*Example 4*

The previous examples refer to very simple structures only, and to illustrate the application of the method to more extensive frames the structure shown in *Fig. 11* will be considered. The loads indicated are the values at collapse, and the plastic moments of resistance and weights per unit length of the sections presumed to be available for the design are shown in Table 3. The complete work-sheet for this example is shown in

TABLE 3

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Full plastic moment | 33 | 54 | 80 | 120 | 180 | 270 | 400 | 600 | 900 |
| Weight per unit length | 5 | 7 | 9 | 12 | 16 | 21 | 28 | 37 | 50 |

*Fig. 10*

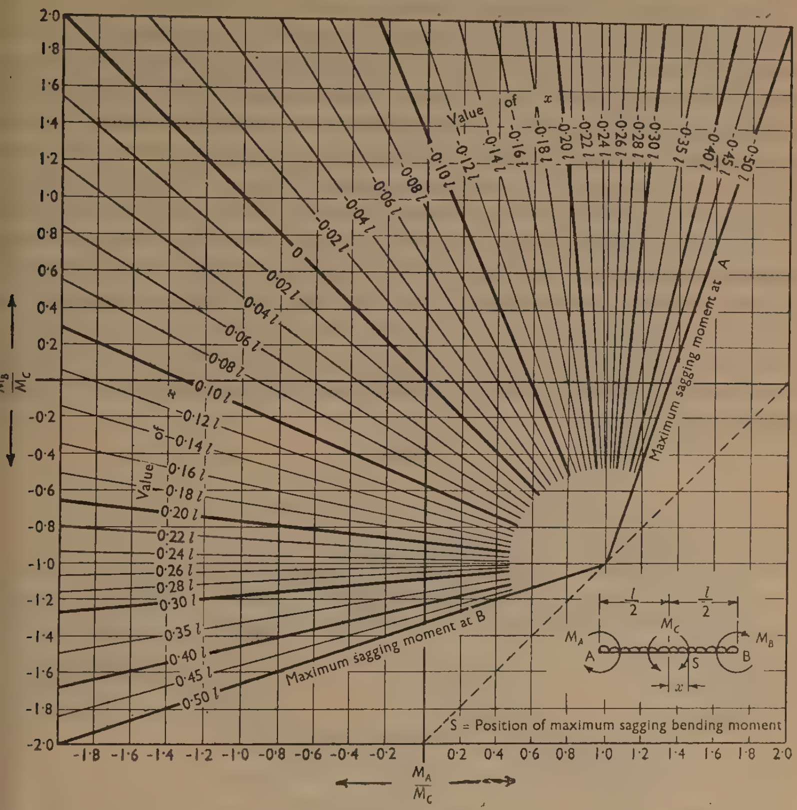

Value of x
0
0·02 l
0·04 l
0·06 l
0·08 l
0·10 l
0·12 l
0·14 l
0·16 l
0·18 l
0·20 l
0·22 l
0·24 l
0·26 l
0·28 l
0·30 l
0·35 l
0·40 l
0·45 l
0·50 l
-0·02 l
-0·04 l
-0·06 l
-0·08 l
-0·10 l
-0·12 l
-0·14 l
-0·16 l
-0·18 l
-0·20 l
-0·22 l
-0·24 l
-0·26 l
-0·28 l
-0·30 l
-0·35 l
-0·40 l
-0·45 l
-0·50 l
Maximum sagging moment at A
Maximum sagging moment at B
$\frac{M_B}{M_C}$
$\frac{M_A}{M_C}$
$\frac{l}{2}$
$M_A$
$M_C$
$M_B$
A
B
S
x
S = Position of maximum sagging bending moment

*Fig. 11*

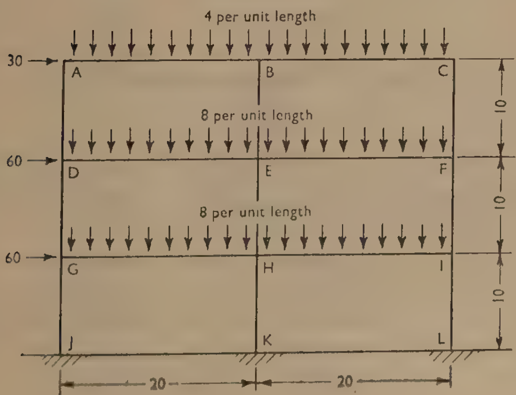
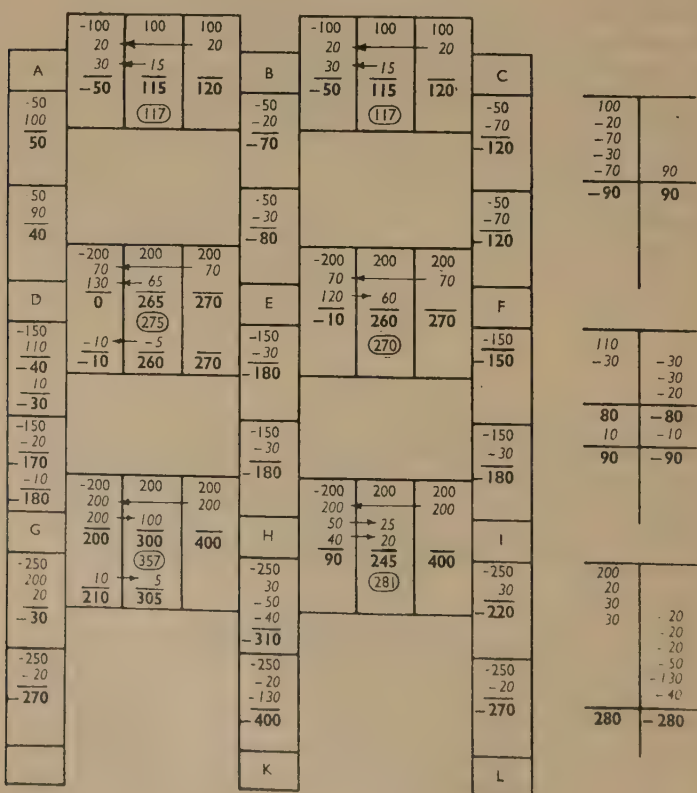

4 per unit length
8 per unit length
8 per unit length
30
60
60
A
B
C
D
E
F
G
H
I
J
K
L
10
10
10
20
20

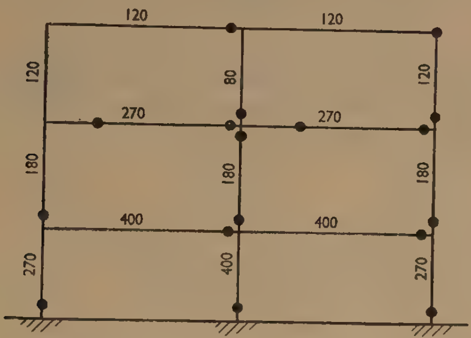
*Fig. 12*, and the resulting full plastic moments and positions of plastic hinges in *Fig. 13*. The form of setting-out the calculations in *Fig. 12* is recommended as more compact than that used for Examples 1, 2 and 3, once the procedure of plastic moment distribution has been fully understood. Joints A, B, C, etc. in *Fig. 11* are represented by corresponding positions in *Fig. 12*, and the moments at the joints are entered in the surrounding panels. Thus the initial moments at joint E are 200 in beam DE; — 200 in beam EF; — 50 in stanchion EB; and — 150 in stanchion EH. During the distribution process, the numerical values of the moments at the centres of the beams are kept slightly below the maximum end moments to allow for the effect of the distributed loads. The maximum sagging moments are calculated (with the help of *Fig. 10*) towards the end of the design process, and are shown ringed in *Fig. 12*. If the value so calculated is greater than the full plastic moment of the beam, the necessary redistributions of bending moments are made, as for beam DE in *Fig. 12*.

*Fig. 12*



The total weight of the structure according to the design given in *Fig. 13* is 3,950 units. This same problem was set to a number of students, who obtained designs varying in weight between 3,770 and 4,030. The range of variation in total weight was thus only 7 per cent, indicating that all the designs were reasonably economical.

*Fig. 13*




It is to be noted that the above procedure does not take account of eduction in full plastic moments due to axial loads, instability of stanchions, nd moments introduced by sway deflexion. Allowance for the reduction f full plastic moments due to axial loads may readily be made during the esign process by reducing the moments at the hinge positions accordingly. onsideration of the other two factors is beyond the scope of the present aper.

## Calculation of Collapse Loads

The plastic moment distribution process just described enables suitable lastic moments of resistance to be assigned to a structure supporting a nown set of loads. The process is a flexible one, and it is generally ossible to derive an infinite number of structures which would just collpse under the given loads. A similar method may be used to determine e loads under which a structure of given moments of resistance would st collapse.

If, for example, it is intended to determine the load factor at collapse a structure with given full plastic moments of resistance $M_A$, $M_B$, c. at sections A, B, etc. respectively, it is possible by means of plastic oment distribution to find a large number of sets of moments of resistce $M_A{}^1$, $M_B{}^1$, etc., which when substituted for the actual moments of sistance would just cause collapse to occur at the working loads. If the sign process described above is then manipulated so that :

$$\frac{M_A}{M_A{}^1} = \frac{M_B}{M_B{}^1} = \ldots \text{etc.} = \lambda,$$

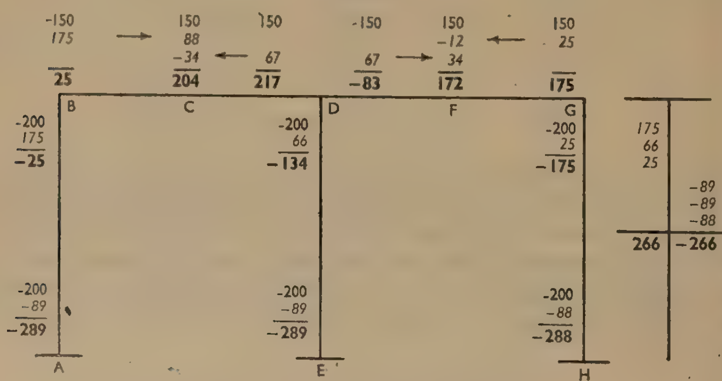
then it is apparent that the actual structure would collapse at $\lambda$ times the working load, and hence $\lambda$ is the collapse-load factor required.

The derivation of collapse loads for a given structure thus entails carrying out the moment distribution in such a way that the moments of resistance finally derived are in constant ratio to the moments of resistance of the given structure at those sections at which plastic hinges occur. The process will be described by reference to a numerical example.

*Example 5*

The frame shown in *Fig. 4* is subjected to the working loads indicated, and has a uniform full plastic moment of 300 ton-inches. It is required to find the load factor at collapse.

*Fig. 14*

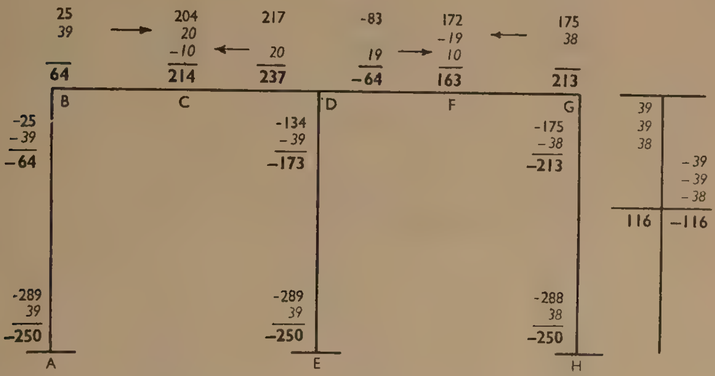



The plastic fixed-end moments (in ton-inches), assuming working loads, are first set down, as shown in plain type in *Fig. 14*. The joints B, D, and G are then balanced. As a general rule, when deriving collapse loads, in the absence of indications to the contrary, out-of-balance moments should be distributed between the members meeting at a joint in proportion to their full plastic moments. Adopting this rule, the joints are balanced as shown by the figures in italics on *Fig. 14*. The balancing moments in the beams are in turn distributed to the centre of each beam whilst the resulting out-of-balance sway moment is distributed equally between the feet of the stanchions. The resulting total moments are shown in bold figures.

Since the structure under consideration has a uniform full plastic moment, the maximum moments in *Fig. 14* are required to be equal, and must represent sufficient plastic hinges for collapse to occur. Actually, the moments at the feet of the stanchions are considerably greater than the other moments, and this indicates that the former should be reduced. Since no other bending moment is greater than 217 ton-inches, it is worth

while trying the effect of reducing the moments at A, E, and H to 250 ton-inches, and for the sake of clarity this step is shown in *Fig. 15*. The resulting out-of-balance sway moments are distributed equally between the tops of the stanchions at B, D, and G. The joints are then balanced this time by taking out-of-balance moments entirely into the beams in order to avoid a further out-of-balance sway moment.

*Fig. 15*




The moments resulting from the above process still fail to represent a state of collapse, since the maximum bending moment of 250 ton-inches occurs only at the three sections A, E, and H. The moment distribution obtained satisfies, however, the conditions (1) and (3) given on p. 52, so long as the load factor is not greater than $\frac{300}{250} = 1{\cdot}200$. Hence if $\lambda_C$ is the load factor at collapse, it is certain that $\lambda_C > 1{\cdot}200$. A closer estimate of $\lambda_C$ might be obtained by further reducing the moments at the feet of the stanchions, and continuing the distribution process until the moments did represent a state of collapse. Alternately, the process may be very much accelerated at this stage by using the derived bending moments to deduce the probable collapse mechanism. It is assumed, for this purpose, that plastic hinges will occur at the sections with the highest ratios of bending moment to moment of resistance of the original structure. In the present case, since the structure has a uniform full plastic moment, it is only necessary to number the bending moments in descending order of magnitude (see *Fig. 16*) until sufficient hinge positions have been found to transform it into a mechanism. It should be noted that the bending moments at the hinge positions must be in directions agreeing with the relative rotations at the hinges of the mechanism, and in order to ensure this, it is convenient to set down, as in *Fig. 16*, the direction of the rotations which are allowable. The present example requires seven hinges, and the mechanism shown in *Fig. 17* is thus obtained. Assuming that this is the

*Fig. 16*

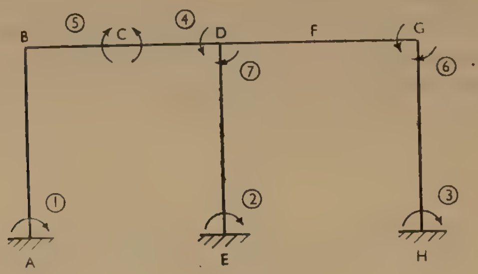
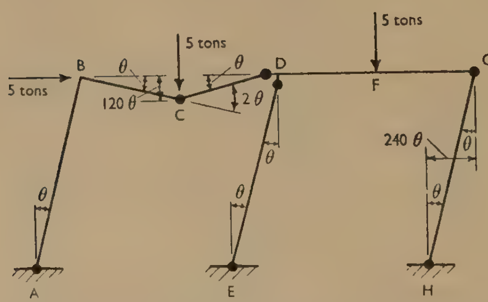



*Fig. 17*




correct mechanism, the application of the work equation to the original structure gives

$$5\lambda \times 240\theta + 5\lambda \times 120\theta + 5\lambda \times 0 = 8 \times 300\theta$$

that is, $$\lambda = 1{\cdot}333$$

The above value of $\lambda$ is based on an assumed mode of failure, and hence gives a value of the loads greater than or equal to the loads at collapse. Hence at this stage it is known that:

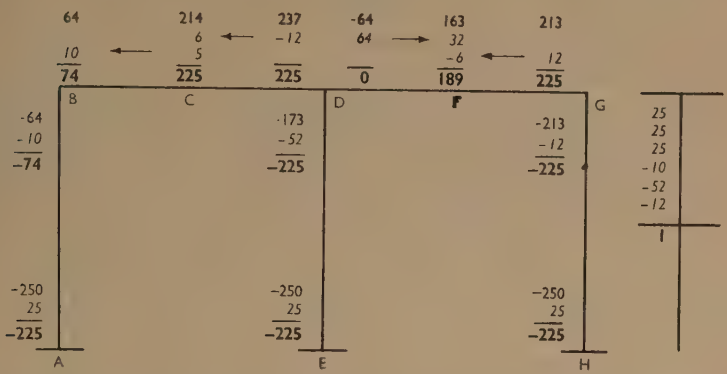
$$1{\cdot}200 < \lambda_C \leqslant 1{\cdot}333$$

The bending moments deduced in *Fig. 15* are now modified to correspond to the mode of collapse given in *Fig. 17*. The plastic hinge value becomes $\dfrac{300}{1{\cdot}333} = 225$ ton-inches, and so all moments at the assumed hinge positions are modified to agree with this value. This process is shown in *Fig. 18*. When all the additional bending moments have been balanced and distributed, it is found that the bending moment nowhere exceeds the full plastic value of 225 ton-inches. Hence the assumed mode of collapse is correct, and $\lambda_C = 1{\cdot}333$.

It will be realized that the above method of calculating collapse loads is not automatic. Some skill is required during the distribution process in order to bring the plastic moments at the hinges into the correct ratio.

The basic distribution process is, however, simple, and the necessary skill is soon acquired after a little practice. It is not usually necessary to carry the distribution process far before a good indication of the correct mode of collapse is obtained. In the preceding example the first trial was successful.

*Fig. 18*



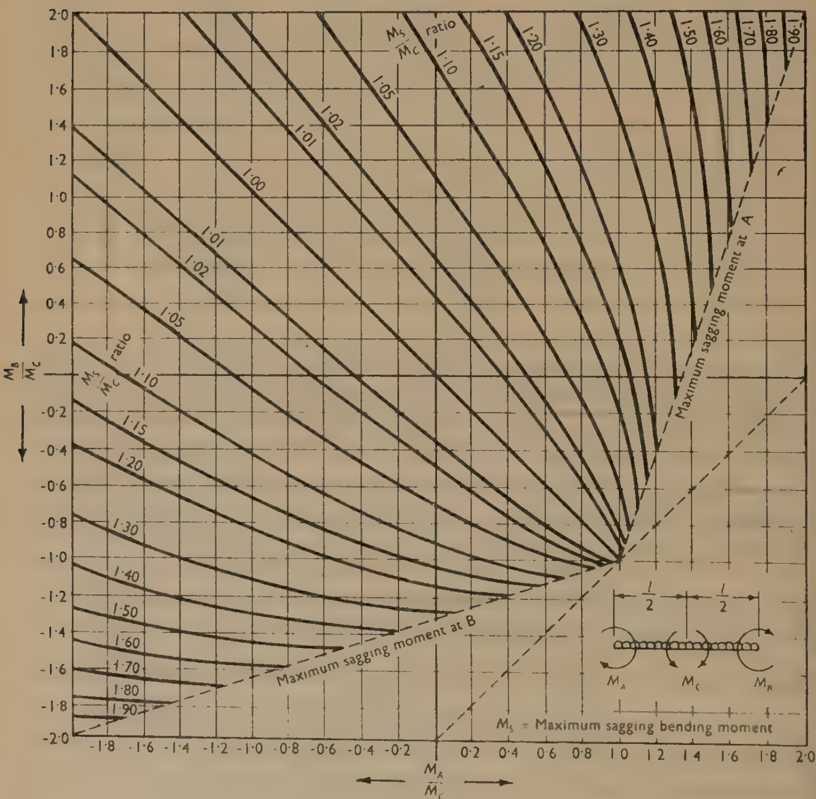

If the mode of collapse assumed is actually incorrect, this will be revealed when the bending moments have been modified by moment distribution to agree with the full plastic moments derived from the work equation. It will then be found that the full plastic moment has been exceeded elsewhere in the structure, and this will lead to a revised estimate of the positions of the plastic hinges. Thus, had an attempt been made to derive the mode of collapse in the above example from the equilibrium moments given in the bold figures in *Fig. 14*, a mode of collapse identical with that shown in *Fig. 8* (*b*) would have resulted, with the exception that no hinge could occur at D in stanchion DE. This gives $\lambda = 1{\cdot}375$ and a hinge moment of $\dfrac{300}{1{\cdot}375} = 218$ ton-inches. When the relevant moments in *Fig. 11* are modified to agree with this value, it is found that the moment at D in stanchion DE is 273 ton-inches. This indicates that there should be a hinge at this section, thus allowing DF to remain horizontal. The correct mode of collapse as given in *Fig. 17* is thus finally obtained.

When it is desired merely to confirm the ability of a structure to support a set of loads at a given load factor, and there is no need to determine the load factor at collapse, it is convenient to use the bending moments corresponding to the factored loads. The distribution is then carried out so that the maximum bending moment in any member is limited to the plastic moment of resistance of that member. If all out-of-balance moments can be balanced and distributed within this restriction, then the structure will support the given loads. Thus, if it had been desired to check the ability of the structure shown in *Fig. 4* to support the given loads at unit load

factor when the full plastic moment of resistance had the uniform value 300 ton-inches, it would be unnecessary to carry the distribution process beyond the stage reached in *Fig. 14*, in which the maximum resultant bending moment is 289 ton-inches.

Distributed loads are dealt with as already described in the section on design of plane frames. It is necessary to calculate the value of the maxi-

*Fig. 19*




mum sagging bending moments when deriving a safe (lower) limit for the load factor, and, for evenly distributed loads, the chart shown in *Fig. 10* may be applied. When deducing the collapse mechanism, it is also necessary to know the position of the maximum sagging moment, and for this purpose, the chart shown in *Fig. 19* may be useful. This also deals with evenly distributed loads. The horizontal and vertical axes have the same scales as in *Fig. 10*, but the contours give the distance of the maximum sagging moment from the centre of the beam, as indicated in the inset diagram.

## Conclusions

The plastic moment distribution process affords a direct means of assigning suitable full plastic moments of resistance to the members of a structure required to support a given set of loads. The process is a flexible one, and the solution obtained depends upon the way in which the out-of-balance moments are distributed. This flexibilty is bound to arise, since it is generally possible to choose the plastic moments of resistance of the members of a structure in an infinite number of ways, such that the structure would just collapse under given loads. It is, however, possible to deduce upper and lower limits for some of the required moments of resistance, and a convenient design procedure is first to assign suitable practical sections to the members concerned. The moments are then redistributed so that full use is made of these members, after which suitable sections may be chosen for other members. This process is continued until the whole structure has been designed.

The above procedure involves the exercise of considerable choice, and it is justifiable to enquire whether it is possible, in any one case, to deduce the design giving the least total weight. This problem has been investigated, and it is found that plastic moment distribution does supply one means of obtaining the most economical design. This application has not been described here for reasons of space. It is found, however, that the loss in economy as compared with the theoretically most economical design is usually small, provided that the full plastic moment is reached in at least one section of each member at the collapse load. The designs derived by the plastic moment distribution process will always satisfy this latter condition, and there will usually be only a secondary advantage in seeking the frame which has the theoretical minimum weight.

When a structure may be subjected to various combinations of loads, plastic moment distribution still provides a direct design method. It is assumed that the various load combinations will not alternate a sufficient number of times to introduce complications caused by shake-down effects. The moment distribution is performed for all loading combinations simultaneously, suitable full plastic moments of resistance being chosen for each member in turn.

The plastic moment distribution process may be used for deducing the collapse loads of given structures. It has the advantage over other methods of analysis that, at any stage before the final result is obtained, it is possible to make a safe estimate of the carrying capacity. The process is much accelerated by using the bending moments derived at any stage to estimate the mode of collapse. The application of the work equation then gives an estimate of the collapse load which is either equal to or greater than the correct value. The accuracy of the derived mode of collapse may then be checked by moment distribution; if the mode of collapse is incorrect, the redistributed moments provide an improved safe estimate of the collapse

load and also reveal the amendments which should be made in the assumed mode of collapse.

The methods described in this Paper have been illustrated by numerical examples which have been kept simple in order to clarify the exposition. These methods are, however, capable of dealing with much more complicated problems. As in the case of elastic analysis by moment distribution, the advantages of plastic moment distribution over other methods of design and analysis are more apparent the more extensive the frame under consideration.

The work described in this Paper was carried out at the Engineering Laboratory, Cambridge University, under the direction of Professor J. F. Baker, Head of the Department of Engineering. It forms part of a general investigation into the behaviour of steel structures in the plastic range being carried out at Cambridge with the assistance of the British Welding Research Association and the Department of Scientific and Industrial Research.

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The Paper is accompanied by 12 sheets of diagrams, from which the Figures in the text have been prepared.

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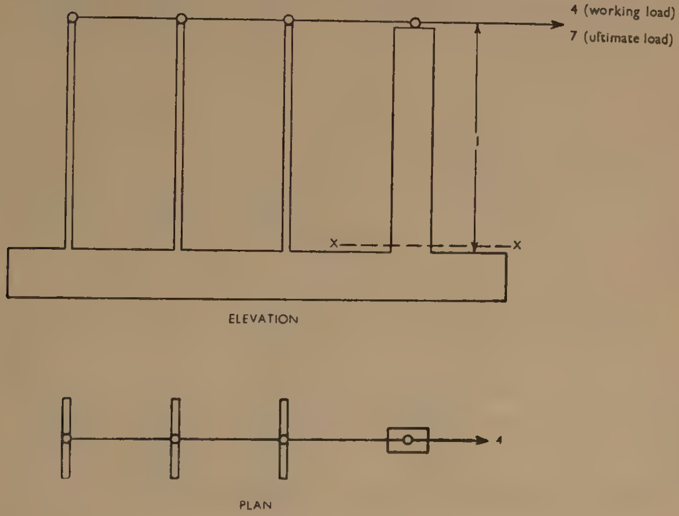
## Discussion

Professor **A. L. L. Baker** observed that the plastic-hinge theory facilitated a true determination of the real strength of a structure and of the real value of the factor of safety; it avoided a number of unreal

assumptions which had to be made in the elastic theory; it simplified calculations and secured economy.

Turning to the Paper itself, he could not yet entirely agree to the abandonment of strain compatibility and a consideration of approximate deformations, deflexions, and stresses under working load. As a simple illustration he sketched *Figs 20*, which showed the elements of a structural

*Figs 20*




| | | | | |
|---|---|---|---|---|
| $M_u$ | 1·75 | 1·75 | 1·75 | 1·75 |
| $I$ | 1 | 1 | 1 | 3 |
| $M_E$ | 0·67 | 0·67 | 0·67 | 2 |

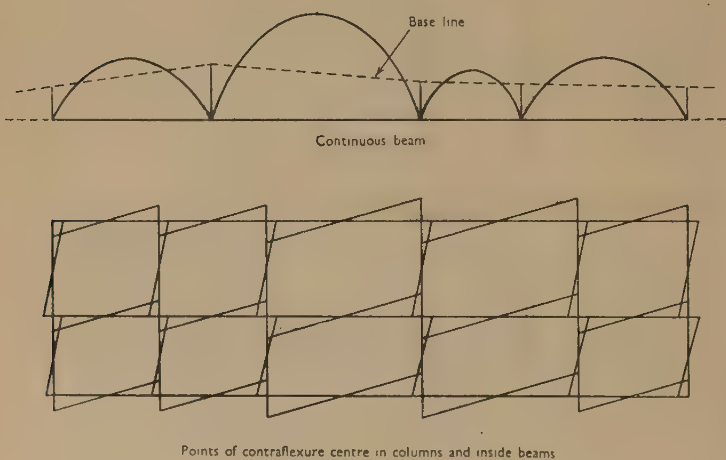
$M_u$ Denotes ultimate moment of resistance
$M_E$ " bending moment for elastic conditions
$I$ " second moment of area

system consisting of four columns. Each column had a load factor of 1·75 and a rectangular cross-section, the fourth one having the longer side at right angles to that of the other three. He would assume that a force of 4 units was applied along the tops of the columns and that the lever arm was unity, whilst the second moments of area were in the ratio 1 : 1 : 1 : 3, so that the elastic moment ($M_E$) distribution would be $\frac{2}{3} : \frac{2}{3} : \frac{2}{3} : 2$. It would be seen that if the load factor was 1·75 under working load the right-hand column would fail, or at least it would yield plastically, and if the force were repeatedly reversed in direction then gradually

there might be failure at the section XX. That was an extreme case, but it was a pointer to the kind of thing that might happen in a complex framework unless the distribution of bending moments and stresses for the elastic case were examined. It was only necessary to be approximate for the purpose.

Reinforced concrete engineers had usually been brought up on or converted to the use of Müller-Breslau's general elastic equations and, adapting those equations for the plastic-hinge case, they could check with the aid of simple tabulation whether hinge rotations were excessive, and also by a simple method of trial and adjustment, derive an approximate elastic

*Figs 21*




solution, and so find out fairly quickly whether there was a risk of the sort of occurrence which Professor Baker had described.

Structural steel was different from reinforced concrete and investigating a framework only at the final stage of failure as a mechanism might be satisfactory, but he was not content. For instance, in the case of a building frame, it seemed to him that once the step had been taken of calling the ultimate load the load which was applied when the structure became a mechanism, there was no arguable case against designing, for instance, a continuous multi-span beam by simply drawing a base-line through the bending-moment diagram for freely supported beams to give the best distribution of moments for economy in design and providing sections accordingly (see *Figs 21*), and, if the beam were one of a framework subject to sway forces, drawing-in the sway bending moments on a basis of central points of contraflexure in the columns and the inside beams (see *Figs 21*). With the correct or greater plastic hinge values at the appropriate sections

that structure would resist the required or greater ultimate load, or was there a fallacy involved?

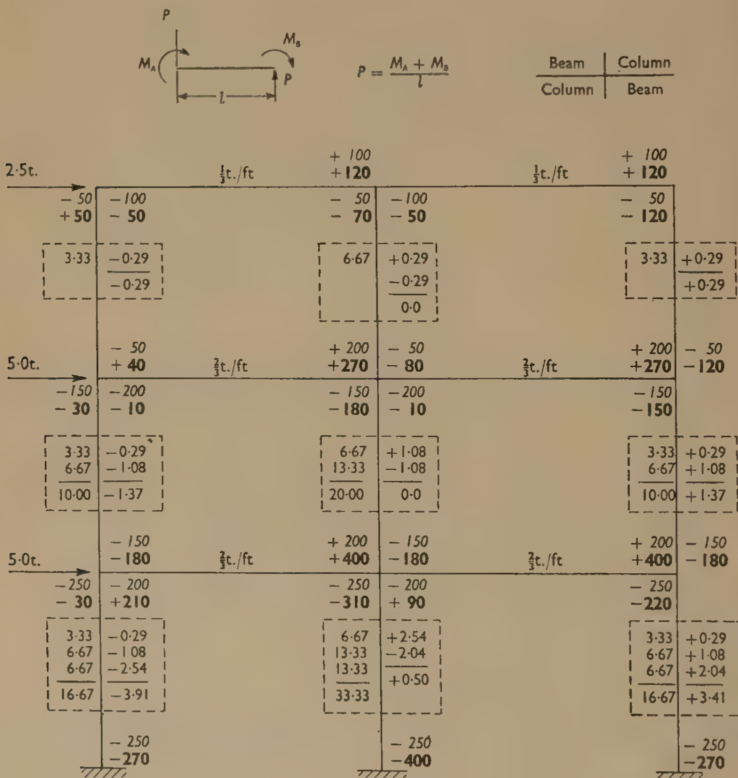
He thought that for a great many frames used commonly in practice, once the step had been taken of ignoring strain compatibility it was possible to make such simple assumptions and design accordingly without using moment distribution. Reinforced concrete engineers in Continental countries were very much in favour of using ultimate load theory, and he thought it must come in the future.

**Mr K. G. Eickhoff** referred to the Author's statement (p. 69) that "Allowance for the reduction of full plastic moments due to axial loads may readily be made during the design process. . . ." It had been well known for some years that the full plastic moment which a member could stand was dependent upon the axial thrust which it carried. It could easily be shown that it was possible to draw a curve relating the fully plastic moment to the end thrust; the moment would be maximum when the end thrust was zero, and zero when the end thrust was equal to the yield stress multiplied by the known area of cross-section.

In order that due allowance might be made in the design process, it was necessary to form some estimate of the load in the stanchions, and that could be done by adding another column of figures to the Author's method. For comparison, Mr Eickhoff had chosen the frame given in *Fig. 11* and had converted the loads into tons. The additional columns of figures were shown within the broken-line enclosures in *Figs 22*. Considering the top storey, where the beams were uniformly loaded and treated initially as encastré, the reactions at each end of a beam were the same, and those reactions were put down in columns on the left of the stanchions. Those could be carried down to the lower stanchions, and similarly for the next storey down and the next. Although starting, for convenience, with a beam with equal end moments, those moments were altered during the distribution process, and the result would be a beam with an additional shear in it equal to the sum of the bending moments at either end, divided by the length of the beam. It was therefore possible from that relationship to determine the additional reaction at each end of the beam corresponding to the alterations in end moments made during the distribution process, and it was convenient to place those additional reactions in the columns to the right of the stanchions. The moments at the ends of a top-storey beam were originally 100 tons-inches and $-$ 100 tons-inches, giving zero sum, and after the distribution process they were $+$ 120 tons-inches and $-$ 50 tons-inches, giving a total of $+$ 70 tons-inches. If that was divided by the length of the beam a figure of 0·29 tons was obtained. That could be done for the other beams and carried down to the other storeys. By adding the contents of the two columns on either side of the stanchions the final load in the stanchions was determined. The process was conveniently left until the distribution process was finished, and the final result would show whether the estimated fully plastic moment of the

section would be adequate when allowance was made for the axial load. In the case considered, taking the central stanchion, the axial load reduced the full plastic moment by 10 per cent, so that it was worth while making the correction.

*Figs 22*




**Mr N. S. Boulton** remarked that the case for plastic moment distribution had been well substantiated in the Paper. The Author had compared his new moment-distribution method with that used for elastic structures and had pointed out that the two methods were not in any way identical. In addition to the points of difference which the Author had mentioned, the plastic method, unlike the elastic method, involved a finite number of steps—quite a small number, judging by the examples given in the Paper—and also difficulties arising from slow convergence, which sometimes arose in the elastic method, were absent in the plastic method. On the other hand, whereas the elastic method was almost automatic, as the Author had pointed out in his introductory remarks, the new method

appeared to require considerable thought and skill throughout the process, since it allowed, as he had shown, a considerable degree of choice.

The great merit of the Author's method was that it was a direct design process in which convenient practical sections could be assigned to the members within a possible range of full plastic moments which was determined in the process. As used for calculating the collapse load of frames of members of given dimensions it seemed less simple, as the Author had mentioned in his introduction, than the method of Neal and Symonds, in which modes of collapse were combined until the least load factor was obtained. However, Mr Boulton felt that a valuable feature of the Author's method was that it could be used to indicate the probable collapse mode before the final solution was obtained. Moreover, the moment distribution Table could be at once used to verify that the full plastic moments were nowhere exceeded, instead of having to calculate the moments at points where plastic hinges did not occur, in order to verify that the final mode of collapse was correct.

In his "Conclusions" the Author had stated that "there will usually be only a secondary advantage in seeking the frame which has the theoretical minimum weight," since the loss of economy was usually small in structures designed by the procedure advocated in the Paper. The saving of even a few per cent in weight, however, might be worth while in a large structure, and it was therefore desirable to have a simple guiding principle to minimize the total weight during the moment distribution process. The Author evidently had such a method, although for reasons of space he did not describe it. It would be of interest if he would indicate the principles of it in his reply to the discussion.

The method described by Dr Foulkes in a recent Paper [9] seemed to be suitable for use with moment distribution, and, as Dr Foulkes could not be present, Mr Boulton would like to mention the essential steps of the method. Having found the full plastic moments of members to support the given loads by the Author's (Dr Horne's) method, those moments were varied two at a time so as to increase the load factor as judged from the virtual work equation while keeping the total weight constant. The collapse mode and the corresponding load factor for those modified moments was then found by the procedure described on p. 70. The full plastic moments were then reduced in a constant ratio so as to restore the load factor to unity. Other modifications to the full plastic moments were made in the same way until the minimum weight was obtained. Would the Author recommend that method of minimizing the weight by moment distribution, or did he know of a better method?

**Dr B. G. Neal** referred to a case which he thought could be treated only with some difficulty by the Author's method. Basically, that

[9] J. Foulkes, "Minimum Weight Design and the Theory of Plastic Collapse." *Quart. J. Appl. Maths*, vol. 10, p. 353 (Jan. 1953).

method was to write down against the frame a set of bending moments which satisfied immediately all the equilibrium requirements except those at the joints. The next step was to balance the joints while preserving the other equilibrium conditions which were already satisfied, and finally further adjustments were made to satisfy all the equilibrium requirements, until a suitable design was found.

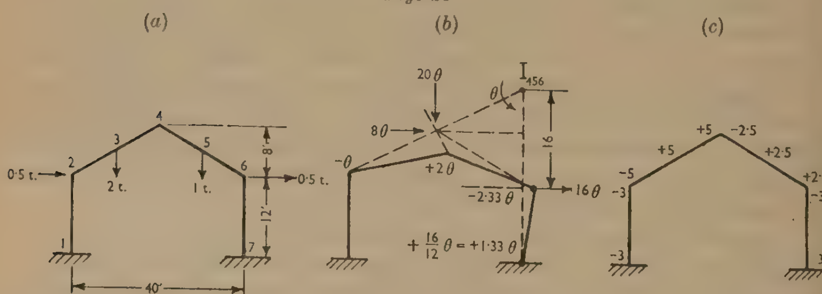
In the structures dealt with in the Paper, which were all of rectangular form, the equilibrium requirements could be stated very simply, as follows :—

1. Equilibrium of each beam.
2. Sidesway equilibrium of each storey.
3. Rotational equilibrium of each joint.

For a pitched roof portal, however, the formulation of the equilibrium requirements was a matter of some difficulty, as he proposed to illustrate by a particular example.

*Fig. 23* (*a*) showed a simple portal problem. The first step was to

*Figs 23*




determine the correct number of equilibrium requirements, and Dr Neal did that by observing that if the bending moments at the seven numbered points in *Fig. 23* (*a*) were given it was possible to draw the bending-moment diagram for the entire frame. A frame of that type had three redundancies, so that there had to be four equilibrium relations between the seven unknown bending moments. Three of those relations could be written down without difficulty, as follows :

$$2M_3 - M_2 - M_4 = 20 \quad . \quad . \quad . \quad . \quad (1)$$

$$2M_5 - M_4 - M_6 = 10 \quad . \quad . \quad . \quad . \quad (2)$$

$$M_2 - M_1 + M_7 - M_6 = 12 \quad . \quad . \quad . \quad . \quad (3)$$

The fourth equation was most conveniently written down as a connexion between the moments of $M_2$, $M_4$, $M_6$, and $M_7$. Dr Neal thought that the easiest way of deriving that relation was by the application of the principle of virtual work. Considering a mechanism with hinges (not

plastic hinges) at the cross-sections 2, 4, 6, and 7, an equilibrium equation could be derived by equating the virtual work done by the loads to the virtual work absorbed by the hinges. The kinematics of the mechanism was best solved by noticing that the apex 4 was constrained to rotate about the end 2 of the rafter member, whereas the top of the stanchion at 6 was constrained to move horizontally. The instantaneous centre of rotation of the rafter member 456 was therefore at $I_{456}$, as was shown in *Fig. 23* (*b*). If the rafter member 456 was given a small rotation $\theta$ about its instantaneous centre, then the top of the right-hand stanchion would move $16\theta$ feet horizontally, and therefore the rotation at the hinge at the bottom of that stanchion would be $16\theta/12 = 1{\cdot}33\theta$. By a similar argument it could be seen that the hinge rotation at cross-section 2 was of magnitude $\theta$. Then, since the left-hand rafter rotated clockwise through an angle $\theta$, and the right-hand rafter rotated anti-clockwise through the same angle, the rotation of the hinge at the apex must be $2\theta$. The vertical downward movement of apex 4 was seen to be $20\theta$ feet.

It was then possible to write down the virtual work expression for that mechanism:

$$-\theta M_2 + 2\theta M_4 - 2{\cdot}33\theta M_6 + 1{\cdot}33\theta M_7$$
$$= (10\theta \times 2) + (10\theta \times 1) + (16\theta \times 0{\cdot}5)$$

which, after rearrangement yielded the fourth equilibrium equation:

$$-M_2 + 2M_4 - 2{\cdot}33M_6 + 1{\cdot}33M_7 = 38 \quad . \quad . \quad . \quad (4)$$

If the procedure outlined in the Paper was to be followed, it would be necessary to write in along the left-hand rafter three moments of magnitude 5, at the ends and at the centre, that value being obtained from equation (1) by making $M_3$, $M_2$, and $M_4$ equal in magnitude. The left-hand rafter would be entered similarly with moments of magnitude 2·5 obtained from equation (2), and finally moments of magnitude 3 would be written against the tops and bottoms of the stanchions, the value 3 being obtained from equation (3), the equation of side-sway equilibrium. Reference to *Fig. 23* (*c*) showed that a complete set of moments had been obtained, but the fourth equilibrium equation was not yet satisfied. Thus the method explained in the Paper was not directly applicable to that type of problem.

In conclusion, Dr Neal made the general point that since it was possible to derive the equilibrium requirements through an essentially kinematic process, as he had done in the example which he had just given, it must follow that there was an extremely close connexion between the statical approach outlined in the Paper and the kinematic approach which had been explained elsewhere,[6] and in a sense it was misleading to think of those approaches as fundamentally different. He thought that they were fundamentally very much the same.

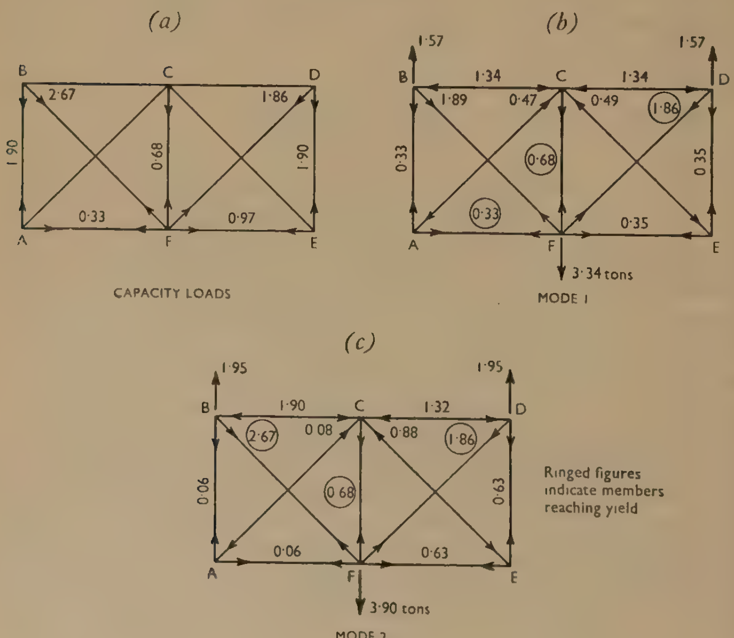
**Dr R. W. Steed** said that many practising engineers welcomed the underlying philosophy of the plastic theory, but ideally that philosophy should embrace all types of structure and not just building frames. For

some time he had been interested in the application of the plastic theory to triangulated structures.

On p. 52 there were given three requirements of a bending-moment distribution representing a state of collapse. When considering triangulated structures those requirements might be re-written thus:

(1) The equilibrium condition (the force distribution to be in equilibrium with the applied loads).
(2) The collapse mechanism condition (existence of sufficient plastic members for either the whole structure or part of it to become a mechanism).
(3) The yield condition (the capacity load of any individual member should nowhere be exceeded).

*Figs 24*




It would be noticed that in the first requirement he had replaced "bending-moment distribution" by "force distribution." In the second, "plastic hinges" had been replaced by "plastic members." In the third requirement, "the full plastic moment" had been replaced by "the capacity load of any individual member."

In connexion with research on the application of plastic theory to such structures he had encountered the following problem. Considering the

triangulated pin-jointed truss shown in *Fig. 24* (*a*) the capacity loads of the tension members were as indicated. The load capacities of the compression members were large in comparison with the load capacities of the tension members and were omitted from the Figure. In *Fig. 24* (*b*) a likely mode of failure was shown which he had called Mode 1. The mode of failure was that the member AF, the member CF, and the member DF had reached yield, and all three requirements which he had given earlier had been fulfilled. The frame was in equilibrium and if the mode of failure was correct, three members had reached yield. Nowhere had the load capacity of any individual member been exceeded and upon the application of further load large deformations of the truss would develop. Was 3·34 tons to be taken as the value of the collapse load for that structure ?

Another mode of failure, Mode 2, was shown in *Fig. 24* (*c*), and there the three members to yield were BF, CF, and DF. That gave a collapse load of 3·9 tons, and the three requirements had also been met. Which was the true collapse load, 3·34 tons or 3·9 tons ? If the two modes were realized, then according to the proposition that a structure would support the maximum load it possibly could, the collapse load was 3·9 tons. That was subsequently found to be correct by testing a small-scale pin-jointed truss to destruction. There was a danger, however, without an experimental test or an elasto-plastic analysis, that only the theoretical Mode 1 would be discovered ; it satisfied the three requirements, and so, incorrectly, 3·34 tons might be given as the collapse load for that truss.

Had the Author any experience of the application of plastic theory to triangulated structures, and was there another requirement, in addition to the three mentioned above, needed for that type of structure ?

**The Chairman** said that the Author had specified three requirements for a bending-moment distribution representing a state of collapse. It did not seem obvious to the Chairman that those conditions could always be satisfied : the Author had pointed out that it might be difficult to satisfy them simultaneously, and had said that they were more readily satisfied in pairs. The Chairman was not sure what was meant by that ; presumably they had to be satisfied simultaneously, and the Author's statement required clarification.

On p. 53 the Author had referred to a structure which might be subjected to two or more different loading combinations and the Chairman found difficulty in understanding the treatment. *Figs 9* showed two different load systems, and the final design had to be a compromise between the requirements of those. Was it not possible that part of the structure could collapse under load system $M_1$ and part under system $M_2$ ? How was that dealt with ? The same problem was presented in the elastic design of a redundant structure for two different load systems. It was possible to design directly for one load system, but he did not think that it was possible to do so for two different load systems.

In the example given in the Paper there were two different load systems,

but the points of application of the loads were the same in both (with an extra side load in one). What happened if a load, such as a crane, travelled across the structure ? Was it possible to deal with that problem ?

The Chairman also asked how, if yield had developed at one hinge, one could be certain of not going beyond the horizontal portion of the stress/strain curve there and so entering part of the curve for which the analysis was incorrect, while the other hinges were developing.

Professor A. L. L. Baker, who evidently found the subject very easy and straightforward, had stated that the Author's method would give economical design. The Chairman could not accept that as a general result, because it depended on the relative values taken for the load factor in " collapse " design and the factor of safety in elastic design. Having designed upon a certain load factor, was there any guarantee that, some portion of the structure would not yield under normal working loads or deflect so much as to cause serious troubles? There was no need for the Chairman to detail the obvious dangers of a too flexible structure in certain types of building.

It was obvious, as the Author had said, that there were an infinite number of solutions to the problem he had tackled, but what criterion would make it possible to choose the best ? Would it be necessary, having designed by the plastic method, to adopt a criterion involving the working conditions of the structure ? It was not sufficient to select the structure of the smallest weight, since that would be the one in which trouble would most likely occur under working conditions.

**Professor J. F. Baker** said that he would like to ask for more information regarding Professor A. L. L. Baker's remark about Continental activity in the field of plastic theory. It was to be hoped that Professor A. L. L. Baker was not starting a Continental hare, in addition to the American hare, by lending support to the suggestion that other countries were always ahead of Britain. No such impression had been gained by the exponents of plastic design, who went to the Continent from time to time to lecture on the subject. There was too great a tendency for national self-derogation in such matters. That was no longer amusing now that it was so important to attract students of technology to Britain.

**Mr R. S. Jenkins** remarked that there was one " aside " of the Author's with which he could not agree, although it did not affect the Paper itself. The Author had stated at the end of the Paper " As in the case of elastic analysis by moment distribution, the advantages of plastic moment distribution over other methods of design and analysis are more apparent the more extensive the frame under consideration." Mr Jenkins had had a fair amount of experience in the old-fashioned sort of analysis, and he thought that moment distribution became very tedious in complicated statically-indeterminate structures, and there were very much better methods. He had been going to say that since the plastic case was quite different, and distribution did not spread all over the structure, the

Author might be right in what he said with regard to plastic design, but, after listening to Dr Neal, Mr Jenkins began to think that that statement might not apply to plastic theory either, and that in complicated structures there might be better ways of doing it.

Mr Jenkins thought that the Author was being a little less precise than usual when he spoke of "elastic analysis"; Mr Jenkins thought a better term would be "linear analysis," because one could have non-linear elasticity.

The premises of linear theory were that the material obeyed Hooke's law and that the displacements were small. Fortunately, structural materials, although elastic, were not like rubber, and their strains within the working-stress range were such small ratios that that small-displacement premise answered for most structures, with some well-known exceptions. When considering plastic hinges, however, it was obviously impossible to accept that small-displacement premise without investigation. In many cases it made no difference, but he asked the Author if that was the qualification referred to on p. 69, where he had stated that he had not dealt with moments introduced by sway deflexion. Had the Author made any investigations on frames of the type to which he referred, where it might be that the structure could not support the load based on small displacements, since other moments were introduced because the displacement became large? That was but one matter among the many about which information was required before the plastic hinge theory for structural steel could be established for general use in practice.

**Mr G. P. Manning** referred to the fact that the Author had opened his introductory remarks with an apology for introducing yet one more method of structural analysis. Mr Manning could assure him that no such apology was necessary, because a variation of the Author's method had actually been in use amongst French engineers 40 years ago.

He thought that the Author should have made it quite clear in the Introduction that the Paper was intended primarily for steel. That was not in fact made clear until the last page, and much of what was said in the Paper would not apply to many structures in reinforced concrete, although the beginning of the Paper was couched in quite general terms. Mr Manning also gathered that it was intended primarily to apply to structures where the members were basically plain steel joists which were necessarily of uniform section from one end to the other, and where the practical criterion was the maximum bending moment at any point. It did not apply exactly to any member where the moment of resistance might be varied, such as a plated joist, where the total weight of the member was a combined function of the maximum bending moment and the total area of the bending-moment envelope.

Various attempts had been made to compare different methods of analysis. Mr Manning's view was that there was no one method of analysis which was best for all problems in all materials. Some were of

more universal application than others; some were very attractive in theory, in the lecture room, but proved to be very disappointing in the drawing-office. He did not know how many statically-indeterminate structures he had designed in his career, but it must run into hundreds. People asked him what methods he had used. He had tried most of them, including variations of the one now under discussion, which he did not much like. Various speakers had mentioned the factor of safety. Speaking as a mere observer, he would say that it was an imaginary number inversely proportional to the number of competitive tenders.

How did he now design the various statically-indeterminate structures? Everyone that evening seemed to be becoming more and more mathematical, but he used what might be termed a method of "exproximation." He first calculated, by his own method, the theoretical moments throughout the frame; he then proceeded to check a few approximate bending moments, $WL^2/16$, $WL^2/12$, and so on, and finished by putting the sections in by eye. There were therefore three distinct stages in his design, each one getting less and less mathematical. He could recommend that method, which had stood him in good stead for many years. The Author's method of assuming hinges and manipulating the bending moments was quite good within its own field, but, like the Chairman, Mr Manning felt that with a complicated frame one might get so lost that one might overlook some possible method of failure. He found that in all engineering problems the difficulty was not so much in solving any one specific problem, but in being able to state exactly what the problem was which had to be solved. It was quite possible in a complicated frame to overlook one of the problems which should be solved, and therefore to produce a frame which had not even a theoretical factor of safety.

**Mr A. L. Moss-Morris** referred to Mr Manning's opinion that possibly true economy did not come from the type of design which assumed that the members had a uniform plastic moment along their length, and said that that was particularly evident in continuous beams, which were subjected to a uniformly-distributed load. In that case, the moments in the span were approximately constant over a considerable length, while at the supports there was a rapid change in moment. By the method of design outlined in the Paper it was not necessary to know the modulus or area of section to be used, and those were actually determined from the design. He felt that if the support moments were generally made larger than the mid-span moments, a considerable saving could be effected. Since beams were normally spliced at their supports, their was no great difficulty in increasing the plastic moment of resistance of those short lengths above that of the rest of the beam. He was engaged on work similar to that of the Author but with reinforced concrete as the material, and even in that case, where the amount of ductility available was considerably less, the required amount of redistribution of moment could be obtained without excessive deformations.

Several speakers had asked if it was possible to be sure that at the design loads the deflexions of the structure would not be excessive. He thought that a simple criterion for that could readily be found. If all but one of the plastic hinges were placed at the joints between two members, and only that hinge required to transform the structure into a mechanism was put into the span of the members, the deflexions in that case must always be less than they would be for a structure made from simply-supported members, or for a framed structure with pin joints. He thought that that was fairly obvious, but, if not, it could be derived from the virtual-work equations.

Would the material be able to withstand the required strains, if, for instance, one plastic hinge was formed at a very low load and the other plastic hinges at considerably higher loads? He thought that the technique for obviating that, at least in beam members, was to make the span-moment/support-moment ratio not less than that for the corresponding encastré beam.

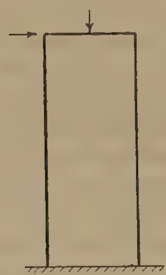
**The Author,** in reply, said that the chairman, Professor A. L. L. Baker, and Mr Moss-Morris had all raised the question of stresses and deformations at working loads. It might of course be necessary to consider such matters, just as in structures designed to a safe stress it was sometimes necessary to modify the design in order to avoid excessive deflexions. The first requirement of any design process—whether elastic or plastic—was that it should set forth some simple criterion of strength or suitability which might be expected to lead in general to a rational economical design. Long experience of orthodox design methods had enabled one to state with some confidence when it was necessary to calculate deflexions under working loads. That experience was not yet generally available for structures designed by the plastic theory, and it might at present be necessary to calculate deflexions at working loads more frequently than in structures designed to an elastic criterion. Since structures designed by the plastic theory used lighter sections, deflexions might be expected to be greater than in similar structures designed by elastic theory. Against that, however, had to beset the high premium placed in plastic theory design on the maximum use of rigid joints, and it was doubtful whether there would be any appreciable net increase in deflexions at working loads.

Mr Moss-Morris had put forward an argument relating to the deflexions of beams with which he was in entire agreement. It would be shown [10] that in multi-storey multi-bay frames with rigid joints and distributed beam loads the maximum bending moments in the beams occurred always at the ends. Plasticity would therefore first occur at the ends of the beams, the rest of the beams remaining elastic practically until the collapse load was reached. There was in fact reason to expect that the deflexions of such beams would be less—not more—than the deflexions of corresponding beams designed, according to orthodox methods, as simply supported.

[10] References 10 to 15 are given on p. 98.

Professor A. L. L. Baker had given an instructive example in which yield stress would be reached, and the full plastic moment developed, at working loads for a structure which had been designed by the plastic theory to a load factor of 1·75. Experience had shown that it was not usual for plastic designs to exceed the yield stress at working loads unless the structure contained, adjacent to each other, members of highly unequal stiffness. An example in which yield stress would most likely be exceeded at working loads was afforded by the tall portal frame shown in *Fig. 25.* In that example, in which yield stress would be exceeded at the feet of the stanchions, the high stresses at working loads would not pass unnoticed because of the obvious necessity of calculating the sway deflexion of the frame. In such a structure, the limiting deflexion, not the ultimate strength, would necessarily be the real design criterion. In those parts

*Fig. 25*



EXAMPLE OF FRAME IN WHICH YIELD STRESS MIGHT BE REACHED AT WORKING LOAD

of a structure where deflexions were not critical, some degree of plastic deformation at working loads might not be objectionable, since the structure would behave elastically after the first application of the maximum load. When the loads were of an alternating character, special considerations arose, but that subject was beyond the scope of the discussion.

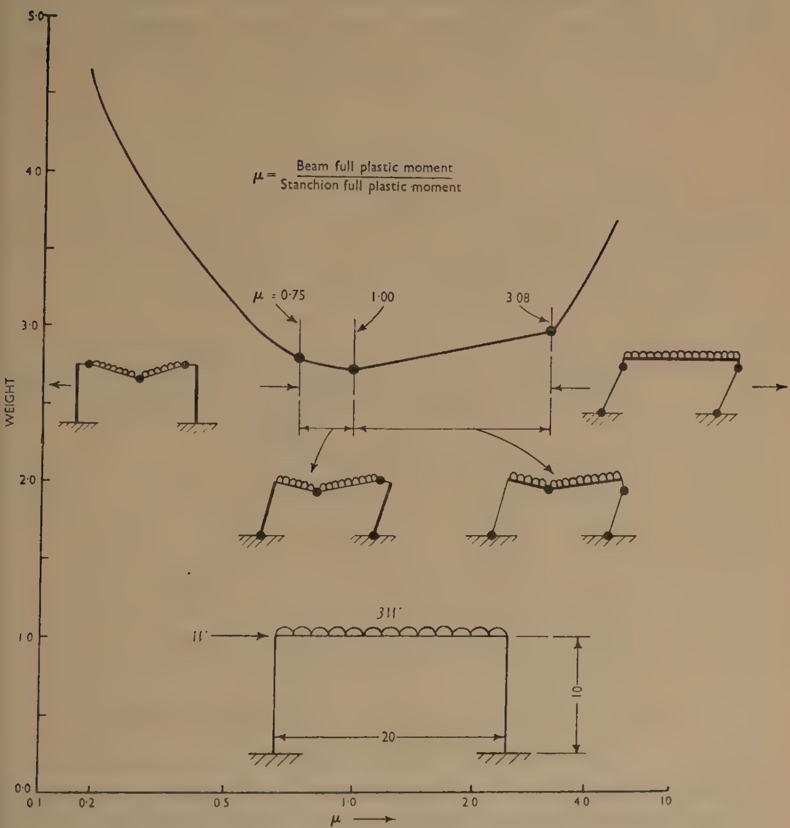
The question of excessive hinge rotations was of great importance in ultimate-strength theories for reinforced-concrete structures, but experience had so far shown that hinge rotations sufficient to enable the loads to reach the theoretical collapse values could take place in structures of mild steel.

Professor A. L. L. Baker appeared to suspect the design process of simply "drawing in" base-lines for continuous beams and multi-storey frames. According to the plastic theory that was quite a legitimate procedure, the only objection being that, for mild-steel structures composed of uniform members, the design would not in general represent a state of collapse at the factored loads, and would not therefore be "economical." The moment-distribution process described in the Paper was proposed as a

logical means of deriving economical structures composed of prismatic members, but it was not suggested that it was the only such method.

Professor A. L. L. Baker, Mr Manning, and Mr Moss-Morris had all raised the question of members of varying sections. Such members were not so readily obtained in mild steel as in reinforced concrete, but even for

*Figs 26*



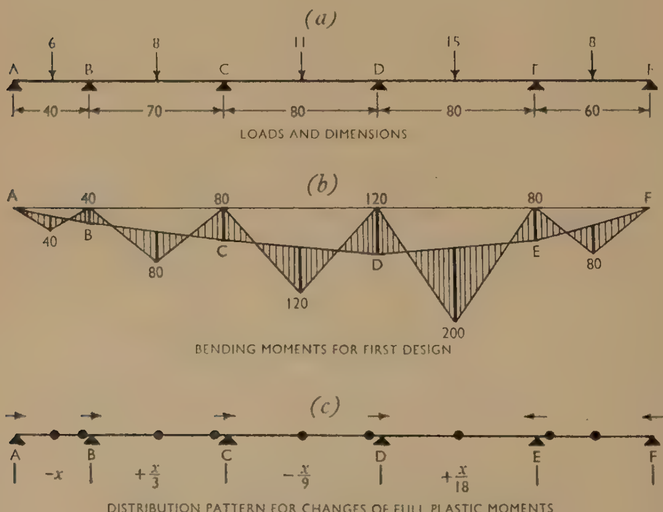

Weight Comparisons for Fixed-Base Portal Frame

teel it was a matter well worth investigation. The Author had himself onsidered the matter in some detail in another paper,[11] and had found hat, as Mr Moss-Morris had stated, considerable economy could be chieved by increasing the moment of resistance of the beams locally at the nds.

The Author said that Mr Eickhoff had made a very useful contribution in his suggested method of recording the axial loads in the stanchions. It was certainly convenient to have a means of recording on the same work-sheet all the essential information required in design.

Mr Boulton had raised the question of obtaining the minimum-weight design. *Figs 26* showed weight comparisons for a rectangular fixed-base portal frame subjected to a distributed beam load of 3 $W$ and a side load of $W$. It had been assumed in the calculations that the weight per unit length of a member was proportional to $M^{0 \cdot 6}$ where $M$ was the full plastic moment. The diagram showed the weight plotted vertically against $\mu$ horizontally, where $\mu$ was the ratio of the full plastic moment of the beam

*Figs 27*




EXAMPLE OF MINIMUM WEIGHT DESIGN BY MOMENT DISTRIBUTION

to that of the stanchions. Within the limits $\mu = 0 \cdot 75$ to $\mu = 3 \cdot 08$, there was very little variation in total weight, and the moment-distribution process would always lead to a design within those limits. That was a typical example illustrating the point made by the Author in the Paper that there was comparatively little advantage in seeking the absolute-minimum-weight frame, since the small potential saving in weight would, in practice, be overshadowed by other important considerations.

At the same time, the Author would agree that the problem of minimum-weight design deserved attention. Mr Boulton's suggestions for using moment distribution to obtain the minimum-weight frame were in

general agreement with the Author's proposals. Those latter proposals could be introduced by considering a continuous beam loaded as shown in *Fig. 27* (*a*). Suppose that moment distribution (or some other method) resulted in the design shown in *Figs 27* (*b*) and (*c*), where *Fig. 27* (*b*) showed the bending-moment distribution and *Fig. 27* (*c*) the positions of plastic hinges at collapse. Wherever a hinge occurred at the end of a span, an arrow was drawn above the support in a direction away from the span in which the hinge occurred. Then if the full plastic moment of the first span AB were decreased by $x$, it could readily be shown that the full plastic moment of the second span BC would, as a result, have to be increased by at least $\frac{x}{3}$. It would then be found that the full plastic moment of span CD could be decreased by $\frac{1}{3} \times \frac{x}{3} = \frac{x}{9}$, and so on, the process being allowed to continue so long as the changes could travel in the direction of the arrow. The "carry-over factor" for change of section was therefore, in general, $-\frac{1}{3}$. The span DE had an arrow in the opposite direction at end E, and the change of section was not carried over the support at E. The carry-over factor to the last span concerned, DE, was $-\frac{1}{2}$ in place of $-\frac{1}{3}$. If changes of full plastic moments according to the resulting "pattern" reduced the total weight of the frame, then a value of $x$ was chosen of sufficient magnitude to change the direction of one of the arrows. It was necessary to establish a number of rules in addition to those, particularly when dealing with multi-storey frames, but sufficient details had been given to make clear the basic principles of the process.

The Author said that, in connexion with the problem of minimum-weight design, some elegant fundamental work had been done by Mr J. D. Foulkes.[12] Much of that work had not yet been published, and had not been completed when the Author had written the Paper then under discussion. The work of Mr Foulkes would render his own treatment out of date, and the Author therefore thought that it was unnecessary to elaborate further on his own method of minimum-weight design.

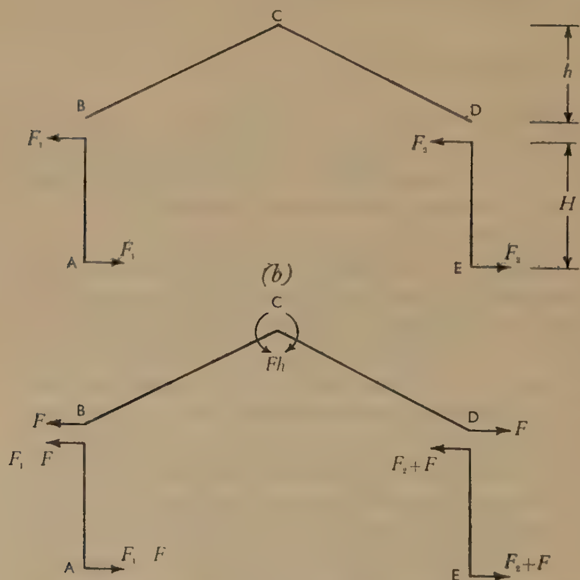
The Author remarked that Dr B. G. Neal had given a clear exposition of the equilibrium conditions for a pitched-roof portal frame. Such structures were very difficult to deal with in elastic analysis, and it was not surprising that some difficulty also arose in calculating the plastic collapse loads. The Author was of the opinion that, for single-span pitched-roof frames, the graphical approach [1] was superior to any other. Dr Neal and Professor Symonds had shown that the method of combined mechanism could be applied successfully to multi-bay pitched-roof frames,[13] and there was also a method of applying the moment-distribution technique to that problem.

In pitched-roof portal frames, the distribution of the total horizontal shear force as between the stanchions directly affected the bending moments

in the rafters. That complication did not arise in frames which contained vertical and horizontal members only. Thus if the shear forces in *Fig. 28* (*a*) were modified so that a shear force $F$ were transferred from stanchion AB to stanchion DE, then a sagging bending moment $Fh$ would be induced at the apex C of the intermediate rafters. At the same time, the sum of the end moments in stanchion AB would be decreased by $FH$, the sum of the end moments in stanchion DE being increased by the same amount. Provided no change occurred in the distribution of shear

*Figs 28*

*(a)*




Horizontal Shear Forces in a Pitched-Roof Portal Frame

forces between stanchions, any change in the bending moments in the two rafters could be treated as though both members constituted together a single horizontal beam, C being at mid-span. Hence any possible changes in the bending moments could be built up from the extended Distribution Table shown in Table 3. Lines (*a*), (*b*), and (*c*) were identical with the first three lines of Table 1, whilst line (*d*) showed how the sum of the top and bottom moments of each adjacent stanchion could be modified to bring about a change of shear-force distribution between the stanchions.

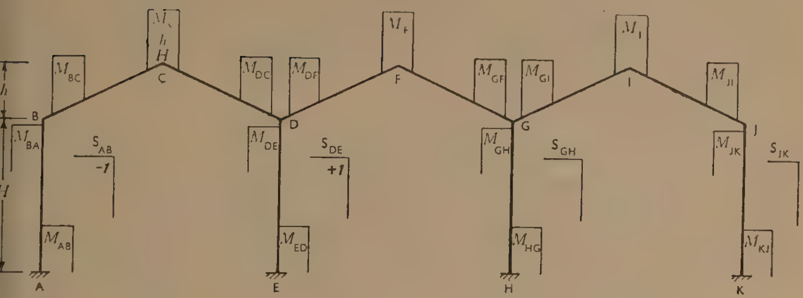
The bending moments which had to be recorded when dealing with a three-bay pitched-roof frame were shown in *Fig. 29*. The columns $S_{AB}$, $S_{DE}$, $S_{GH}$, and $S_{JH}$ were individual sway columns for each stanchion,

TABLE 3.—DISTRIBUTION TABLE FOR PITCHED-ROOF FRAMES

| | Left-hand sway | Left-hand end moment | Apex moment | Right-hand end moment | Right-hand sway |
|---|---|---|---|---|---|
| a | — | 1 | $\frac{1}{2}$ | 0 | — |
| b | — | 0 | $-\frac{1}{2}$ | 1 | — |
| c | — | 1 | 0 | 1 | — |
| d | −1 | — | $\frac{h}{H}$ | — | 1 |

$H$ denotes height to eaves.
$h$ denotes height from eaves to apex.

*Fig. 29*




TREATMENT OF A THREE-BAY PITCHED-ROOF PORTAL FRAME BY MOMENT DISTRIBUTION

eplacing the combined sway column for rectangular frames. The sum of he stanchion moments in any one stanchion had to agree with the moment 1 the corresponding sway column. The sum of the sway moments in all he columns $S_{AB}$, $S_{DE}$, $S_{GH}$, and $S_{JK}$ had to remain constant, but the istribution between the columns could be altered by using the last line f Table 3, as shown for the first bay of the frame in *Fig. 29*.

Dr Steed had raised the question of triangulated structures, and lthough this was strictly outside the scope of the present discussion, the uthor would do his best to answer the queries raised. Dr Steed had orrectly interpreted the three conditions to be satisfied in a triangulated ructure at collapse, provided an over-riding assumption, which he had ot stated, was justified. That assumption was that any member should, t the ultimate load, be capable of offering a constant resistance durg deformation. Such an approximation could not be justified for

compression members in general The Author would therefore deprecate any easy assumption that the philosophy of the simple plastic theory as applied to building-frame structures could be extended to triangulated frames. At the same time, the general plastic theory did afford a means of studying the behaviour of triangulated mild-steel structures beyond the elastic limit, provided one resisted the temptation to jump to easy conclusions, and it should certainly be possible, for example, to throw light on the much discussed problem of secondary stresses.

Setting aside the query regarding the behaviour of compression members, the principles stated by Dr Steed would be found adequate to solve any problem without ambiguity. In the case of Dr Steed's example, Mode 1 (*Fig. 24* (*b*)) did not constitute a mechanism. If members CF and DF were assumed to extend, all other members except AF remaining of constant length, the mechanism shown in *Fig. 30* was obtained. According to that mechanism, the member AF was required to contract, whereas it had been assumed to be yielding in tension. Similar false modes of failure could be obtained for building frames if one did not take the precaution of drawing the collapse mechanisms, and the Author would recommend a similar procedure as being essential when dealing with triangulated structures.

The Author felt that the Chairman had perhaps misunderstood the significance of the three conditions to be satisfied at the collapse load. Any one structure would collapse at a definite intensity of loading, and at collapse, the three conditions of equilibrium, mechanism, and yield would necessarily be satisfied. The difficulty of obtaining a bending-moment distribution satisfying the three conditions was only an analytical one, but had to be overcome in order to calculate the collapse load. A bending-moment distribution satisfying the equilibrium and yield conditions but not that of mechanism gave not the collapse load but a load of lower intensity. A bending-moment distribution which satisfied the equilibrium and mechanism condition but not that of yield would correspond to a load intensity greater than the value at collapse.

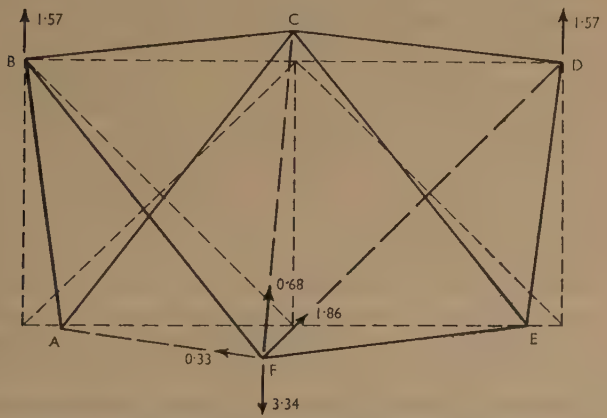
The Author had hoped that in presenting the example of a design for two different systems of loads, he had demonstrated the versatility of the moment-distribution technique. In a rigid-pointed structure, under a system of loads A, one part X of the structure might be giving support to another part Y. Under a different system of loads B, under which part X was more severely loaded, part Y could be called upon to give extra support to part X. In elastic design, it was usually impossible to make more than accidental use of interaction of this nature, whereas it could be used to the full by using plastic design methods.

The Author was able to inform the Chairman that some study had been made of the effect of moving loads on structures designed by the plastic theory.[5] The question of the optimum design of members of varying cross-section under loading conditions of that nature had also received

some attention.[14] With regard to the strain-hardening range, it was true that, at collapse some plastic hinges would, in actual structures, show stresses above the horizontal portion of the stress/strain curve. It had, however, been found that that effect was insufficient to modify significantly the distribution of bending moments, and some theoretical work [15] had shown that that result was to be expected.

Mr Jenkins had expressed doubts as to the superiority of moment-distribution methods in the elastic range. The Author would agree that multi-storey frames were tedious to analyse by moment distribution

*Fig. 30*




Deformation of Truss according to the Erroneous Model (see *Fig. 24 (b)*)

because of the sway problem, but his own opinion held to the general superiority of moment distribution. Mr Jenkins was of course quite correct in his criticism of the term " elastic analysis " when reference should have been made to " linear analysis."

Mr Jenkins had raised the question of the effect of change of geometry at collapse on the conditions of equilibrium. In the type of structure under consideration, the only important effect of change of geometry was that additional stanchion moments could be introduced by sway deflexion, and the Author had in his treatment assumed that such moments could be neglected. There certainly would be some structures in which the effect would be considerable—particularly those structures in which deflexions at working loads were appreciable. The Author's general impression was that, in structures for which ultimate strength rather than deflexion at working loads was the criterion for design, the change of geometry at the point of collapse was insufficient to reduce the collapse load significantly below the value given by simple theory. That was, however, a problem

which demanded further study, and in fact some as-yet-unpublished work had already been done at Cambridge on the subject.

The Author said that he had been somewhat at a loss to understand what Mr Manning was seeking in the three stages of his design method. Mr Manning had himself remarked that in engineering the difficulty was more often to state the problem which had to be solved than to solve it when it had been stated. The conception of ultimate strength could in general be more nearly related to what was required of a structure than the conception of safe stresses. It had been shown by the work of the Steel Structures Research Committee, and more recently by work at the Building Research Station, that calculated elastic stresses often bore but little relationship to stresses actually induced. Hence the devotion of laborious hours to elastic analysis might sometimes fulfil no useful purpose, and may merely serve to exemplify the lack of any clear perception of the nature of the problem facing the designer. The plastic theory, based as it was upon a thorough investigation, both experimental and theoretical, into the behaviour of structures right up to the ultimate load, did contain a clear perception of the problem which had to be solved. There was of course no pretence that the plastic theory itself—still less the particular technique suggested in the Paper—could take account of all the important factors which had to be considered in design. The elastic theory of structures had still to be regarded as an important tool to be used where necessary. There was also a great deal which could be learned only by experience, but there had to be a general conception of design which could be communicated. If there was to be any consistent advance in structural design, one could scarcely allow the individual to pick and choose his basic criteria of suitability, as Mr Manning would appear to advocate.

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12. J. Foulkes, "Minimum Weight Design and the Theory of Plastic Collapse." *Quarterly J. App. Math.*, vol. 10 (1953), p. 347.
13. B. G. Neal and P. S. Symonds, "The calculation of Plastic Collapse Loads for Plane Frames." *The Engineer*, vol. 194 (1952), pp. 317 and 363.
14. M. R. Horne, Discussion, Final Report, 4th Congress, Intl Assn Br. & Struct. Engng, 1952, p. 119.
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## PUBLIC HEALTH ENGINEERING DIVISION MEETING

10 November, 1952

Mr G. M. McNaughton, Member, Chairman of the Division, in the Chair

The following Lecture was delivered and, on the motion of the Chairman, the thanks of the Division were accorded to the Lecturer.

# "Atmospheric Pollution : Causes, Effects, and Prevention"

**by**

**Albert Parker, C.B.E., D.Sc.**

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### Introduction

)N average we each breathe in a day 30 to 40 lb. of air, whilst we consume nly 3 to 4 lb. of water and only about $1\frac{1}{2}$ lb. of dry matter as food. Yet re pay far more attention to the purity of our drinking water and food han to the air we breathe.

Gross contamination of the air of many of our large towns and of the vers and streams that flow through them is part of the price that has so ır had to be paid for the development of industry, to enable Great Britain ) support a rapidly increasing population at a rising standard of living. ur small island has an area less than one-five-hundredth of the total land rea of the world, whilst our population is nearly one-fortieth of world opulation, so that the density of our population is about twelve times the verage for the world.

Development of industry in Great Britain has been based on the use of oal as the major source of heat and power ; and it is mainly from the use -or rather misuse—of this indigenous fuel that the air of our towns is so eavily polluted. During the past 50 years, there has also been an creasing amount of pollution from the use of petroleum oils for furnaces ıd road transport.

Coal has been used in Britain for at least a thousand years, at first adually, and then more rapidly, displacing wood. Though the quantity coal burned was not more than a few million tons a year until about 0 years ago, there were serious complaints as early as the thirteenth ntury about pollution of the air from the burning of coal in London. For ample, in the reign of Edward I, when the annual consumption of coal

was probably no more than one million tons, there was a proclamation prohibiting the use of coal in London while Parliament was in session. There was a similar proclamation in the reign of Queen Elizabeth; and in 1648, Londoners went so far as to ask Parliament to prohibit the import of coal into the City from Newcastle. In 1661, John Evelyn wrote his famous pamphlet entitled "Fumifugium, or the Inconvenience of the Aer and Smoake of London Dissipated, together with some Remedies."

With the development of industry in Great Britain since the beginning of the eighteenth century, there has been a rapid rise in the use of coal. The great changes which have occurred within the past 250 years are well shown by the approximate figures for coal production and inland utilization given in Table 1. During this period, the population rose from 6 or 7 millions in the year 1700, to 10 or 11 millions in 1800, 37 millions in 1900, and about 50 millions in 1951.

TABLE 1.—ANNUAL COAL PRODUCTION AND UTILIZATION IN GREAT BRITAIN

| Year | Production: millions of tons: | Inland use: millions of tons |
|---|---|---|
| 1700 | 3 | 3 |
| 1800 | 12 | 12 |
| 1853–1862 | 70 | 64 |
| 1913–1922 | 241 | 184 |
| 1923–1932 | 233 | 166 |
| 1933–1942 | 221 | 180 |
| 1943–1949 | 198 | 185 |
| 1950 | 216 | 199 |
| 1951 | 223 | 206 |

At the beginning of the present century, the quantity of petroleum and petroleum products imported was small. During the past 30 years, the quantity imported has increased rapidly, as shown by the figures in Table 2, which also gives the quantities used within Great Britain, after allowing

TABLE 2.—ANNUAL IMPORTS AND UTILIZATION OF PETROLEUM AND PETROLEUM PRODUCTS IN GREAT BRITAIN

| Year | Imports: millions of tons | Inland use: millions of tons |
|---|---|---|
| 1920 | 3·4 | 3·1 |
| 1930 | 8·9 | 7·6 |
| 1938 | 11·7 | 9·0 |
| 1948 | 17·9 | 12·8 |
| 1950 | 19·2 | 15·3 |
| 1951 | 26·7 | 16·9 |

for re-exports, oils for ships' bunkers in foreign trade and shipping, losses in refining, etc.

In the earlier years of the nineteenth century, when the great acceleration in industrial development began, the boilers and furnaces burning coal were not nearly so efficient as they are to-day. The gas industry was in its infancy, electricity was no more than a possibility, and by-product coke ovens were unknown. Large quantities of smoke must have been emitted from every ton of coal burned. By 1819, the smoke nuisance was increasing to such an extent that Parliament appointed a Committee to enquire how far persons using engines and furnaces could erect them in a manner less prejudicial to public health and comfort. In 1843, another Select Committee recommended legislation to deal with nuisances from steam engines and furnaces, but on the problem of smoke from domestic fires they went no further than to express the hope that it might eventually be prevented. Since that time the desirability of greatly reducing the pollution of the air from the burning of coal has been stressed by many authorities. During recent years various aspects of the problem have been considered by several authorities, including the Heating and Ventilation (Reconstruction) Committee of the Building Research Board of the D.S.I.R. under the Chairmanship of Sir Alfred Egerton, the Fuel and Power Advisory Council of the Ministry of Fuel and Power under Sir Ernest Simon (now Lord Simon of Wythenshawe), and the Departmental Committee on National Fuel Policy of the Ministry of Fuel and Power under Lord Ridley. The reports of these authorities were published by H.M.S.O. in 1945, 1946, and 1952 respectively. In July 1953, a Committee on Air Pollution, under the Chairmanship of Sir Hugh Beaver, was appointed by the Ministers of Housing and Local Government and of Fuel and Power, and the Secretary of State for Scotland, " To examine the nature, causes and effects of air pollution and the efficacy of present preventive measures ; to consider what further preventive measures are practicable ; and to make recommendations."*

Since 1912, there has been an Atmospheric Pollution Research Committee, which is now a Committee of the Fuel Research Board of the D.S.I.R. This Committee, which has issued many reports, advises on the work of the Fuel Research organization in relation to measurements of pollution, factors affecting dispersion, and investigations of methods of preventing pollution.

There have also been legal enactments directed towards the mitigation of pollution. Of these the most recent is the Public Health (Smoke Abatement) Act, 1926, the main provisions of which are included in the Public Health Act, 1936. Under this Act local authorities are empowered to make appropriate by-laws, subject to confirmation by the Minister of Housing and Local Government, regulating the emission of smoke of such

* Since this lecture was delivered, an Interim Report of the Committee on Air Pollution has been published by H.M.S.O. in December 1953 (Cmd. 9011).

colour, density, or content as may be prescribed. There are broadly similar provisions in the Public Health (London) Act, 1936, and the Public Health (Scotland) Acts, 1897 to 1939. These Acts apply to industrial smoke and not to smoke from domestic chimneys ; and the provisions of the Acts may not be applied so as to obstruct or interfere with the working of mines or with a number of operations in iron and steel works. Some local authorities have also obtained powers to constitute certain areas as smokeless zones. In addition, there are clauses in lease agreements for factories and similar premises on certain estates, with the object of restricting the emission of smoke and grit.

Legislation and restrictive clauses in lease agreements, however, though helpful, cannot solve the problem of preventing pollution, unless there are practicable methods at reasonable cost of producing the necessary heat and power without the emission of smoke, grit, and harmful gases and vapours. The position is that it is not yet practicable entirely to prevent pollution of the atmosphere from the use of fuels and their products. Within the past 50 years, there has been improvement in some areas, with the development and application of more efficient methods of burning coal and oils, and the extended use of gas, coke, and electricity to replace coal in many factories and houses. But other areas have suffered as they have become industrialized and more thickly populated. On average, the quantity of smoke discharged from each ton of coal has decreased, but the total quantity of coal used has increased, with the result that there has been no very great reduction in pollution by smoke for the country as a whole. Pollution by gaseous oxides of sulphur has increased from the increased use of coal, accentuated by the tendency for a rise in the sulphur content of the coals available. The use of residual fuel oil from petroleum, which often contains a considerable amount of sulphur, is also adding to air pollution by sulphur. There must have been a rapid rise in the pollution of the air near ground level in towns by the exhaust gases from road vehicles ; these gases from spark-ignition engines are rich in carbon monoxide.

So far, reference has been made only to air pollution arising from the use of fuels, which is the main subject of this lecture, for the greatest single contribution to air pollution arises from the burning of fuels. In many areas, there are chemical and other manufacturing processes from which are discharged polluting gases, vapours, and dusts not directly related to the combustion of fuel. In the middle of the nineteenth century there was a public outcry as a result of the establishment of processes for making alkali (sodium carbonate) from common salt. From these processes large volumes of hydrochloric acid were discharged. A Royal Commission was appointed, and following its first report, the first Alkali, etc., Works Regulation Act was passed in 1863. Later, the Act was extended on several occasions. It was consolidated in 1906, but further additions have been made by Alkali Orders published in 1928, 1935, 1939, and 1950. The

provisions of the Act, which are administered centrally by Alkali Inspectors under the Ministry of Housing and Local Government, now relate to more than nineteen hundred processes at more than a thousand works in Great Britain. Since the Alkali Inspectors are all men of scientific training and technical experience, they have gained the confidence and co-operation of the industrialists operating the processes over which they have jurisdiction. The progress of the work under the Alkali, etc., Works Regulation Act, which is based on the " best practicable means " is described in the annual reports of the Chief Alkali Inspectors. A summary of the work is given in a Paper [1] by Mr W. A. Damon (Chief Alkali Inspector).

### Nature and Amount of Pollution from Use of Fuels

When coals and fuel oils are completely burned with air, the principal products entering the chimney are gaseous carbon dioxide, nitrogen and excess air, water vapour, and gaseous sulphur dioxide. Most of the mineral matter with coal is ordinarily left as ash or clinker, though a proportion, which varies with the conditions of firing, is carried forward as grit with the chimney gases. If the fuel is burned under conditions which do not ensure complete combustion, the products include not only those already mentioned, but also gaseous carbon monoxide, hydrogen and hydrocarbons, and smoke in the form of soot and sticky tarry matter. The products of complete combustion of coke are the same as with coal and oil; with imperfect combustion carbon monoxide also is produced, but there is little or no smoke. Gas made at gas works for public supply is purified before distribution to remove all particles of tar, and most of the sulphur compounds. On complete combustion, which is easily achieved, it forms carbon dioxide and water vapour, with only very small quantities of gaseous oxides of sulphur.

Table 3.—Polluting substances from the combustion of coal and gas and their products

| | |
|---|---|
| 1. Solid | Particles of carbon or soot causing black smoke, and particles of coal, coke, and ash carried forward as grit in the chimney gases. |
| 2. Liquid or semi-solid carbonaceous matter | Particles of tarry matter causing yellowish brown smoke. |
| 3. Unburned and partially-burned gases | Hydrocarbons and carbon monoxide. |
| 4. Sulphur oxides | Gaseous sulphur dioxide and sulphur trioxide mist, which with water give sulphurous and sulphuric acids. |

[1] W. A. Damon, "The Treatment of Waste Gases in Chemical Industry." *Trans. Instn Chem. Engrs*, vol. 31 (1953, No. 1), p. 26.

Of the various products of combustion and partial combustion which have been mentioned, those causing serious pollution of the atmosphere are given in Table 3.

In estimating the quantities of these polluting substances discharged into the atmosphere, it is first necessary to obtain information on the main uses of the fuels and their products, on the quantities so used, and on the amount of each polluting substance discharged relative to the weight of fuel used. Statistics are available to show the main uses of coals and oils as fuels, with their quantities, in Great Britain, and information has been obtained by the Fuel Research organization of the D.S.I.R. and by others on the amounts of the polluting substances discharged in relation to the weight of fuel consumed. Conditions of use of fuels vary so widely that estimates so obtained can only be approximate.

In 1951, a typical recent year, the total inland consumption of coal, excluding coastwise bunkers, was 205 million tons. This amount was divided among the main broad uses as shown by the figures in the second column of Table 4.

TABLE 4.—MAIN USES OF COAL AND ESTIMATES OF POLLUTION FROM SUCH USES IN GREAT BRITAIN IN 1951

| Type and use of fuel | Quantity of coal: millions of tons per annum | Pollution produced: millions of tons per annum | | |
|---|---|---|---|---|
| | | Smoke | Grit | Sulphur dioxide |
| Coal: | | | | |
| Domestic use | 37·5 | 0·9 | 0·1 | 0·9 |
| Electricity works | 35·4 | small | 0·2 | 1·0 |
| Railways | 14·4 | 0·4 | 0·1 | 0·4 |
| Other industrial uses, including collieries | 66·7 | 0·8 | 0·2 | 1·8 |
| Coke and gas: | | | | |
| Coke ovens and use of coke | 23·6 | small | small | 0·5 |
| Gas industry | | | | |
| At gas works | 27·4 | small | small | 0·1 |
| Use of gas | — | nil | nil | small |
| Use of coke | — | nil | small | 0·3 |
| Totals | 205 | 2·1 | 0·6 | 5·0 |

Of the total of 205 million tons, 51 million tons, or one-quarter, was carbonized in coke ovens, gas works, and low-temperature carbonization plants to provide coke or solid smokeless fuel, gas, tar, motor spirit, fertilizers, and raw materials for chemicals. The remainder, 154 million tons, was burned as raw coal with various efficiencies, and with the production of large quantities of smoke and other polluting substances. It

included 37·5 million tons (including 5·1 million tons miners' coal) used in household grates, or between one-fifth and one-sixth of the inland consumption of 205 million tons.

In Table 4, estimates to the nearest 0·1 million tons are included for the amounts of each of the main polluting substances discharged into the air from the use of coal, coke, and gas. The figures show that the weight of the carbonaceous and tarry matter in the smoke produced was about 2 million tons a year. Nearly one-half of this smoke was derived from domestic grates, though they used less than one-fifth of the coal consumed. The total pollution by oxides of sulphur was about 5 million tons, and the weight of grit discharged was roughly 0·6 million tons. About one-fifth of the sulphur and one-sixth of the grit arose from domestic appliances burning raw coal, and one-fifth of the pollution by sulphur came from electricity generating stations.

Information on the quantities of carbon monoxide discharged in the waste gases from the many types of industrial and domestic equipment used for burning coal, coke, and gas is insufficient to provide more than a very rough estimate of the total amount of carbon monoxide emitted into the atmosphere from these sources. From the limited information available, it seems that of the carbon in the fuel burned in domestic fires, a proportion of the order of 10 per cent appears as carbon monoxide, and the remainder mainly as carbon dioxide; though the concentration of the carbon monoxide in the waste gases from domestic fires may be of the order of only 0·1 per cent or less owing to the high dilution of the combustion gases with large volumes of air drawn up the chimney. With industrial equipment burning coal and coke, it is probable that, as an overall average, the proportion of the carbon converted to carbon monoxide is in the region of 3 to 5 per cent. On these bases, it can be calculated that the amount of carbon monoxide discharged from domestic chimneys in Great Britain is in the region of 10 millions tons a year, and that the quantity from the other uses of coal and coke is also of the order of 10 million tons a year, or an overall total in the region of 20 million tons a year. Since carbon monoxide has a density about the same as that of air, and the chimney gases are discharged well above ground level, the carbon monoxide is readily dispersed and mixed with the air under usual meteorological conditions, and the concentration in the atmosphere is in consequence ordinarily very small.

Over a long period of years there has been a steady growth in the use of gas for domestic and industrial requirements, and a more rapid increase in the use of electricity. These increases in demand have been met, mainly (1) by improvements in the efficiency of manufacture of gas and of generation of electricity, and (2) by using larger quantities of coal at the gas works and electricity generating stations. There have also been increases in the quantities of gas bought by the gas industry from the coke-oven industry and in the quantity of gas-oil used, and in the generation of

electricity from water power. The coal carbonized at gas works rose from 16 million tons in 1921 to 19 million tons in 1938, while over the same period the quantity of coal used at electricity generating stations increased from 6·5 to 15 million tons. Since 1938, the rates of growth in the use of coal for the production of gas and electricity have been increased by the conditions, including domestic coal rationing, arising from the war, while there has been a marked reduction in the quantity of raw coal burned for domestic needs, from more than 50 million tons to less than 40 million tons a year. As a result, the total quantity of smoke discharged into the atmosphere was probably rather less in 1951 than in 1938, while the quantities of grit and oxides of sulphur were probably a little greater.

Since the domestic grate burning raw coal produces more smoke per ton of coal than does the average industrial coal-consuming equipment, it is interesting to note from the approximate figures in Table 5 the changes that have taken place in the domestic use of coal, coke, gas, and electricity from 1938 to 1951.

TABLE 5.—DOMESTIC CONSUMPTION OF COAL, GAS, COKE, AND ELECTRICITY IN 1938 AND 1951

| | | Millions of tons |
|---|---|---|
| 1938 | Coal, anthracite and boiler fuel . . . . | 50·4 |
| | Coal in making 2·5 million tons of coke and 925 million therms of gas . . . . | 7·4 |
| | Coal in providing 5,360 million units of electricity . . . . . . . . . | 4·1 |
| | Total domestic coal consumption | 61·9 |
| 1951 | Coal, anthracite, and boiler fuel . . . | 37·5 |
| | Coal in making 4 million tons of coke and 1,390 million therms of gas . . . | 12·0 |
| | Coal in providing 16,370 million units of electricity . . . . . . . . . | 12·0 |
| | Total domestic coal consumption | 61·5 |

From the figures in Table 2 it is seen that the inland consumption of petroleum products in Great Britain in 1951 was 16·9 million tons. Figures are given in Table 6 to show the quantities of the different products and their main uses.

To the figure of about 4·3 million tons of fuel oil from petroleum, there should be added 0·6 million tons of creosote-pitch mixture from the coal-carbonization industries, to give a total quantity of fuel oils of 4·9 million tons. If these oils were burned with reasonable efficiency, they should have given rise to relatively little pollution by smoke or by carbon mon-

TABLE 6.—QUANTITIES AND MAIN USES OF PETROLEUM PRODUCTS IN GREAT BRITAIN IN 1951

| Petroleum products and main uses | Quantity: millions of tons |
|---|---|
| Fuel oil | 4·29 |
| Aviation spirit | 0·33 |
| Motor spirit | 5·45 |
| Kerosene | 1·78 |
| Gas and Diesel oils: | |
| For road vehicles | 1·13 |
| Gas works | 0·56 |
| Other inland requirements | 1·16 |
| Industrial spirit, white spirit, lubricating oils, bitumen, wax, etc. | 2·20 |
| Total | 16·90 |

oxide. The amount of smoke was probably much less than 0·1 million tons, and the amount of carbon monoxide was probably less than 0·25 million tons. There is no reliable information about the average sulphur content of this total of 4·9 million tons of fuel oil, but it was probably between 2 and 3 per cent. On this assumption the combustion of the 4·9 million tons gave rise to the production of 0·2 to 0·3 million tons of sulphur dioxide discharged with the chimney gases.

To the figures of 0·33 and 5·45 million tons of aviation spirit and motor spirit, respectively, from petroleum, there should be added a total of about 0·3 million tons of motor benzole and other spirit from the coal-carbonization industries, and from the hydrogenation of creosote, to give an overall total of motor and aviation spirit of about 6·1 million tons. Air pollution from aircraft is so small that it can be neglected. There is no doubt, however, that the exhaust gases from the use of about 5·7 million tons of motor spirit in road transport must cause considerable pollution by carbon monoxide, particularly at points of dense traffic in the larger towns. Since the sulphur content of motor spirit is very small (well below 0·05 per cent), pollution by oxides of sulphur from petrol engines is negligible.

The composition of the exhaust gases from a petrol-driven vehicle depends on a number of factors, including the mechanical condition of the engine, and the duty (that is, whether the engine is idling, accelerating, cruising, or decelerating). Combustion is far from complete during idling and decelerating. In general, the exhaust gases contain from 3 to 10 per cent of carbon monoxide by volume. Assuming an overall average of 5 per cent of carbon monoxide in the exhaust gas, and a normal relationship of volume of exhaust gas to weight of motor spirit consumed, 5·7 million tons of spirit would give rise to about 4 million tons of carbon monoxide.

Kerosene, which contains very little sulphur, is used for a variety of purposes, many of which discharge only relatively small amounts of

carbon monoxide. Internal-combustion engines burning kerosene give rise to appreciable quantities of carbon monoxide, but in comparison with other sources, pollution from kerosene can be neglected in this broad review of the position.

During the past 25 years, there has been a steady increase in the use of compression-ignition or Diesel engines for road transport, and the quantity of oil consumed by such engines is now more than 1 million tons a year. Provided that the Diesel type of engine is maintained in good condition, the exhaust gas consists almost entirely of nitrogen, carbon dioxide, and oxygen, with no more than 0·1 per cent of carbon monoxide, roughly 400 parts of oxides of nitrogen per million, and aldehydes up to 10 or 12 parts per million. The amount of sulphur in Diesel-fuel oil may be in the region of 0·5 to 1·0 per cent, so that 1 million tons of the oil used in Diesel engines would produce from 10,000 to 20,000 tons of sulphur dioxide, which is small in comparison with other sources of pollution by sulphur. At the same time, it must be admitted that the odour of Diesel exhaust gases is often objectionable, and if the engine is in poor condition, there may be discharged much partially burned oil in the form of clouds of dense smoke.

Pollution of the atmosphere from the use of gas oils at gas works, gas and Diesel oils for other inland requirements, and the uses of the 2·2 million tons of the various oils in the last item of Table 6 is relatively unimportant.

Summarizing the information in the preceding paragraphs, the total quantities of the main pollutants arising from the use as fuel of coal, oil, and their products, are of the order of the figures given in Table 7.

TABLE 7.—ESTIMATES OF THE QUANTITIES OF THE MAIN POLLUTANTS DISCHARGED PER ANNUM FROM THE USE OF COAL, OIL, AND THEIR PRODUCTS IN GREAT BRITAIN IN 1951

| Pollutant | Quantity: millions of tons |
|---|---|
| Smoke | 2·1 |
| Grit | 0·6 |
| Sulphur dioxide | 5·3 |
| Carbon monoxide | 24·0 |

## DISTRIBUTION OF POLLUTION AND ITS MEASUREMENT

The figures so far given in this Lecture for the quantities of the main polluting substances arising from the uses of fuels are for the country as a whole. But this is not the full story. Much more coal is consumed in the winter than in the summer, and very much larger quantities of coal and oil are consumed per square mile in thickly-populated industrial areas than in

rural districts. Further, the dispersion of the polluting substances from their points of discharge is affected by many factors, including the height of the chimney or other point of discharge, the velocity and amount of the discharge, the velocity and direction of the wind, and various other meteorological and topographical conditions. Some polluting substances are more rapidly deposited or otherwise removed from the atmosphere than are others, and some are more readily dispersed and mixed in the atmosphere. In consequence, the incidence of pollution varies greatly from one locality to another, with time of year, and with changes in meteorological conditions. It is important, therefore, to have systematic records of pollution at numerous places at different times of the year over a long period of years. Without such records it is impossible to gauge the magnitude of the problems to be solved, or to measure the effects of the many changes that occur over the years.

*D.S.I.R. Co-operative Scheme for Measurements*

The importance of systematic measurements of the incidence of the major forms of pollution has long been recognized in Great Britain. In 1912, there was set up an Advisory Committee on Atmospheric Pollution, which a few years later became a Committee under the Meteorological Office. In 1927, the work of this Committee was transferred to the Department of Scientific and Industrial Research; and in 1945 the Atmospheric Pollution Research Committee was reconstituted as a Committee of the Fuel Research Board of the Department to facilitate closer co-operation with the work of the Fuel Research Station, particularly with the many investigations leading to greater efficiency in fuel utilization, with reduction in the amount of smoke and other polluting matter discharged. There is also close co-operation with the Meteorological Office, the Ministry of Housing and Local Government, the Ministry of Fuel and Power, the Ministry of Health, the Ministry of Agriculture and Fisheries, the Medical Research Council and other medical authorities, local authorities, and industry.

From the beginning of this work, the scheme of systematic measurements has received the active and increasing support of local health authorities and other co-operating bodies. The D.S.I.R. now receive the co-operation of about 180 local authorities and other organizations, who make regular measurements in their areas by methods laid down by the Department, so that the results are comparative. The Department is responsible for developing and standardizing the methods. Advice is given also on the selection of suitable sites for the instruments in the various localities. All the results of the observations are received regularly at the Fuel Research Station, where they are collated. Each month, an Atmospheric Pollution Bulletin, containing a summary of the results of the observations and a bibliography and abstracts of scientific and technical Papers of interest, is prepared, and copies are circulated to all the

co-operating organizations. In addition, the results are periodically analysed statistically and correlated in a form suitable for inclusion in published official reports. Each monthly Bulletin now gives about 3,000 results from about 900 observation sites.

The number of instruments in regular use by the co-operating organizations is about 1,350, which is more than four times as many as 6 or 7 years ago ; and the increase continues. In Table 8, figures are given to show the numbers of the main types of instrument in regular use by the co-operating organizations.

TABLE 8.—OBSERVATIONS OF ATMOSPHERIC POLLUTION TYPES AND NUMBERS OF INSTRUMENTS IN REGULAR USE BY CO-OPERATING ORGANIZATIONS

| Type of instrument | Number |
|---|---|
| Deposit gauges . . . . . . . | 573 |
| Sulphur dioxide instruments (two types) . . . . . . . . . | 672 |
| Smoke filters (two types) . . . . | 93 |
| Total . . . . . | 1,338 |

In addition, the D.S.I.R. has in operation about a hundred instruments for special investigations and surveys, making a total for all instruments of about 1,450.

Some instruments are operated on a daily basis, others monthly, and some (deposit gauges) provide samples from each of which several results of value are obtained. In consequence, the results of measurements are now being accumulated at the Fuel Research Station at a rate of about 120,000 per annum. Even so, the available information is far from sufficient to provide answers to the many modern problems of atmospheric pollution with the various factors involved, and the wide variations in local and meteorological conditions. But it is worthy of mention that no other country is in possession of so much information about its prevailing levels of pollution.

*Some of the Results of the Measurements*

The standard deposit gauge (*Fig. 1*, facing p. 112) is ordinarily exposed to collect the total deposited matter over a period of one month. It collects rain-water containing various soluble substances, and insoluble particulate matter which is sufficiently coarse to settle. From the analyses of the soluble matter, the amounts of pollution by acid radicles (for example, sulphates, chlorides) and basic radicles (for example, calcium) are commonly determined. The insoluble matter is usually analysed for tar, carbonaceous matter, and ash. Deposited matter varies widely in quan-

tity, and composition according to the quantity and nature of local and distant emissions, and climatic conditions. In areas where much coal or other sulphur-containing fuel is burnt, the soluble deposits contain sulphates, often in excess of the basic constituents, so that the deposited rainwater is slightly acid. Coarse particulate matter, consisting largely of partially burnt fuel and ash, process dusts, etc., is emitted mainly by industrial plant, and generally causes only a local nuisance, whereas very fine particulate matter such as smoke, and gases such as sulphur dioxide, may also cause a nuisance farther from their source. (See *Fig. 2.*)

Amounts of solid matter that are deposited vary considerably from place to place, even over short distances. As a rough indication, it may be said that pollution by deposited matter in some heavily industrialized areas may be equivalent to more than 500 tons per square mile per year, and may reach as much as a rate of 2,000 tons per square mile per year, though such large amounts of deposits do not often occur over so large an area as a square mile. In large cities and some urban areas, total deposits are commonly in the region of 200 to 400 tons per square mile ; in rural areas and country towns the amount is normally less than 100 tons, occasionally as low as 10 tons per square mile per year.

Particles which are so small that they show little or no tendency to settle are referred to as smoke. Mostly they consist of finely divided carbon or carbonaceous matter. Concentrations of smoke at or near ground level range from zero to a little over 1 milligramme per cubic metre of air, which is a value frequently reached in a large city. The concentration usually varies considerably from hour to hour, often showing peaks in the morning and evening. Concentrations may be greater or less at week-ends than during the week, depending on the proportions of domestic and industrial smoke which comprise the whole. In areas in which the smoke is mainly from domestic fires, the amounts in the winter months are often several times as great as in the summer months.

Sulphur dioxide is generally considered to be the most important of the gaseous pollutants because of the large quantities emitted (see Table 7), and its undesirable effects on structures and other materials, agriculture, and probably on health. In the air of towns at or near ground level, the concentration of sulphur dioxide in winter is in the range of 0·05 to 1·0 milligramme per cubic metre (0·017 to 0·35 parts per million by volume), with much lower concentrations in the summer. It is interesting to observe that at or near ground level, the amount of sulphur dioxide in the air of towns is generally lower than the amount of smoke, though according to the estimates in Table 4 the total quantity of sulphur dioxide discharged is more than twice as great as the total quantity of smoke. There may be several reasons for this difference in concentration near ground level relative to the quantity discharged. Firstly, domestic chimneys, which are much lower than industrial chimneys, discharge nearly one-half of the smoke, but only about one-fifth of the sulphur. Secondly, smoke consists

of fine particles suspended in the air, while sulphur dioxide is a gas more likely to be mixed with the air at higher levels, and thus diluted and dispersed. Thirdly, sulphur dioxide is soluble in water (see *Fig. 3*), and is therefore more readily removed by absorption in rain, mist, on wet buildings, and by soil.

There is far from sufficient information about the concentration of carbon monoxide in the air of towns, particularly in areas of dense traffic. Measurements in London streets have been made by the Department of the Chemist-in-Chief of the London County Council. Concentrations as high as 0·0034 per cent by volume have been obtained in winter under what might be termed normal meteorological conditions. But it may be that the concentration is much higher at points of congested traffic and when natural ventilation is unusually low.

Observations during periods of fog, particularly during the severe fog of December 1952, have shown that as a result of the reduced natural ventilation and possibly also from other causes, the concentrations of smoke and sulphur dioxide in the air of towns can rise to several times their amount on a normal day at the same time of the year. In the fog of December 1952, the concentrations of smoke and sulphur dioxide in some parts of London were of the order of 10 times as great as on a normal winter day. It may be that the concentrations of carbon monoxide were also much greater than usual, particularly because motor vehicles had to travel very slowly and under conditions conducive to the discharge of large amounts of carbon monoxide in the exhaust gases.

*Special Investigations of Distribution of Pollution*

In addition to the work on methods of measurement, the Fuel Research Station undertake special surveys and investigations of the factors affecting the dispersal of pollutants in the atmosphere. For example, over a period of 3 years, an intensive survey was carried out in and around Leicester with the object of obtaining basic information on the factors influencing the distribution and dispersion of pollution in a city. Since 1945, the staff have undertaken detailed surveys of pollution by sulphur dioxide in several areas in London, with the object of providing information on the effects of various factors, including the proximity of large electricity generating stations and other industrial undertakings.

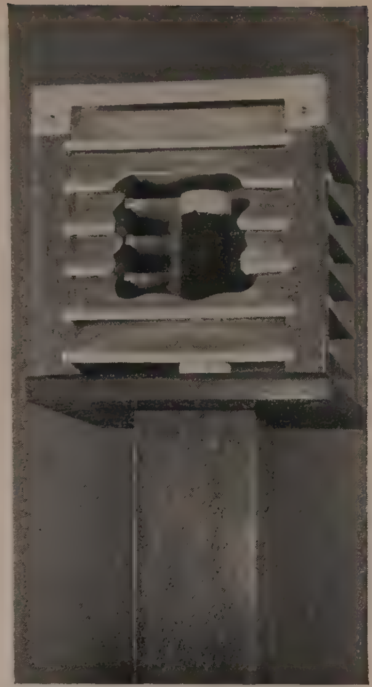
Investigations are also in progress on the dispersion of sulphur dioxide from the chimneys of a power station in an area receiving very little pollution from other sources ; and there are instruments in the areas of some new towns to observe the changes as the towns are developed.

In another investigation undertaken in co-operation with the iron and steel industry, it has been shown that the rate of corrosion of exposed metal increases with increase in the pollution of the air by sulphur dioxide.

*Fig. 1*

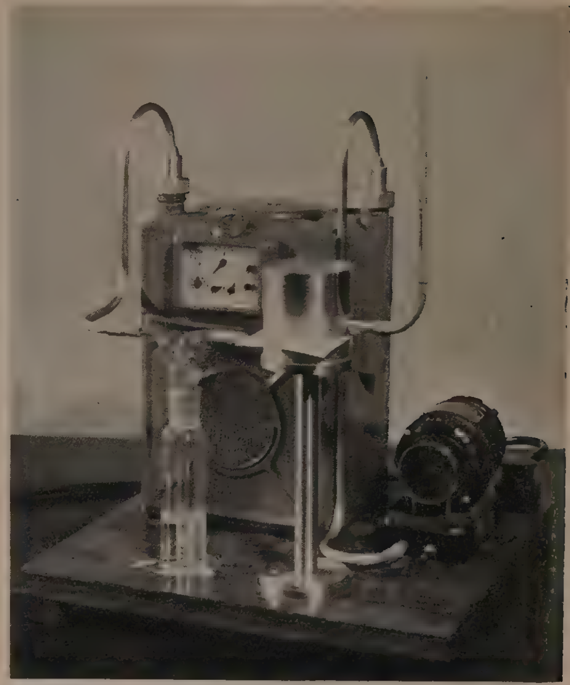
Standard Deposit Gauge for measuring deposited Solid Matter

*Fig. 2*



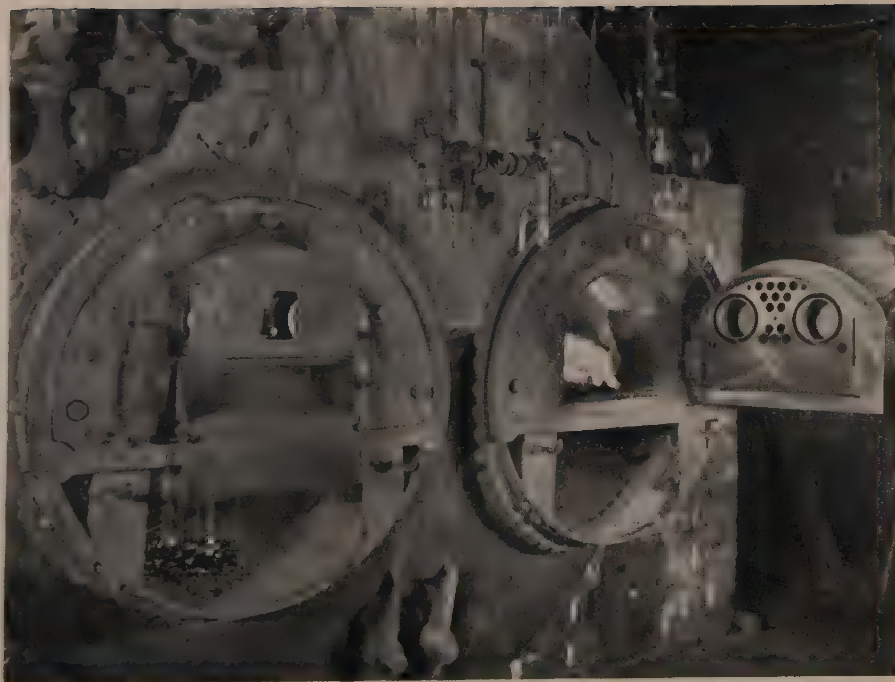
Lead-Peroxide Candle in Louvred Box (cut-away) for measuring Pollution by Sulphur Dioxide

*Fig. 3*



Equipment for Daily Measurement of Pollution by Smoke (absorbed on Filter Paper) and by Sulphur Dioxide (absorbed in Solution of Hydrogen Peroxide)

*Fig. 4*



Lancashire Boiler equipped with F.R.S. Smoke-Eliminator Doors

*Fig. 5*

Pilot Plant at Fuel Research Station for Experiments on Removal of Sulphur Dioxide from Flue Gases by means of Ammonia

## Detrimental Effects of Pollution

The detrimental effects of air pollution by smoke, grit, oxides of sulphur, and carbon monoxide from the use of coal and oil and their products are of many kinds. They include damage to structures and materials, involving extra labour and materials in cleaning and painting buildings and equipment; more frequent laundry work and more frequent replacement of materials; damage to agriculture; and detriment to health. It is difficult accurately to assess the extent of the damage. From estimates that have been attempted, it seems that the money equivalent at present prices, of the damage to buildings, equipment, fabrics, and agriculture, together with the direct waste of fuel, is in the region of £100 million to £150 million a year; and these estimates do not include damage to health.

## Abatement of Pollution

It is not possible in the present state of knowledge and of development of many fuel-burning appliances entirely to prevent pollution of the atmosphere from the combustion of coal and oils and their products for the generation of heat and power; but it is practicable to effect an appreciable reduction in the pollution by smoke, grit, and carbon monoxide. It is much more difficult to reduce the pollution by sulphur dioxide.

### *Coal Cleaning*

In the first place, coal should be cleaned so far as is economically practicable before delivery to the consumer. In deciding what is economically practicable, the overall cost of providing the heat or energy required must be considered, and not merely the price per ton or per unit thermal value of the coal as delivered. This means including not only the added cost of cleaning, but also such items as the reduction in costs of transport, handling, and the removal and disposal of ash and grit. By efficient cleaning, much of the shale, pyrites, and other foreign matter unavoidably brought up with the coal from below ground can be removed. In this way the quantity of mineral matter in the coal as supplied can be reduced, thereby reducing the quantity of ash and dirt to be removed from the furnace, and the quantity of grit discharged with the gases moving to the chimney when the coal, and coke made from it, are burned. Coal cleaning also reduces the quantity of sulphur in the coal as supplied, though in general the amount of sulphur that can be removed in this way is small in relation to the quantity remaining with the coal. Cleaning can only remove much of the adventitious mineral matter; it cannot remove the inherent ash and sulphur.

The National Coal Board have already done much to improve the efficiency of their cleaning plants and in installing new cleaning equipment; and further equipment is to be provided. But advances in methods of coal cleaning do not necessarily mean that over a period of years there will

be a reduction in the average ash and sulphur contents of the coals delivered to consumers, as the more easily worked seams of the better coals are steadily being worked out. In fact, it seems likely that the overall average sulphur content will rise.

*Efficiency of Utilization*

If productivity is to be increased in Britain, there must be a greater mechanization in industry. This means more power and thus a greater expenditure of fuel unless there is a corresponding increase in the efficiency of utilization of fuel and power. It is important for many reasons that all fuels and sources of energy should be used with maximum practicable and economic efficiency. More efficient utilization means that less fuel is needed to meet a given requirement, and to that extent at least the amount of atmospheric pollution is reduced.

Most of the 37 million tons of coal used in 1951 for domestic requirements was burned in open fireplaces and kitchen ranges of traditional design, whereby most of the heat of combustion of the coal was wasted. Within recent years, efforts have been made by many manufacturers to design more efficient domestic heating appliances burning solid fuel. Open fires have been introduced to provide not only radiant heat, but also convected heat from air warmed by passages around the fire, and many also include a boiler to provide hot water. Then there are the closed and openable stoves, designed for continuous burning, to provide space heating and hot water, and improved appliances to provide for cooking, hot water, and space heating. Most of these modern appliances will satisfactorily burn the solid smokeless fuels, anthracite, low-volatile steam coal, and coke; and with such fuels many of them achieve a heating efficiency at least twice as great as that of the traditional simple open coal-burning fire. The efficiency of these improved appliances, though usually higher with bituminous coal than the old open fire, is not so high as with the solid smokeless fuels; and with bituminous coal they emit smoke and cause the deposition of soot and tarry matter in the chimney. In general, the appliances which can satisfactorily burn the solid smokeless fuels have a thermal efficiency about one-third greater with coke than with bituminous coal.

The rate of production of the more efficient appliances has increased considerably during the past few years. Large numbers have been installed in new houses, and supplies are available for other householders. To aid housing authorities in selecting appliances that are efficient and not too expensive, a list of "approved appliances" is prepared by the Ministry of Fuel and Power in consultation with other Government Departments. Only those appliances are placed on the list that have been shown by tests to reach certain standards of performance. The Fuel Research Station at Greenwich, which also has a branch testing laboratory at Thorntonhall, near Glasgow, is the authoritative testing organization. Other labora-

tories have assisted in the work, and tests are also undertaken at manufacturers' works by officers of the Fuel Efficiency Advisory Service of the Ministry of Fuel and Power. The Coal Utilisation Joint Council issue a list of recommended appliances, including not only those on the official list, but also more costly appliances.

During the last few decades, there has been a great increase in the efficiency of generation of electricity at power stations, which now burn more than 36 million tons of coal a year. In 1921, the average quantity of coal consumed to produce one unit of electricity was 3·4 lb., whereas it is now only about 1·3 lb. At the most efficient base-load stations less than 1 lb. of coal is required for each unit generated. Even with this great improvement, the overall average thermal efficiency of generation is only about 22 per cent. Improvement continues, and it seems likely that within the next 5 or 10 years the overall thermal efficiency of generation of electricity will have reached about 25 per cent. Though the efficiency of generation is low, the efficiency of utilization of electricity is usually high, often approaching 100 per cent; and electricity generating stations can use coals that are unsuitable for many other purposes.

Though attention has been given to fuel economy by railway undertakings, the overall efficiency of use of coal for locomotive transport is low, not because of undue waste in raising steam, but owing to losses in producing power from the steam under difficult conditions. The overall average efficiency is only 5 or 6 per cent. Here is scope for great improvement. Electrification of the railways increases thermal efficiency for the longer journeys to between 10 and 15 per cent, and thereby saves fuel and reduces air pollution. It is uneconomic, however, unless the density of traffic is reasonably high. Efficiency in shunting operations can be increased by extending the use of Diesel engines for shunting. The policy of the railway authority seems to be gradually to extend electrification of suitable lines, and to make greater use of Diesel engines for shunting.

The efficiency of carbonization of coal in coke ovens and at gas works to produce coke, gas, and various by-products, has reached the high figure of 75 to 80 per cent, so that there is not great scope for improvement in this respect, but improvement can still be achieved. Though coke can be used with high efficiency in industrial boilers and for making producer gas, the efficiency with many furnaces is low. In general, the efficiency of gas-fired furnaces is higher than that of corresponding appliances fired with coal or coke. Even so, there is often much waste of heat, particularly when furnaces are far from fully loaded with the materials to be heated.

The direction in which more attention could be given by the coal-carbonization industries with national advantages is in the provision of larger quantities of coke of suitable size, quality, and combustibility for domestic heating appliances. Most gas works of any size use coke of high quality for making water-gas and for heating the retorts or carbonization chambers. More intensive investigation and development work is needed

with the object of using small coke (coke breeze) in place of the larger coke now used at the works. More of the larger sizes could then be released for sale.

Coals of certain properties have to be selected to produce satisfactory coke and gas, and the reserves of such coals are not so great as those of other coals. If the carbonization industries are to expand sufficiently to provide enough coke to replace a large proportion of the coal now used in domestic appliances, a wider selection of coals will have to be used by these industries by blending other coals with those now used for making coke and gas. It will probably also be necessary to increase the quantity of solid smokeless fuel made by blending and briquetting some coals of small size before carbonization.

Most of the quantity of 66·7 million tons of coal given in the fourth item of Table 4 as used for "other industrial purposes" was used for such purposes as raising steam for processes and power, and for heating furnaces. Though the efficiency of generation of steam in large modern boiler installations is often as high as 80 per cent, this high figure can only be maintained if the equipment is kept in first-class condition and is operated under skilled supervision. In many works, little attention has been given to such equipment as boilers and furnaces. Far too much heat, which could have been usefully employed, has been carried away in hot gases and liquids, large quantities of steam have been wasted, and there has been unnecessary loss of heat owing to inattention to the insulation of equipment and steam piping. With some furnaces for heating metals, no more than 5 per cent of the heat value of the fuel is put to useful purpose. Though there has been improvement in recent years, largely as a result of the advisory service of the Ministry of Fuel and Power and the extension of facilities for training operators, there is room for much further improvement.

*Smoke*

The domestic open fire burning bituminous coal produces much more smoke per ton of coal than any other fuel-burning equipment in general use. The problem has been studied by many investigators, but there is no early prospect of designing an open fire that will burn bituminous coal without emitting an appreciable amount of smoke. With modern open firegrates of suitable type, there is no difficulty in burning anthracite, low-volatile coal, and coke, provided that the fuel is in pieces of suitable size. If sufficient quantities of the solid smokeless fuels suitable for domestic heating could be made available, and if most householders could be persuaded to replace their old open fires by the modern appliances designed to burn coke, and to use only solid smokeless fuels in place of bituminous coal, there would be a great reduction in pollution by smoke, and greater fuel efficiency in domestic heating. Unfortunately, there is not available nearly enough suitable solid smokeless fuel to replace the bitu-

minous coal now required by householders. To improve the position, the coal-carbonization industries must be induced to pay more attention to the production of coke for the domestic market, appliances to burn coke must be made as attractive as practicable in appearance, and householders must be educated to use solid smokeless fuels in the new appliances.

Even with a determined effort, the change to the situation when the vast majority of householders can and will replace bituminous coal by the smokeless fuels will take many years—perhaps a generation ; but this is no reason why a determined effort should not be made. With greatly increased production of coke there will be larger quantities of gas and tar for sale, and there must be developed a balanced market to ensure that all the products can be sold at such prices that there is an overall economic return. This will take time and require heavy capital expenditure. As already mentioned, the thermal efficiency of coke in suitable domestic appliances is in general one-third greater than that of coal. The calorific value of coke is somewhat less (about 10 per cent) than that of the coal from which it is made. This means that suitable coke is directly worth 15 to 20 per cent more per ton than coal as a heating agent to the householder, without any allowance for the fact that it is a smokeless fuel. One disadvantage of coke is that its bulk density is only about one-half that of coal. To overcome this disadvantage, the householder must be provided with large space for fuel storage to meet the main heating load in the winter.

It is common practice with many blocks of flats and other large buildings to provide the space heating and hot water from a central boiler. With boilers of this kind, which are much larger than for individual houses, it is practicable to burn bituminuous coal with relatively high efficiency and little smoke. This principle of central heating has been extended in some areas in Britain and abroad to provide district heating for groups of large buildings and for housing estates. With housing estates in this country, a uniform charge for the heating service is added to the rent of the house, for there has not been developed an entirely satisfactory method of measuring the heat taken by each house from the service. As a result, householders have not been so economical in their use of space heating and hot water as in houses with individual heating appliances. Investigations are needed with the object of devising and developing satisfactory heat meters for district-heating schemes, if the application of such schemes is to be extended on any large scale. Consideration has been given to the possibilities of thermal-electric stations in which steam is first used to generate electricity, and then the exhaust steam from the turbines is used to provide central heating and hot water in neighbouring areas ; this system has been tried abroad and is being tried in one district in this country. Though there might well be an extension of district heating schemes of one kind or another in suitable areas, it is unlikely that it will be economic to provide heating by such schemes for more than a small proportion of the dwellings of the country.

With large modern industrial boiler installations, such as those at electricity-generating stations, there is no difficulty in burning bituminous coal with the emission of little or no smoke, provided that the equipment is kept in order and properly operated.

In the past, with the thousands of hand-fired boilers burning bituminous coal at numerous industrial works, it has been difficult to avoid the emission of smoke, especially for a period immediately after stoking. The difficulties in avoiding smoke emission have arisen partly from inadequate arrangements for controlling the amount of secondary air admitted to ensure complete combustion of the combustible gases and tarry vapours, and partly from inadequate training of the boiler firemen. As a result of extensive work by the Fuel Research Station, equipment has been developed to replace the firedoors ordinarily fitted to hand-fired marine boilers and boilers of the Lancashire type, where by the emission of smoke can be almost eliminated. The equipment is not expensive, and is simple to construct and operate. It provides at the right time and in the right way the extra air required for the necessary period after stoking to burn the smoky volatile matter evolved from the fresh charge of coal. Tests on the full-scale marine and Lancashire boilers at the Research Station, and tests on the boilers of a merchant ship during a voyage of several weeks, have shown that with average coal not only is smoke eliminated, but the quantity of coal consumed for the same amount of steam raised is reduced by 5 or 6 per cent. These smoke-eliminator doors (*Fig. 4*) are now being produced by several manufacturers, and many hundreds of them have been fitted to industrial boilers.

There is no doubt that methods and equipment have been developed whereby most of the boilers in industry could be operated without undue emission of smoke, and with greater efficiency in the use of the fuel. What is required is extended application of the equipment and methods, and, in general, better training and supervision of boiler operators.

With certain types of metallurgical furnaces using coal, satisfactory methods of avoiding the emission of smoke, without detriment to the quality of the metal, have not yet been developed. This is a difficult problem requiring extensive scientific and technical investigation, and a new outlook on the part of those controlling the furnaces in the older metallurgical industries. A beginning has been made in attacking this problem by a small team under Professor R. J. Sarjant at the University of Sheffield, as part of the programme of the Fuel Research Station of the D.S.I.R., and with some assistance from the Sheffield, Rotherham and District Smoke Abatement Committee.

*Grit*

The gases entering the flues of industrial boilers and other furnaces fired with solid fuel carry appreciable quantities of ash or grit, including some solid carbonaceous matter, particularly if the furnaces are fired with

pulverized fuel, or are operated with forced draught. Most of this grit can be removed by grit catchers such as those installed at many large electricity generating stations. Modern designs of cyclone dust extractors remove most of the larger particles and can achieve an overall efficiency of dust extraction of the order of 80 per cent. Electrostatic precipitators can take out the smaller particles and bring up the efficiencies to more than 90 per cent. The application of efficient dust extractors in industry generally should be extended.

*Sulphur Dioxide*

On average at the present time, British coals and the cokes made from them contain about 1·5 per cent of sulphur. Some coals as supplied to consumers contain less than 1 per cent of sulphur, many contain 1 to 2·5 per cent, and some have more than 2·5 per cent. Residual fuel oils from petroleum generally contain from 2 to 4 per cent. When the coal, coke, or oil is burned, whether in industrial or domestic appliances, most of the sulphur is converted to sulphur dioxide to be discharged with the chimney gases. Even with highly efficient boiler installations achieving complete combustion without unduly large quantities of excess air, however, the concentration of sulphur dioxide in the flue gas is ordinarily no more than about 0·1 per cent by volume. Since the volume of flue gas produced for each ton of fuel burned is in the region of 350,000 to 500,000 cubic feet, the volume discharged each day from a large power station using 2,000 tons of coal a day is between 700 million and 1,000 million cubic feet. The problem of removal of sulphur dioxide, therefore, is that of bringing enormous volumes of gas into intimate contact with some solvent or reagent which will rapidly dissolve or react with the sulphur dioxide to give a solution or other product easily separated from the gas. Further, the process must not give rise to an objectionable solid, liquid, or gas which cannot be disposed of without causing damage or nuisance, and the overall cost must not be too high.

The only processes that have been developed to the stage of application on a large scale are those that have been operated at the electricity generating stations at Battersea and Bankside, and at Fulham. No other processes have been operated at any other places either in Britain or abroad.

In the Battersea process, the flue gas is washed with Thames River water with the addition of small quantities of chalk; and the solution so obtained is oxidized by air to convert the dissolved sulphite into sulphate. The water containing the sulphate is then discharged to the river. The process, which can remove 85 or 90 per cent of the sulphur dioxide, is dependent on large supplies of water containing sufficient alkali to neutralize most of the acid oxides of sulphur, and on ensuring that the liquid effluent does not unduly pollute the body of water into which it is discharged. A quantity of water in the region of 35 tons is required for every

ton of coal burned under the boilers, or about 15 million gallons a day for an installation burning 2,000 tons of coal a day. There are few inland sites in Great Britain where the necessary quantity of water could be obtained, and where the liquid effluent could be discharged without serious difficulty. The process has been found to be troublesome in operation, causing corrosion of scrubbers and flues and other difficulties. It also greatly reduces the temperature of the flue gas from about 120° C. to about 50° C., thereby reducing the "effective" height of the chimney and so increasing the tendency of the gases to descend to near ground level under certain meteorological conditions. Experience, combined with investigations and observations, have led to the conclusion that any process that so greatly lowers the temperature of the flue gas may have disadvantages at least equal to the advantages, so far as air pollution near ground level in the neighbourhood is concerned, unless it continuously removes not less than 85 per cent of the sulphur dioxide in the flue gas treated. With new plant at present prices of labour and materials, the overall cost of the process is equivalent to adding 7*s*. or 8*s*. to the cost of every ton of coal burned.

During the war, the flue-gas-washing process was stopped at Battersea, because it gives rise to an easily visible plume of water vapour from the top of the chimney. Since the war steps have been taken to reinstate the process, and at the present time about two-thirds of the flue gas from Battersea electricity station is being so treated.

Arrangements have now been made to treat the gases from the new electricity generating station at Bankside (opposite St Paul's Cathedral) by the Battersea process, modified in the light of experience. At the new station at Bankside the boilers are being fired with oil containing 3 to 4 per cent of sulphur. It is unlikely that the process will be operated at any riverside stations in London in addition to those at Battersea and Bankside because of the possible effect on the river.

By the Fulham (Howden—I.C.I.) process the flue gas is scrubbed with water to which sufficient chalk is added to convert the sulphur dioxide into calcium sulphite, which is then oxidized by air to calcium sulphate. From the washing water, calcium sulphate is separated as a sludge, and the water is then re-used with the addition of more chalk and some water to make up for loss by evaporation. In this process, though there is little or no liquid effluent, there is a large quantity of troublesome sludge for disposal. At present prices, the overall cost of the process is equivalent to an addition of about 10*s*. to the cost of every ton of coal burned.

Since this process also gives rise to an easily visible plume of water vapour, it was stopped during the war and much of the equipment was moved for use for other purposes elsewhere. This flue-gas-washing process at Fulham has not been reinstated partly because of the materials and labour involved and the cost, and partly because there were many difficulties in its operation.

A few years ago, at the request of the Electricity Commissioners, the D.S.I.R. set up a Working Group, under the chairmanship of the Director of Fuel Research, to study the problem of removal of sulphur dioxide from the flue gases of electricity generating stations. Various processes that have been suggested as possibilities have been considered by the Group, and several have been investigated experimentally. Consideration has also been given to the possibilities of improving and reducing the cost of the processes used at Battersea, Bankside, and Fulham, but there seems to be little prospect of effecting any substantial reduction in the overall cost of these processes.

The most promising new process suggested involves the use of water to which ammonia is added. If fairly pure ammonia is used, the final product is ammonium sulphate, which is valuable as a fertilizer for agriculture. If the ammonia is added in the form of concentrated crude gas liquor, the final products are ammonium sulphate (more than 90 per cent by weight) and sulphur (nearly 10 per cent by weight), for the gas liquor contains not only ammonia but also some sulphur as sulphide and polysulphides.

Investigations of this ammonia process for the removal of sulphur dioxide from flue gases had been undertaken on a small scale by the staff of the power station at Fulham in co-operation with Simon-Carves, Ltd. (See *Fig. 5.*) These investigations could not be continued, so arrangements were made by the D.S.I.R. to continue the work at the Fuel Research Station. In addition to experiments on a laboratory scale to obtain fundamental information, the work at the Research Station has included long series of experiments with two small pilot plants, one treating 1,000 cubic feet of flue gas per hour, and the other treating 25,000 cubic feet per hour. Concentrated gas liquor from gas works has been used in most of the experiments, in which about 90 per cent of the sulphur dioxide has been removed. From estimates that have been made on the basis of the results of the experiments, it seems likely that the overall net cost of this process on a full-scale at a power station would be in the region of 2*s.* 6*d.* for each ton of coal burned under the boilers, with coal containing about 1·2 per cent of sulphur. It may be that with coals containing more sulphur (say 2 per cent or more) the income from the products would cover the whole cost of the process. Pilot-plant experiments with this Fulham/Simon-Carves process on a much larger scale are now desirable. If the process can be developed to the stage of successful operation on a full commercial scale, it has the advantage that it would recover the sulphur in useful form instead of as a waste liquid or solid, and it should be much less costly than those in operation at Battersea and Bankside and previously operated at Fulham. It should be mentioned, however, that the total quantity of concentrated gas liquor that could be produced at gas works at the present time would be sufficient for the flue gases from only about six million tons of coal a year, or about one-sixth of the present consumption of coal by the British Electricity Authority.

*Carbon Monoxide*

There have never been any serious efforts substantially to reduce the quantities of carbon monoxide discharged into the atmosphere from the combustion of solid and liquid fuels, and the problem is beset with many difficulties. The quantities discharged from the chimneys of domestic and industrial establishments are large, but the discharges are well above ground level. Improvements in the efficiencies with which the fuels are used in such establishments must cause a reduction in the quantities of carbon monoxide produced.

The most important problem from the aspect of public health is the discharge of carbon monoxide in the exhaust gases of spark-ignition engines of road vehicles, for the discharges are at low level. It might be possible so to re-design such engines and their carburettors that there is much less carbon monoxide in the exhaust gas, but it is likely that the rates of acceleration would be substantially reduced. Compression-ignition engines in good condition, as already mentioned, give relatively little carbon monoxide, but they give other deleterious products such as oxides of nitrogen and aldehydes ; and when the engines are in bad condition they emit smoke and unpleasant odours.

Very much more information is needed on the concentrations of carbon monoxide in the air in different places at different times of the day and year, particularly near ground level, and the effects of such factors as changes in meteorological conditions, and density of traffic, before it can be decided to what extent pollution by carbon monoxide should be reduced.

## Summary and Conclusions

The estimated quantities of the main air pollutants discharged from the use of coal, oil, and their products as fuels in Great Britain are given in Table 7.

Measurements of the intensity of air pollution in very many parts of the country over a number of years have shown that the pollution is heavy in thickly-populated industrial areas, especially during the winter months ; it is serious in the thickly-populated areas at times of fog in the winter.

The detrimental effects of air pollution from the use of fuels are of many kinds. They include damage to structures and materials, involving more frequent replacement, extra labour and materials in cleaning and painting buildings and equipment, more frequent laundry work and replacement of fabrics, damage to agriculture, and detriment to health. From estimates that have been made, it seems that the money equivalent of the damage to buildings, equipment, fabrics, and agriculture, together with the direct waste of fuel in unburnt products, is in the region of £100 million to £150 million a year ; and this estimate excludes damage to health.

On the basis of existing knowledge there could be obtained within a reasonable time a considerable reduction in the quantities of smoke and grit discharged from most of the industrial uses of fuels. Progress will not be made at a satisfactory rate unless there are determined efforts on the part of all concerned. Industry should make greater use of competent fuel technologists, and operators of fuel-using equipment should be adequately trained and supervised.

Appreciable reduction in emission of smoke from domestic appliances in individual houses is dependent either on a great increase in the supplies of suitable solid smokeless fuels and the installation in much larger numbers of appliances to burn these fuels in place of coal, or, on the development of appliances satisfactory to the British householder to burn coal smokelessly. The prospects of developing appliances acceptable to householders to burn coal smokelessly are not promising. Attention should be directed, therefore, to the provision of greatly increased quantities of solid smokeless fuels suitable for household grates designed to burn such fuels. This means an extension of the coal-carbonization industries, with the use by these industries of a wider range of coals than at present. A development in this direction can only take place gradually, because of heavy capital expenditure, and the need to create a balanced market not only for the coke but also for gas and the other products of carbonization.

The scientific and technical investigations in progress on the use of a wider range of coals and on the most appropriate systems of carbonization to produce cokes suitable for domestic appliances should be intensified. Much more attention should be given to the use of coke breeze for heating retorts and for making water-gas and producer gas at gas works, so that the larger sizes of coke now used for these purposes can be released for sale.

There might well be some extension of district heating in selected thickly populated areas in which the cost would be reasonable, but there is first required a satisfactory method of measuring the heat from the service that is used by each householder ; investigations should be intensified with the object of developing a really satisfactory heat meter. District-heating schemes efficiently operated would reduce pollution by smoke.

Greater use of gas and electricity in place of bituminous coal brings about a reduction in smoke emission, but, for reasons of cost, gas and electricity are unlikely to displace solid fuel in meeting the major part of the main heating load in the winter. A well balanced division in the use of solid fuel, gas, and electricity should ensure the maximum practicable load factors for gas and electricity supplies.

Though it is not known with certainty, it seems probable that the deleterious effects of the other air pollutants, particularly oxides of sulphur, would not be so great in the absence of pollution by smoke (especially tarry smoke) and grit, as in their presence.

There are practicable methods of greatly reducing the quantity of grit discharged into the air ; their use should be extended.

There has not yet been developed to full commercial scale any satisfactory method at reasonable cost of preventing pollution by oxides of sulphur from the combustion of coal, coke, and fuel oils. For installations using very large quantities of these fuels, the effects of this pollution at or near ground level can to some extent be mitigated by coal cleaning, by judicious selection of coals in relation to sulphur content, by careful selection of sites for such installations, and by discharging the gases through chimneys that are high in relation to surrounding buildings and to the topography of the locality. Chimneys at power stations are now usually built to a height of 2½ times that of nearby buildings. Meanwhile, investigations of methods of removal of sulphur dioxide, with recovery of the sulphur in useful form, should be continued and intensified.

Much more information is required on the intensity of different forms of pollution, including not only smoke, grit, and sulphur, but also carbon monoxide, fluorine, oxides of nitrogen, hydrocarbons, aldehydes, etc., on the many factors affecting their dispersion, and on methods of prevention or reduction of the pollution. There is also required much more information on the detrimental effects of pollution, particularly on health. Investigations in these fields of research should be greatly intensified.

Paper 5909

# "The Design of Simply Supported Prestressed Concrete Beams for Ultimate Loads"

by

**Frederick William Gifford, B.Sc.(Eng.), Ph.D.**

(*Ordered by the Council to be published with written discussion*)†

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SYNOPSIS

The ultimate-load theory is developed for rectangular and flanged beams, taking strains into account, and assuming that the distribution of strain across a section at failure is linear. These equations are combined with working-load equations to enable the factor of safety to be introduced, and for the more usual case of beams failing by concrete crushing, a simple method, using the steel stress/strain curve, is given to enable either the factor of safety to be specified and the steel stresses to be obtained, or vice-versa.

Using the equations and taking suitable values of the various factors involved, curves have been plotted of factor of safety against steel stress at working load for ranges of various factors concerned, to enable their influence to be assessed.

Experimental values of these factors have been collected and considered, and recommendations for values to be used are given.

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It can be shown [1] that if limiting concrete working stresses are specified, then the concrete section and location of steel may be found to satisfy the loading conditions; in addition, the steel *force* required is fixed, and if the steel stress is arbitrarily chosen the area of steel can readily be calculated. No reference will have been made to the factor of safety, which can be calculated; the following method, however, enables the factor of safety to be assumed, and the steel area and stress found to suit.

It has been shown [1] that for a general concrete section subject to full working-load moment $M_L$, if the top fibre stress is $c_{tL}$ and the bottom fibre stress is $c_{bL} = 0$, then:

$$\frac{M_L}{BD^2} = c_{tL}\left\{\frac{1}{K} + (1-j)(1-j-f)\right\} \quad . \quad . \quad . \quad (1)$$

$$p_{tL} = c_{tL}(1-j) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

† Correspondence on this Paper should be received at the Institution by the 15th August, 1954, and will be published in Part III of the Proceedings. Contributions should be limited to about 1,200 words.—Sec. I.C.E.

[1] The references are given on p. 143.

*where* $D$ denotes depth of concrete section

$BD$ ,, area of net concrete section

$\frac{1}{K}BD^3$ ,, second moment of area of net concrete section about the centroid

$p$ ,, $\frac{A_s}{BD}$ where $A_s$ denotes area of prestressing steel

$jD$ ,, depth of centroid of net concrete section below top flange

$fD$ ,, height of centroid of steel from bottom flange

$t_L$ ,, steel stress

A section subject to bending may fail in two ways :

(1) crushing of the concrete, when the concrete reaches its ultimate stress and strain ; or

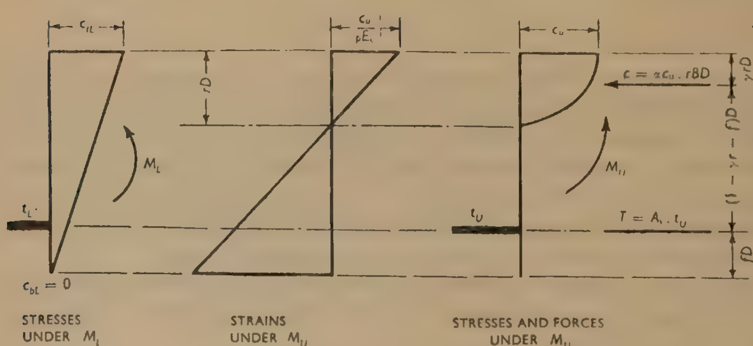
(2) the steel reaching its ultimate stress.

There is also the possibility that these may combine to cause failure. In almost every case, beams (whether bonded or unbonded) will fail in practice by the concrete crushing, the exception being for beams of bonded construction having a very low proportion of steel with a very high prestress.

*Rectangular Beams—Failure by Concrete Crushing*

Referring to *Figs 1*, which show the working-load stress-distribution,

*Figs 1*




and the stresses, forces, and strains under $M_U$, the ultimate moment of the beam :

$c_u$ denotes the concrete stress in the top fibre, this being the crushing stress of the concrete in the beam

$\frac{c_u}{{}_pE_c}$ ,, the strain corresponding to $c_u$. ${}_pE_c$ is the plastic $E$ of the concrete

$t_U$ denotes the stress in the steel at the ultimate load of the beam
$t_{ult}$ ,, the ultimate stress of the steel
$\alpha c_u$ ,, the average concrete stress over the depth of the compression zone
$rD$ ,, depth of neutral axis below the top fibre
$\gamma rD$ ,, depth of the centre of compression in the concrete below the top flange.

For working loads, equations (1) and (2) become :

$$\frac{M_L}{BD^2} = \tfrac{1}{2}ct_L(\tfrac{2}{3} - f) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$pt_L = \tfrac{1}{2}ct_L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

For ultimate loads, considering horizontal forces and moments :

$$pt_U = \alpha rc_u \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

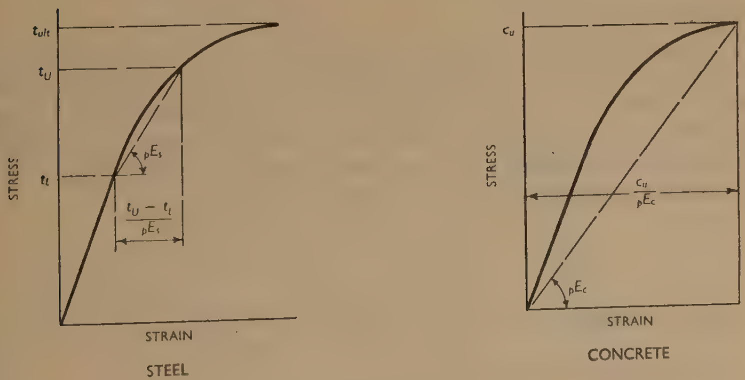
$$\frac{M_U}{BD^2} = pt_U(1 - \gamma r - f) = \alpha rc_u(1 - \alpha r - f) \quad . \quad (6)$$

*Unbonded Beams*

By equating the total elongation of the concrete adjacent to the steel for its whole length, and the corresponding elongation of the steel, when

*Figs 2*

(*a*) (*b*)




STRESS/STRAIN CURVES

moment changes from $M_L$ to $M_U$, and eliminating the length-of-steel term from both sides, it can be shown that :

$$\frac{t_U - t_L}{_pE_s} = \frac{c_u(1 - r - f)F}{_pE_c \,.\, r} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where $\frac{t_U - t_L}{{}_pE_c}$ is the change of strain in the steel from $t_L$ to $t_U$ (see *Fig.* 2 (*a*)).

$F$ is a strain factor introduced because, in unbonded beams, the change of strain in the steel is approximately constant over the length of the steel (since there is no bond and only frictional forces are present to cause variation), whereas the effective strain in the concrete adjacent to the steel has a high value at the section of failure but will have small values (often negative) at the supports where the section remains uncracked and the bending moment is small. It is therefore necessary to equate the total elongations along the line of the steel, or in effect the average strain changes. The $F$-factor for unbonded beams is that which relates the average effective concrete strain change to its value at the section of failure ; it has a value of the order of 0·2 for common cases, the effective strain in the concrete adjacent to the steel in zones where cracking has occurred being the strain calculated in terms of the strains of the compressive zone—which are those governing the main deformations of the member and hence the elongation of the steel.

It will be observed that whilst equation (7) is based on strain changes, the concrete strains are absolute values. The reason for this is that a term representing the average concrete strains adjacent to the steel under working load, being small, has been neglected. Also the steel stresses are, strictly, horizontal components where the cable slopes, necessitating frictional forces to be present.

*Bonded Beams*

With perfect bond, equation (7) applies if $F = 1$. However, at a crack a local slip must occur, and this may cause the steel-strain change at the point of failure to be a little higher than its average value, which is equal to the effective concrete-strain change adjacent to the steel. This can be accommodated by using an $F$-factor which would be greater than unity.

Combining equations (4) and (5), and (3) and (6) :

$$\beta = \frac{t_L}{t_U} = \tfrac{1}{2} \cdot \frac{c_{tL}}{\alpha c_u r} \qquad (8)$$

$$\text{Factor of safety (FS)} = \frac{M_U}{M_L} = 2r\,\frac{\alpha c_u(1 - \gamma r - f)}{c_{tL}(\frac{2}{3} - f)} \qquad (9)$$

Eliminating $r$ from (7), using (8) :

$$\frac{t_U - t_L}{{}_pE_s} = X\beta - Y \qquad (10)$$

where

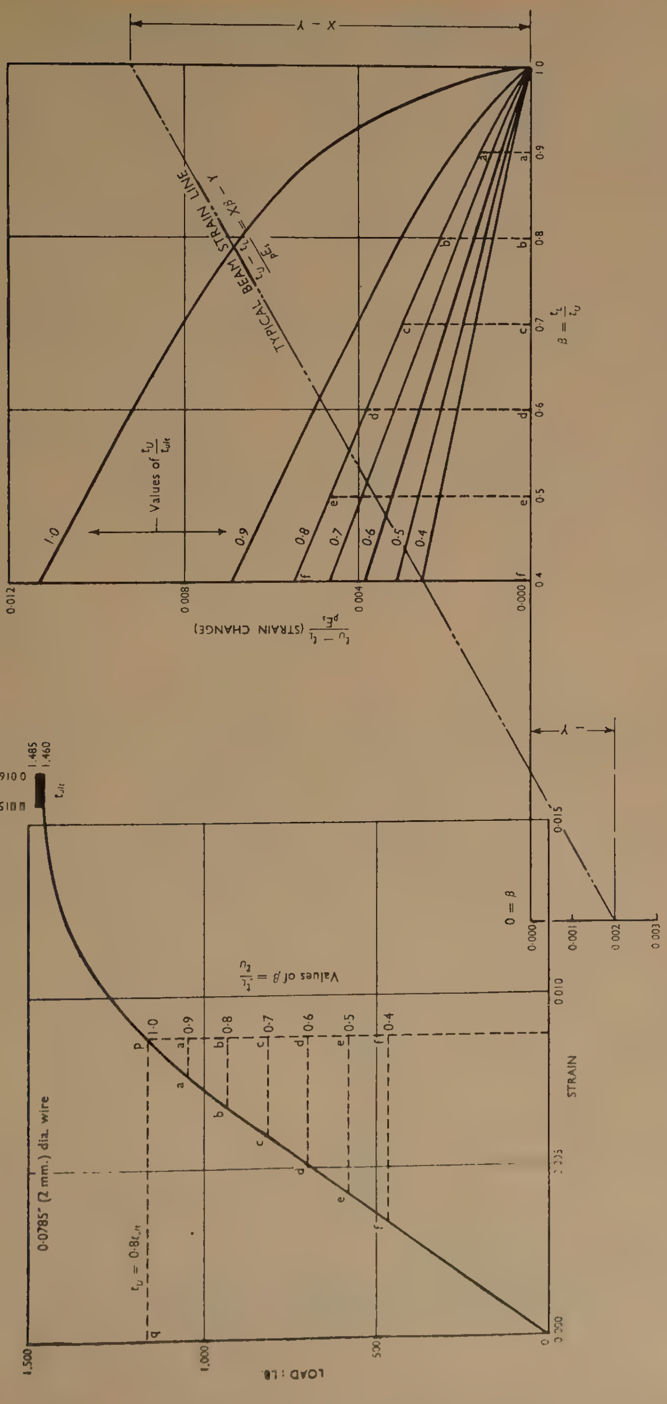
$$X = F\,\frac{c_u}{{}_pE_c}(1 - f)\frac{2\alpha c_u}{c_{tL}} \qquad (11)$$

and

$$Y = F\frac{c_u}{{}_pE_s} \qquad (12)$$

*Figs 3*

(a) (b)




LOAD/STRAIN CURVE

DERIVED STRESS-RATIO/STRAIN-CHANGE CURVES

Equations (8), (9), and (10) are those required for design, and, of the terms present, $c_{tL}$ and $f$ are known from working load conditions; $\gamma, \alpha c_u, \frac{c_u}{_pE_c}$ and $F$ can be assessed; and the factor of safety is specified for ultimate load conditions. The terms $t_L$, $t_U$, $_pE_s$, and $r$ are not known except that the first three must be related by the stress/strain curve of the steel. From equation (9), $r$ may be calculated, and from (8), $\beta = \frac{t_L}{t_U}$. If the stress/strain equation of the steel were linear, $_pE_s$ would be known as the normal of $E$ of the steel, and there would be two equations to solve for $t_L$ and $t_U$. Since this is not the case, a graphical solution is necessary.

Equation 10 relates the change of strain in the steel as the stress changes from $t_L$ to $t_U$ and the stress ratio $\beta = \frac{t_L}{t_U}$ for the beam, and it is necessary to find the stresses which will satisfy these values for the steel. This may be readily done by plotting the same relationship for the steel.

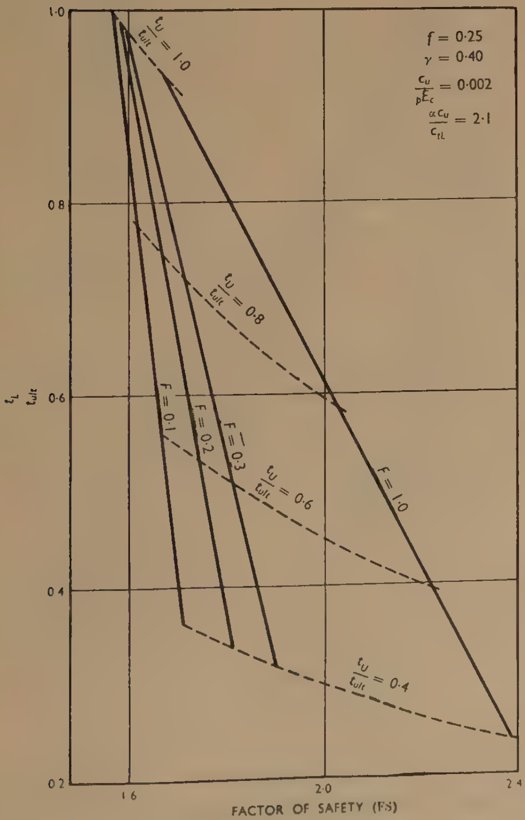
*Fig. 3(a)* shows a load/strain curve for the 2-millimetre steel available at Imperial College, and *Fig. 3 (b)* is the $\frac{(t_U - t_L)}{_pE_s} : \frac{t_L}{t_U}$ curve obtained from it. It will be seen to be a family of curves for various values of $\frac{t_U}{t_{ult}}$, and is used by plotting the point corresponding to the $\frac{t_L}{t_U}$ and $\frac{(t_U - t_L)}{_pE_s}$ values obtained from equations (8) and (10), and reading off the corresponding $\frac{t_U}{t_{ult}}$ value. The ultimate stress of the steel, $t_{ult}$, is known, and from that may be found $t_U$, then $t_L$, and finally $p$ (and $A_s$) from equation (4).

Alternatively, a reverse approach may be made, this being particularly useful for plotting the influence of various factors on the factor of safety (FS). Plot the strain equation of the beam, that is, equation (10), on the $\frac{t_U - t_L)}{_pE_s} : \frac{t_L}{t_U}$ curves of the steel (this being done readily by plotting strain change of $- Y$ at $\beta = 0$, and $X - Y$ at $\beta = 1$, and joining by a straight line), and read off the values of $\frac{t_L}{t_U}$ corresponding to the intersection of the straight line with the lines for $\frac{t_U}{t_{ult}} = 1{\cdot}0, 0{\cdot}9, 0{\cdot}8, 0{\cdot}7$, etc. Using these values, calculate $t_U$, $t_L$, $r$, and (FS), and plot the factor of safety against $\frac{t_L}{t_{ult}}$.

The method of plotting the $\frac{(t_U - t_L)}{_pE_s} : \beta$ curves of the steel is straightforward, and need only be done once for a given specimen of steel. With

reference to *Figs 3*, to plot the curve of, say $\frac{t_U}{t_{ult}} = 0{\cdot}8$, on the stress/strain curve of the steel, draw the horizontal line pq at stress $t_U = 0{\cdot}8\ t_{ult}$, and drop perpendicular pr. Draw horizontal lines aa, bb, cc, dd, etc. at values of $\frac{t_L}{t_U} = 0{\cdot}9, 0{\cdot}8, 0{\cdot}7, 0{\cdot}6$, etc., and transfer these values, plotting them vertically on the $\frac{(t_U - t_L)}{pE_s} : \beta$ graph at the appropriate value of $\beta = \frac{t_L}{t_U}$.

*Fig. 4*

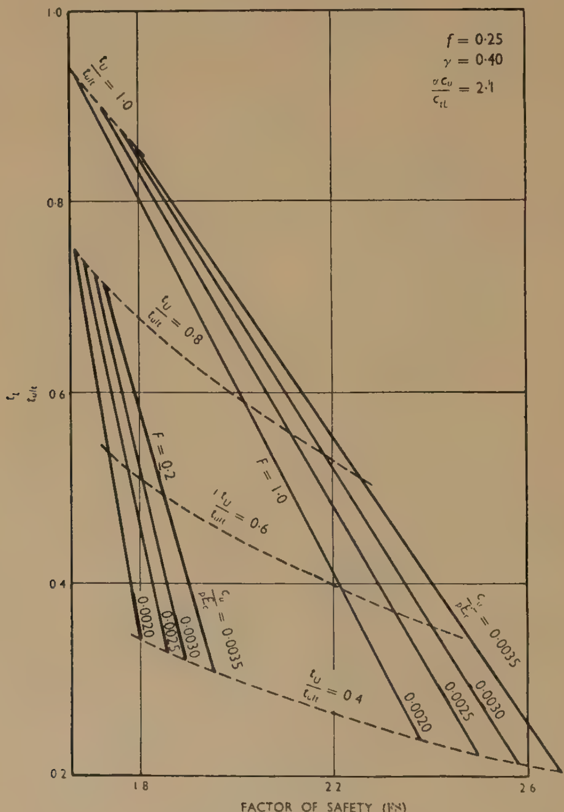



VARIATION OF (FS) WITH $F$

With a view to studying the influence of various factors on the factor of safety, a series of (FS) : $\frac{t_L}{t_{ult}}$ curves have been plotted for rectangular

beams, using the stress/strain curve for the 2-millimetre steel given earlier. These curves are reproduced as *Fig. 4* (variation of $F$), *Fig. 5* $\left(\text{variation of } \frac{c_u}{pE_c}\right)$, *Fig. 6* $\left(\text{variation of } \frac{\alpha c_u}{c_{tL}}\right)$, and *Fig. 7* (variation of $f$). The values oı

*Fig. 5*



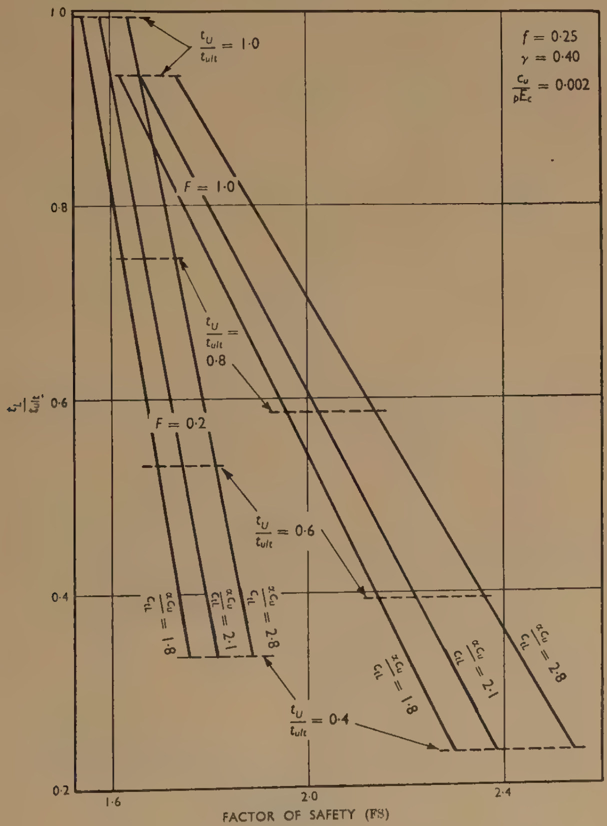

VARIATION OF (FS) WITH $\frac{c_u}{pE_c}$

$\frac{\alpha c_u}{c_{tL}}$ are based on $\alpha c u = 0{\cdot}6$ and $0{\cdot}7$, and $c_{tL}$ = one-third and one-quarter of the cylinder strength. The value $\frac{c_u}{pE_c} = 0{\cdot}002$ corresponds to a safe limiting concrete crushing strain measured in tests.

Inspecting the curves, it is noticeable that they are all approximately

linear. Taking values at $\frac{t_L}{t_{ult}} = 0{\cdot}5$, variation of $F$ from 0·2 to 1·0 (*Fig. 6*) gives (FS) = 1·73 to 2·11, indicating clearly the fundamental difference between an unbonded and a bonded beam, the former having a low (FS). Over the range of $F = 0{\cdot}1$ to 0·3 (FS) = 1·68 to 1·81, showing that the exact

*Fig. 6*

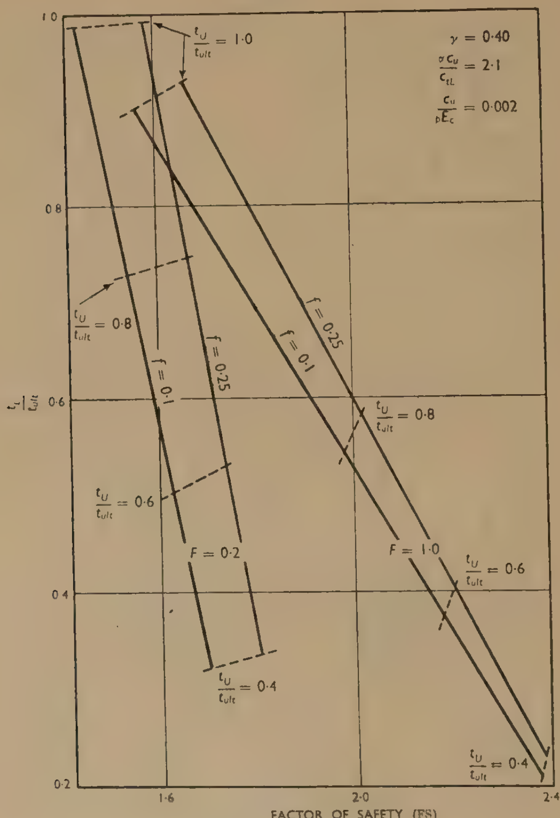



VARIATION OF (FS) WITH $\frac{\alpha c_u}{c_{tL}}$

value of $F$ is not of great concern provided that the order is correct. From *Fig. 5*, $\frac{c_u}{pE_c} = 0{\cdot}002$ to 0·0035 gives (FS) = 2·11 to 2·27 for $F = 1$, and 1·75 to 1·85 for $F = 0{\cdot}2$; the higher concrete strains do improve the factor of safety, but not to a very large extent. From *Fig. 6*, variation of $\frac{\alpha c_u}{c_{tL}}$

from 2·1 to 2·28 gives (FS) = 2·10 to 2·23 for $F = 1{\cdot}0$, and 1·75 to 1·82 for $F = 0{\cdot}2$, showing that decrease in the working stress $c_{tL}$ increases the factor of safety, but not in inverse proportion. From *Fig.* 7, $f = 0{\cdot}1$ to 0·25 gives (FS) = 2·03 to 2·11 for $F = 1{\cdot}0$, and 1·63 to 1·75 for $F = 0{\cdot}2$; it is noticeable that the increase is greater for the lower $F$-value.

*Fig. 7*




Variation of (FS) with $f$

It would appear that even under favourable conditions, the factor of safety with $\dfrac{t_L}{t_{ult}} = 0{\cdot}5$ is not likely to exceed 2·0 for an unbonded beam, and 2·5 for a bonded beam, unless an $F$-value appreciably in excess of unity can be obtained for bonded beams. This will be understood readily when it is realized that the factor of safety can come from two sources only —increase of steel stress and increase of lever arm.

*Sections with Rectangular Top Flanges—Failure by Concrete Crushing*

For working load, $M_L$, equations (1) and (2) apply. The width of the top flange is $'B$, its thickness $bD$; where the web is in compression under ultimate load, its effect is neglected.

*Neutral Axis within the Top Flange*

By analogy with equations (8) to (12) it can be shown that:

$$\beta = \frac{t_L}{t_U} = (1-j)\frac{c_{tL} \, . \, B}{\alpha c_u{}' \, . \, B} \cdot \frac{1}{r} \qquad (13)$$

$$(\mathrm{FS}) = \frac{M_U}{M_L} = \frac{\alpha c_u \, . \, 'B(1-\gamma r - f)}{c_{tL} \, . \, B\left\{\frac{1}{K} + (1-j)(1-j-f)\right\}} \, . \, r \qquad (14)$$

$$\frac{t_U - t_L}{{}_pE_s} = X\beta - Y \qquad (15)$$

$$X = F \, . \, \frac{(1-f)\alpha c_u \, . \, 'B \, . \, c_u}{(1-j)c_{tL} \, . \, B \, . \, {}_pE_c} \qquad (16)$$

and

$$Y = F \frac{c_u}{{}_pE_c} \qquad (17)$$

The use of equations (13) to (17) is exactly the same as for a rectangular section. It will be realized that, if $r$ exceeds $b$, then the neutral axis below the top flange, and of course the equations do not apply.

*Neutral Axis below the Top Flange*

It is more convenient here to assume the centre of compression under ultimate load to be at $\gamma bD$ below the top flange, and not $\gamma rD$ as before. The relevant equations are:

$$\beta = \frac{t_L}{t_U} = (1-j)\frac{c_{tL} \, . \, B}{\alpha c_u \, . \, 'B} \cdot \frac{1}{b} \qquad (18)$$

$$(\mathrm{FS}) = \frac{M_U}{M_L} = \frac{\alpha c_u \, . \, 'B(1-\gamma b - f)}{c_{tL} \, . \, B\left\{\frac{1}{K} + (1-j)(1-j-f\right\}} \cdot b \qquad (19)$$

$$\frac{t_U - t_L}{{}_pE_s} = F\frac{c_u(1-r-f)}{{}_pE_c \, . \, r} \qquad (20)$$

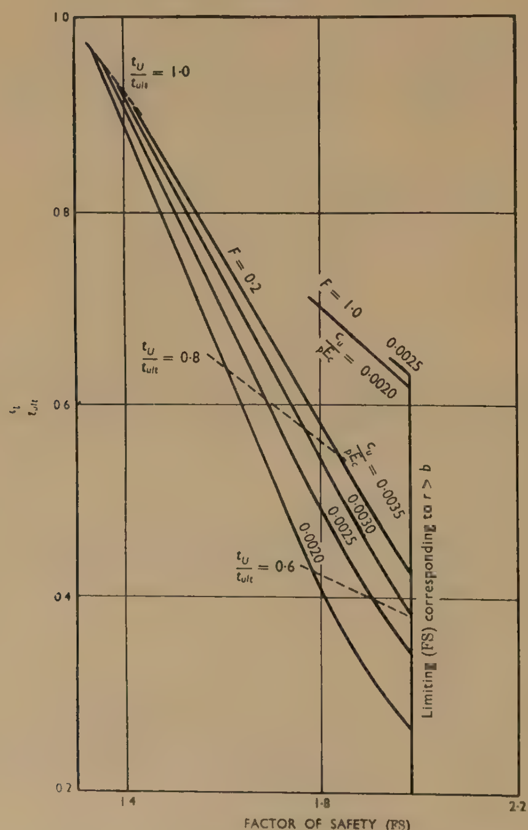
Equations (18) and (19) clearly show that for this condition the factor of safety and the steel stress ratio are fixed—clearly owing to the fact that the compression in the top flange is independent of the depth of the neutral axis.

since $r > b$, from (20):

$$\frac{t_U - t_L}{{}_pE_s} < F\frac{c_u(1-f)}{{}_pE_c \, . \, b} - F\frac{c_u}{{}_pE_c} \qquad (21)$$

The maximum value of $\dfrac{t_U - t_L}{pE_s}$ will be that corresponding to $r = b$, and this or any lesser value can be used. Referring to the $\dfrac{t_U - t_L}{pE_s} : \beta$ diagram for the steel, the point corresponding to the maximum steel-strain change and the particular $\beta$-value can be plotted, and quite clearly this

*Fig. 8*




VARIATION OF (FS) WITH $F$ FOR FLANGED SECTION

maximum value corresponds to the most economical case, since it gives maximum $t_L$ (and hence minimum $p$). If the point lies above the $\dfrac{t_U}{t_{ult}} = 1$ line, then the lower strain corresponding to this value must be used, otherwise the steel would fail before the concrete, producing a lower factor of safety.

*Fig. 8* shows the $\frac{t_L}{t_{ult}}$ : (FS) curves for a typical flanged section having $a = 1$, $b = 0{\cdot}2$, $c = 0{\cdot}2$, $f = 0{\cdot}15$, with $\gamma = 0{\cdot}4$, $\frac{\alpha c_u}{c_{tL}} = 2{\cdot}1$, for $F = 0{\cdot}2$ and $1{\cdot}0$, and $\frac{c_u}{{}_pE_e} = 0{\cdot}20$ to $0{\cdot}35$.

*Note.*—$a\,.\,'B$ denotes bottom flange width
$bD$ ,, ,, ,, thickness
$c\,.\,'B$ ,, web thickness

The low $F$-value of 0·2 for unbonded construction produces curves somewhat similar to those for the rectangular section, whilst for $F = 1$ for bonded construction, a governing consideration is clearly the limitation of the factor of safety owing to the restriction of the compression zone.

From the above, the most satisfactory way of dealing with the problem of a member with a rectangular top flange appears to be to assume first that $r = b$, and calculate the factor of safety using equation (19), this being the maximum value obtainable with the given section, since the compression cannot be increased. If this value is more than adequate, select a suitable value and proceed as for the case of $r < b$. If the factor of safety for $r = b$ is adequate, proceed as for $r \geqslant b$, using the maximum steel-strain change possible, and if not, the section must be re-designed for working load, either lowering the stress $c_{tL}$ or increasing the top flange thickness.

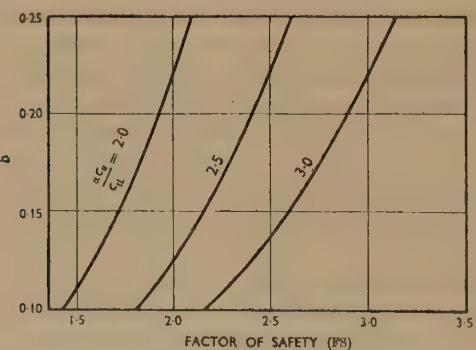
Under working load $M_L$, with bottom fibre stress zero, the width of the bottom flange is of little importance since it is at low stress and is at little distance from the steel. The top flange thickness is conversely of appreciable importance, and the web somewhat less. Thus, if a symmetrical I-section is selected, with a reasonable mean web thickness, then the working-load moment $M_L$ calculated from it will be approximately that for all sections of the same depth which have the same top flange. Similarly, the ultimate-load moment for the section for $b = r$ will be the same for all sections with the same top flange, and in consequence the maximum factor of safety will also be approximately constant. Thus, using a symmetrical I-section, (FS) may be plotted against $b$ for various $\frac{\alpha c_u}{c_{tL}}$ values, for a given value of $f$ and $\gamma$. *Fig. 9* shows the curves obtained for the range of $b = 0{\cdot}10$ to $0{\cdot}25$, for $\frac{\alpha c_u}{c_{tL}} = 2{\cdot}0$, $2{\cdot}5$, and $3{\cdot}0$, for $f = 0{\cdot}15$, $\gamma = 0{\cdot}40$, $c = 0{\cdot}2$; the curves are sufficiently accurate for any section, and can therefore be used in the initial selection of section to ensure that the $b$ and $\frac{\alpha c_u}{c_{tL}}$ values chosen can provide an adequate factor of safety. Interpolation for $\frac{\alpha c_u}{c_{tL}}$ is linear, as is evident from the governing equation (19).

*Failure by Steel Fracturing without Concrete Crushing*

It is evident that failure by steel fracturing without concrete crushing can occur only with a lower proportion of steel ($p$) and a higher $\frac{t_L}{t_{ult}}$ value than that for simultaneous failure of concrete and steel, which is indicated on *Figs 4* to *8* for $\frac{t_U}{t_{ult}} = 1$. It is clear that the factors of safety will be low and the $\frac{t_L}{t_{ult}}$ values high.

A direct solution for a given factor of safety is not possible since two

*Fig. 9*




Approximate Values of Maximum Factor of Safety, for Flanged Beams

graphical relations are involved—the stress/strain relationships of the concrete and of the steel; these are most useful in the form of $\alpha c_U : \frac{c_U}{pE_c}$ and $\frac{(t_{ult} - t_L)}{pE_s} : \frac{t_L}{t_{ult}}$. The concrete stress $c_U$ in the top fibre of the beam under $M_U$ is less than $c_u$, the corresponding stress for concrete failure; for the steel, $t_U = t_{ult}$, and hence the curve corresponding to this is that of $\frac{t_U}{t_{ult}} = 1$ on the $\frac{(t_U - t_L)}{pE_s} : \frac{t_L}{t_U}$ curves $\left(\text{which is in effect the } \frac{(t_{ult} - t_L)}{pE_s} : \frac{t_L}{t_{ult}} \text{ curve}\right)$.

For a rectangular section the equations corresponding to (8) to (12) are:

$$\beta = \frac{t_L}{t_{ult}} = \frac{1}{2}\frac{c_{tL}}{\alpha \,.\, c_U} \,.\, \frac{1}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

$$(\text{FS}) = \frac{M_U}{M_L} = 2r\frac{\alpha c_U}{c_{tL}} \,.\, \frac{(1 - \gamma r - f)}{(\frac{2}{3} - f)} \quad . \quad . \quad (23)$$

$$\frac{t_{ult} - t_L}{{}_pE_s} = X\beta - Y \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where
$$X = F\frac{c_U}{{}_pE_c}(1-f)\,2\frac{\alpha c_U}{c_{tL}} \quad . \quad . \quad . \quad . \quad . \quad (25)$$

and
$$Y = F\frac{c_U}{{}_pE_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

If a value of $\alpha c$ is guessed, and its corresponding value of $\frac{c_U}{{}_pE_c}$ is obtained from the stress/strain relationship of the concrete, $\frac{(t_{ult} - t_L)}{{}_pE_s}$ may be calculated from (24) to (26), and hence $\frac{t_L}{t_{ult}}$ may be obtained from the curve of the steel; equation (22) solves for $r$, which may then be put into (23) to obtain the corresponding factor of safety.

Sections with rectangular top flanges may be treated in a similar manner, it being apparent that $r$ will normally be less than $b$.

*Experimental Values of Ultimate Load Coefficients*

The sources of the data quoted below are given in references 2 to 6, in all cases applying to beams in which the concrete crushed.

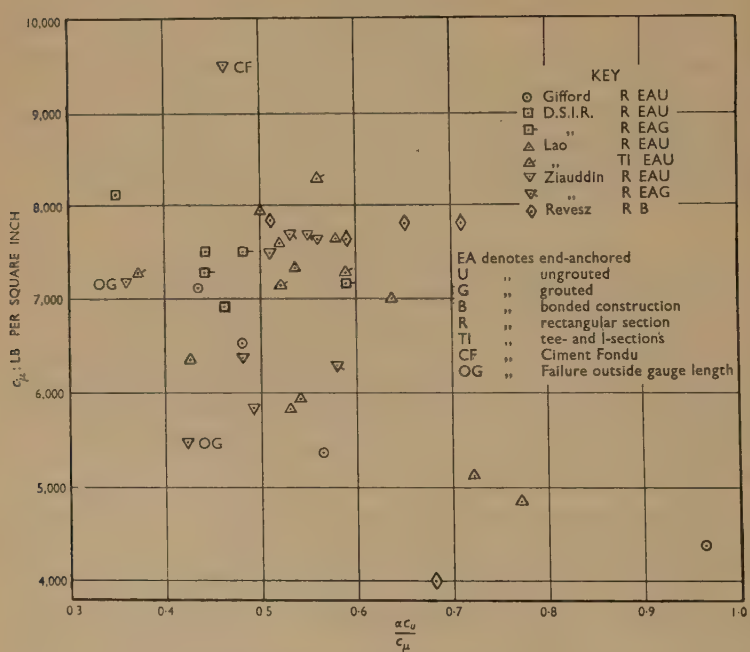
$\gamma$.—The value of $\gamma$ cannot be derived with a high degree of accuracy because it is sensitive to small changes of steel force; conversely, $\gamma$ has a small effect on $M_U$. The absolute range is $\frac{1}{3}$ to about $\frac{1}{2}$ and it generally appears to lie within the range 0·40 to 0·45; 0·40 may be taken as a safe limiting value.

$\alpha c_u$.—The value required in the equations is $\frac{\alpha c_u}{c_{tL}}$. Since $c_u$ is obtained arbitrarily, it is of more concern to relate both $\alpha c_u$ and $c_{tL}$ to the cube strength $c\mu$ and derive from experiments the values of $\frac{\alpha c_u}{c\mu}$. It would be expected that this factor would vary with the cube strength (since higher-strength concretes exhibit less plasticity), and also with the proportions of the test specimen, and in consequence the experimental values of $\frac{\alpha c}{c\mu}$ have been plotted against $c\mu$ (see *Fig. 10*). Even for beams of similar proportions and properties, for which the same symbol has been used, there was too much scatter to enable any relationship to be established. However, of the thirty-nine results, only three lie below 0·42; twenty-nine lie between 0·42 and 0·60; and seven above 0·60. The average of thirty-eight (neglecting the value above 0·9) is 0·525. For practical purposes, a range of 0·45 to 0·60 may be taken, the 0·45 being used as a reasonable "safe limiting value," and for purposes of calculating the expected factor of

safety (as distinguished from the minimum possible) the average value of 0·525 can be used.

$\frac{c_u}{{}_pE_c}$ (*per cent*).—This was also expected to be related to the cube strength, and has been plotted against $c\mu$ on *Fig. 11.* Again, considerable scatter is evident and no relationship can be established; however, inspection shows that of the forty-three results plotted, seven lie below 0·20 (of which two are strains adjacent to, but not at, point of failure); thirty

*Fig. 10*



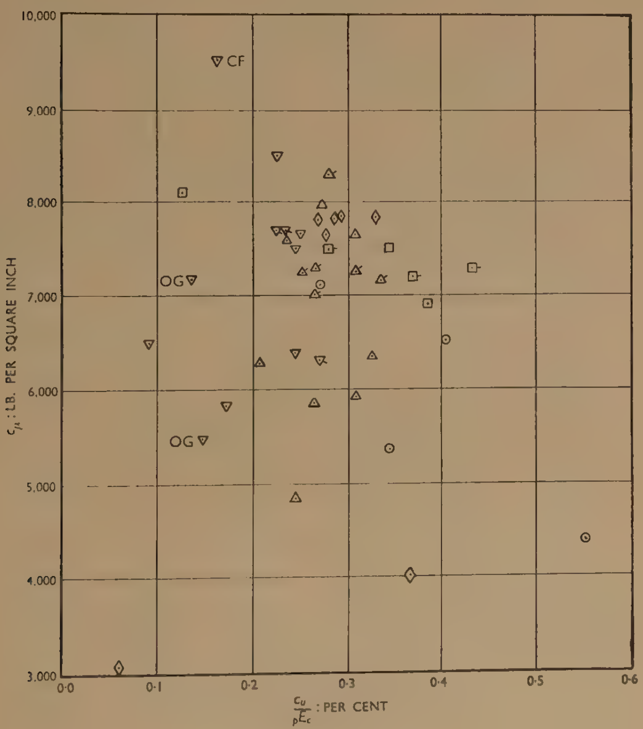

EXPERIMENTAL VALUES OF $\frac{aC_u}{C\mu}$

lie between 0·20 and 0·35; and six are above 0·35. For practical purposes the range may be taken as 0·20 to 0·35, 0·20 being the "safe limiting value," and 0·275 the value for calculating the expected factor of safety. The actual average of the points plotted is 0·272.

*F for unbonded beams.*—Theoretical investigation [2] indicated that, for rectangular beams of specific bending moment and steel eccentricity, the value of $F$ bore an approximately linear relation to $r$; this enabled the Author to predict low $F$-values, of the order of 0·2, which have subsequently been substantiated by tests. Taking into account the location of the

steel, $F$ has been plotted against $\dfrac{r}{(1-f)}$ (see *Fig. 12*). Whilst slightly different relationships would be expected for the different types of beams, it is noticeable that, of the twenty-two points plotted, seventeen lie in the range of $F : \dfrac{r}{(1-f)} = \frac{2}{3}$ to $1\frac{1}{2}$ and that the relation $F = \dfrac{r}{(1-f)}$ is a reasonable average. Whilst nine of the points are for $F$-values from 0·10

*Fig. 11*




Experimental Values of $\dfrac{cu}{pE_c}$

(For key, see *Fig. 10*)

to 0·20, it has been shown earlier that this variation will not lead to much variation of (FS), and in consequence a value of $F = 0{\cdot}20$ is considered a reasonable value to assume ; subsequently, when $r$ has been calculated, if $F : \dfrac{r}{(1-f)}$ lies within $\frac{2}{3}$ to $1\frac{1}{2}$, the assumed value of $F$ may be considered

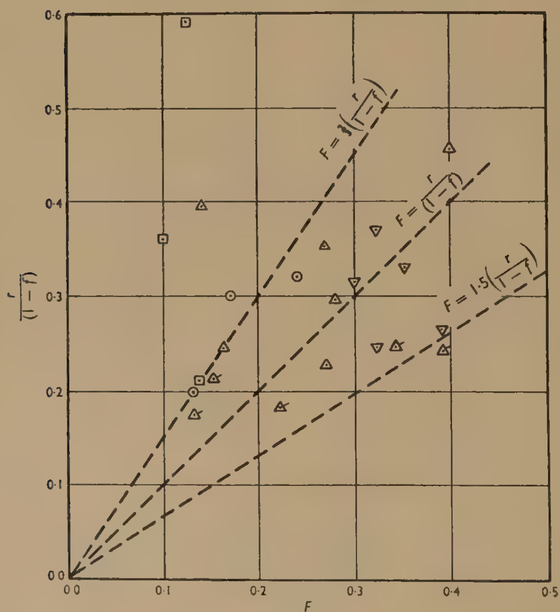
satisfactory. Since the loading and eccentricity of steel will effect the $F$-values, the following brief details of the test beams are given.

All beams were rectangular, of constant section and constant steel eccentricity, except where noted.

*Gifford.*—Loading uniformly distributed, steel parabolic, zero eccentricity at ends, $f = 0{\cdot}1$ at centre.

*Directorate of Scientific and Industrial Research.*—Two equal loads, $0{\cdot}33L$ from ends. Steel straight between loads turned up straight to ends; $f$ at centre $= 0{\cdot}2$, $f$ at ends $= 0{\cdot}35$.

*Fig. 12*




EXPERIMENTAL VALUES OF $F$
(For key, see *Fig. 10*)

*Lao.*—Two equal loads, $0{\cdot}25L$ from the ends. $f = 0{\cdot}3$.

*Lao.*—T- and I-beams with thick webs. Two equal loads, $0{\cdot}3L$ from ends. $f = 0{\cdot}28$.

*Ziauddin.*—Two equal loads, $0{\cdot}37L$ from the ends. $f = 0{\cdot}3$.

*F for bonded beams.*—It has been suggested [7] that in a bonded beam it is possible that the local strain in the steel at a crack is higher than the mean strain as measured over a gauge length on the concrete, and also that the ultimate stress of the steel in the beam can exceed that obtained in a

simple tension test. At the present, further information is required to substantiate these suggestions—the grouted-beam tests of the D.S.I.R. and by Ziauddin indicate that an *F*-value of unity is reasonable, and this value is recommended, for the present, as being safe.

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7. K. Hajnal-Kónyi, *idem*, reply to Discussion, p. 128.

The Paper is accompanied by nine sheets of diagrams, from which the Figures in the text have been prepared.

Paper No. 5914

# The Resistance to Flow of Water along a Tortuous Stretch of the River Irwell (Lancashire)—an Investigation with the Aid of Scale-Model Experiments

by

**Professor Jack Allen, D.Sc., M.I.C.E., and Aziz Shahwan, M.Eng.**

(*Ordered by the Council to be published with written discussion*) †

SYNOPSIS

The Paper describes experiments made on a model of the River Irwell having a horizontal scale of 1 : 500 and a vertical scale of 1 : 96. Although no attempt was made to adjust the roughness after the original moulding in cement mortar, the water levels for various discharges were found to be in fair agreement with those observed in nature.

Analysis of observations in the model and on the river itself indicates that the bends and changes of section account for one-quarter to three-quarters of the total resistance, depending upon the portion of channel and the discharge considered. This subject is also discussed in terms of a wider channel which was tried in the model and, further, results are given concerning the effect on upstream water levels of raising or lowering the level at the downstream end of the model.

The conclusions reached regarding the effect of textural roughness in the model are based not upon *assumed* friction coefficients but upon values obtained by experiment in straight channels of similar section made with similar cement mortar.

## INTRODUCTION

THE results of an investigation into the resistance to flow along a tortuous stretch of the River Mersey have been presented in a previous Paper.[1] It was established that although the river-section was equivalent to a rectangular channel roughened with projections about 6 inches high, its textural roughness accounted for only about one-quarter of its total resistance at a flow of 350 cusecs. Even with a discharge as high as 7,000 cusecs, the bends and changes of section were responsible for one-half of the total loss of head.

These conclusions were thought to be of interest because they were probably the first quantitative demonstration of the comparative unimportance of surface roughness in such a case, and because of the bearing

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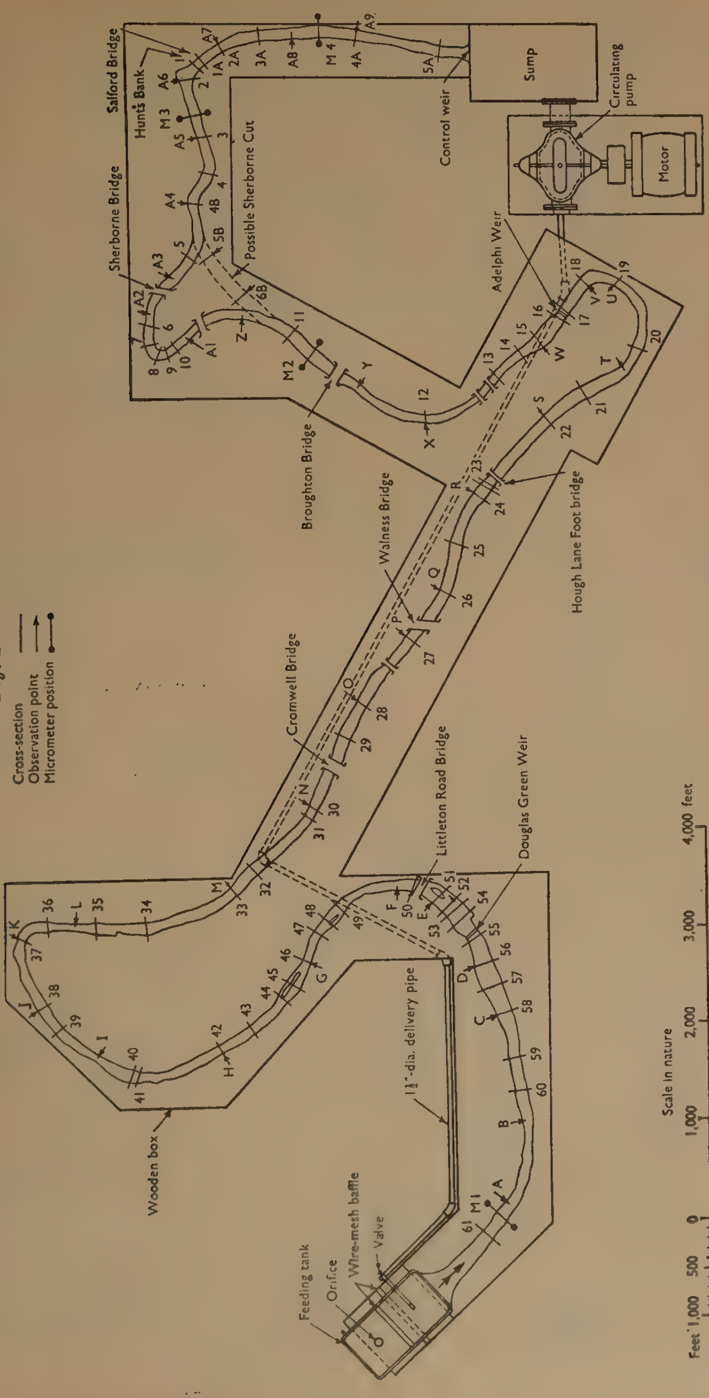
[1] The references are given on p. 161.




RIVER IRWELL. SKETCH-PLAN SHOWING MAIN FEATURES OF PORTION INCLUDED IN MODEL

(To avoid confusion with cross-sections and observation points, some bridges down-stream of section 3 have been omitted from the plan.)

of this fact upon the practicability of scale-model experiments. Indeed, it was found that in a model having a horizontal scale of 1 : 800 and a vertical scale of 1 : 120, the losses of head were almost identical with hot and cold water (viscosity changing by 60 to 100 per cent) despite the Reynolds numbers being as low as $vm/\nu = 36$.

Recently, the Authors have had an opportunity of carrying out a similar investigation in respect of the River Irwell, which flows through Salford and Manchester into the Manchester Ship Canal. The stretch under consideration is shown in *Fig. 1*; it contains two weirs (Douglas Green and Adelphi) and the coefficient of discharge under various heads of one of these—Adelphi Weir—was found by scale-model experiments made some years ago at Manchester University under the direction of Professor A. H. Gibson. The Authors have made experiments on a scale model of this stretch of the Irwell in the Engineering Department at Marischal College, University of Aberdeen.

## Model of River Irwell

The model was moulded in cement mortar containing 1 part by volume of Portland cement to 10 parts of 40- to 100-mesh sand. It was painted with a thin wash of neat cement.

| | |
|---|---|
| Horizontal scale | 1 : 500 |
| Vertical scale | 1 : 96 |
| Vertical distortion of scale | 5·21 |
| Velocity scale | $1 : \sqrt{96} = 1 : 9{\cdot}80$ |

Scale of discharge (volume per unit time)

$$1 : 500 \times 96 \times \sqrt{96} = 1 : 4{\cdot}70 \times 10^5$$

Discharges of any desired rate could be supplied to the top of the model through one of a series of calibrated orifices. Water levels at salient points were read by means of needles or pointers attached to micrometer heads; in repeating experiments it was found that different observers could obtain the same values to within $\pm$ 0·005 inch, which represents $\pm$ 0·48 inch in nature. Water levels at intermediate points were also measured by means of a pointed scale held against a straight-edge resting on the levelled top of the box containing the model. Zero readings were frequently checked by observing the gauges on a static water surface.

*Measurements of Model Resistance and Estimate of Proportion caused by Skin Friction*

Let $v_A$ denote mean velocity in the cross-section at a station A
$v_B$ ,, ,, ,, ,, ,, ,, B
$l$ ,, distance between A and B, measured along the axis of the channel

$m$ denote average hydraulic mean depth (area of section/wetted perimeter)
$v^2$ ,, mean square of velocity between A and B
$f$ ,, coefficient of friction arising from textural roughness or skin friction
$h_1$ ,, drop in water surface between A and B due to skin friction
$h_2$ ,, drop in water surface between A and B caused by bends and changes of width and depth
$h_v$ ,, velocity head between A and B
H ,, total drop in water surface between A and B

Then $H \doteq h_1 + h_2 + h_v$

$$= \frac{flv^2}{2gm} + h_2 + \frac{v_B{}^2 - v_A{}^2}{2g} \text{ sensibly} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

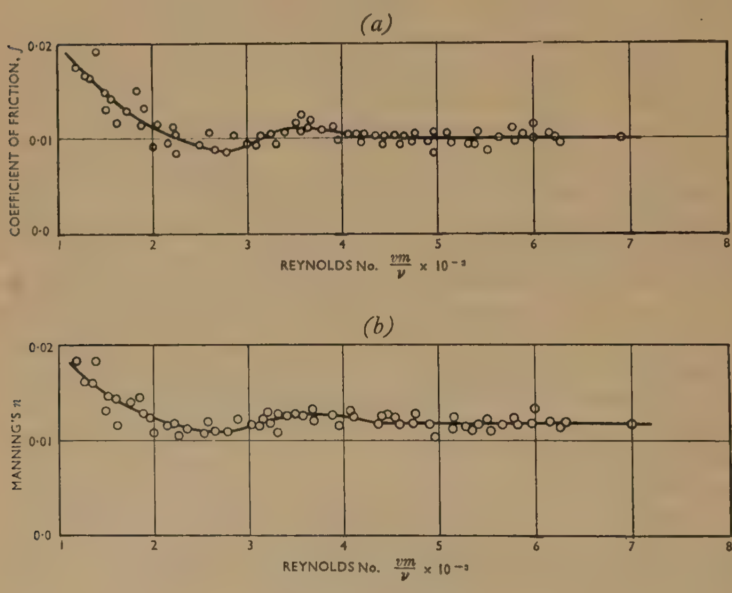
In order to establish the term $h_1$, experiments have been made on a *straight* length of channel moulded in a mixture of cement and sand as near as practicable the same as that adopted for the Irwell model. The straight channel was rectangular in section, with a width of $2\frac{1}{2}$ inches and an overall depth of 4 inches. The width was chosen as being approximately the average width of the channel in the river model. The straight channel was 12 feet long, and a central gauge length of 6 feet 6 inches was adopted for measuring the fall in the water surface with various discharges. Check measurements were also made with two additional gauges dividing the 6-foot-6-inch stretch into portions 2 feet, 2 feet 6 inches, and 2 feet long respectively. The zero readings of the micrometer gauges were found, in relation to one another, by adjusting their pointers so as to touch a static water surface in the channel. The bed of the channel was levelled with the aid of a straight-edge and spirit-level, and the actual bed-level under each of the four micrometers was finally obtained by adjusting their pointers so as to touch the top of a machined steel block placed on the bed of the channel. Widths of the channel were measured by callipers, and wire-mesh screens were inserted at the entrance to smooth out the flow into the channel; a control weir at the exit enabled various depths to be used for any given rate of flow. In interpreting the results, allowance was made for the change of kinetic head between the gauge-points. The values of the friction coefficient, $f$, and of Manning's roughness factor, $n$ (in $v = \frac{1 \cdot 49}{n} m^{\frac{2}{3}} i^{\frac{1}{2}}$) for the straight $2\frac{1}{2}$-inch-wide channel are shown in *Figs 2*. In evaluating these, the hydraulic mean depth, $m$, has been taken as the average of the $m$'s at four cross-sections in the gauge-length of 6 feet 6 inches, $v^2$ as the average of the (mean velocity)$^2$, at the four sections, and $v$ in the Reynolds number $vm/\nu$ as the average of the $v$'s at the four sections.

As a matter of general interest, it is found that in this straight channel, the flow may be described as fully turbulent (resistance proportional to

the square of velocity) if $vm/\nu$ exceeds about 4,000. It has previously been shown [2] that under such conditions, the value of Chezy's coefficient $C\left(=\sqrt{\frac{2g}{f}}\right)$ is related to the hydraulic mean depth, $m$, and the effective height, $k$, of the roughening protuberances by the formula :

$$C = 37{\cdot}6 + 32{\cdot}1 \log_{10}\frac{m}{k} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

*Figs 2*




NARROW CHANNEL

Adopting the observed values of $C$ and $m$, the corresponding values of $k$ in this instance are 0·035 inch, within $\pm$0·005 inch, provided $vm/\nu$ exceeds 4,000. This serves to describe the sort of finish imparted to the mortar of the straight channel and of the river model : it confirms that no attempt was made to provide a plaster-smooth surface.

Considering next the Irwell River model, the results from it have been interpreted as follows :

(1) Two stretches have been considered, namely,

(*a*) Between observation points G and S (Sections 46 and 22), *Fig. 1.* This is between Douglas Green Weir and Adelphi Weir, and the length, measured along the axis of the channel, is 24·9 feet.

TABLE 1.—STATIONS G TO S

| Discharge : cusecs | | Velocities in model : ft./sec. | | | | Mean $m$ in model : feet | $\frac{v_m}{\nu}$ | $f$ | $h_1$ : feet | $h_v$ : feet | Measured $H$ : feet | Estimated $h_2 = H-(h_1+h_v)$ | $\frac{h_2}{H-h_v}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| In model | In nature | $v_G$ | $v_S$ | Mean $v$ | Mean $v^2$ | | | | | | | | |
| * 0·0340 | 16,000 | 0·940 | 0·792 | 0·801 | 0·651 | 0·0719 | 5,100 | 0·0100 | 0·0350 | −0·0040 | 0·0625 | 0·0315 | 0·47 |
| * 0·0294 | 13,800 | 0·863 | 0·782 | 0·774 | 0·610 | 0·0680 | 4,850 | 0·0100 | 0·0345 | −0·0020 | 0·0689 | 0·0364 | 0·51 |
| * 0·0255 | 12,000 | 0·829 | 0·769 | 0·756 | 0·578 | 0·0652 | 4,540 | 0·0100 | 0·0343 | −0·0015 | 0·0761 | 0·0433 | 0·56 |
| 0·0217 | 10,200 | 0·793 | 0·707 | 0·725 | 0·533 | 0·0622 | 3,730 | 0·0110 | 0·0362 | −0·0020 | 0·0708 | 0·0366 | 0·50 |
| 0·0164 | 7,700 | 0·738 | 0·664 | 0·652 | 0·431 | 0·0580 | 3,200 | 0·0103 | 0·0297 | −0·0016 | 0·0750 | 0·0469 | 0·61 |
| 0·00935 | 4,400 | 0·519 | 0·479 | 0·508 | 0·262 | 0·0510 | 2,400 | 0·0095 | 0·0186 | −0·0006 | 0·0761 | 0·0581 | 0·76 |

TABLE 2.—STATIONS X TO $A_9$

| Discharge : cusecs : | | Velocities in model : ft/sec. | | | | Mean $m$ in model : feet | $\frac{v_m}{\nu}$ | $f$ | $h_1$ : feet | $h_v$ : feet | Measured $H$ : feet | Estimated $h_2 = H-(h_1+h_v)$ | $\frac{h_2}{H-hv}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| In model | In nature | $v_X$ | $v_{A9}$ | Mean $v$ | Mean $v^2$ | | | | | | | | |
| * 0·0340 | 16,000 | 0·915 | 0·960 | 0·945 | 0·913 | 0·0684 | 5,780 | 0·0100 | 0·0342 | + 0·0013 | 0·0979 | 0·0624 | 0·65 |
| * 0·0294 | 13,800 | 0·844 | 0·853 | 0·882 | 0·793 | 0·0648 | 5,300 | 0·0100 | 0·0315 | + 0·0002 | 0·0916 | 0·0599 | 0·66 |
| * 0·0255 | 12,000 | 0·810 | 0·794 | 0·845 | 0·739 | 0·0630 | 4,930 | 0·0100 | 0·0298 | − 0·0004 | 0·0844 | 0·0550 | 0·65 |
| 0·0217 | 10,200 | 0·750 | 0·748 | 0·717 | 0·571 | 0·0602 | 3,610 | 0·0110 | 0·0267 | − 0·0001 | 0·0812 | 0·0546 | 0·67 |
| 0·0164 | 7,700 | 0·722 | 0·590 | 0·672 | 0·466 | 0·0570 | 3,240 | 0·0105 | 0·0220 | − 0·0027 | 0·0667 | 0·0474 | 0·68 |
| 0·00935 | 4,400 | 0·585 | 0·443 | 0·518 | 0·285 | 0·0501 | 2,400 | 0·0095 | 0·0138 | − 0·0023 | 0·0562 | 0·0447 | 0·76 |

* Observations both in the model and in nature reveal that the river overflows its banks when the discharge reaches the order of 12,000 to 13,000 cusecs. For the purpose of the present experiments, the model was provided with artificial flood banks to keep the water within the channel even with 16,000 cusecs (that is, with 0·0340 cusec in the model).

(b) Between X and A9 (Sections 12 and 4A), *Fig. 1.* This stretch is situated downstream of Adelphi Weir and measures 16·5 feet.

(2) Water levels were observed at G, H, I, J–S, and at X, Y, $M_2$, Z–$A_8$, $M_4$, $A_9$.

(3) From the resulting longitudinal water-surface profiles, areas, hydraulic mean depths, and velocities were found for the 25 cross-sections 46–22 in stretch (a) and for the 16 cross-sections 12–4A.

(4) As in the straight-channel tests, water temperatures were measured by means of a mercury thermometer reading to within 0·1° C.

(5) The discharge was given by the observed head over the calibrated orifice in the water-supply (feeding) tank.

(6) The value of $f$, appropriate for substitution in equation (1), was chosen as that found from the straight-channel tests at the same Reynolds number. For this purpose, the smooth curve through the plotted points in *Figs 2* was used.

The " balance-sheets " of heads thus observed and computed are presented in Tables 1 and 2.

The last columns of Tables 1 and 2 show that the bends and changes of section, as distinct from textural roughness, account for 47 to 76 per cent of the total resistance in the model between Douglas Green Weir and Adelphi Weir. Below Adelphi Weir, the proportion is between about 65 and 76 per cent.

These conclusions are reached by estimating the skin-friction effect from the values of $f$ shown in *Figs 2.* To be more precise, however, account should be taken of the variation of $f$ with hydraulic mean depth ; in other words, the curve of $f$ in *Fig. 2* (a) is really an *average* for the values of $m$ covered by the straight-channel tests : these ranged between $m = 0{\cdot}045$ feet and $m = 0{\cdot}075$ feet approximately. As an alternative, therefore, the skin friction in the river model has also been estimated by using the values of Manning's $n$ as indicated by the straight-channel experiments. The basic equation is then :

$$v = \frac{1{\cdot}49}{n} m^{\frac{2}{3}} i^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $i$ is the gradient required to overcome friction in uniform flow ; then $h_1 = il$.

Table 3 shows the sets of results obtained by the two methods.

The difference between the two assessments is, therefore, small. It is interesting to note, however, that the proportionate importance of the bends and changes of section is generally far greater in the region X to A9 (below Adelphi Weir) than from G to S (between Douglas Green Weir and

TABLE 3

| Discharge : cusecs | | Percentage of resistance caused by bends and changes of section | | | |
|---|---|---|---|---|---|
| | | G to S | | X to A9 | |
| In model | In nature | Using $f$ | Using $n$ | Using $f$ | Using $n$ |
| 0·0340 | 16,000 | 47 | 47 | 65 | 64 |
| 0·0294 | 13,800 | 51 | 52 | 66 | 66 |
| 0·0255 | 12,000 | 56 | 56 | 65 | 66 |
| 0·0217 | 10,200 | 50 | 53 | 67 | 72 |
| 0·0164 | 7,700 | 61 | 59 | 68 | 69 |
| 0·00935 | 4,400 | 76 | 72 | 76 | 75 |

Adelphi Weir). It will be seen from *Fig. 1* that this is reasonable. It will be seen, also, that the influence of skin friction is even less apparent with a relatively low flow than with a heavy flood : the same was found to be true for the River Mersey.[3]

As a further check, the same method of analysis has been applied to the reach of the model between micrometers 2 and 3. The length of channel there involved is 9·06 feet, and *Fig. 1* shows that it is particularly tortuous with a very sharp bend in the region of Sections 6 to 10. In this reach, the velocities and other quantities included in the calculations have been computed for ten cross-sections, and it appears that not more than 40 per cent of the resistance can be attributed to textural roughness. Table 4 summarizes the results based upon the appropriate Manning's $n$.

TABLE 4

| Discharge : cusecs | | $\frac{h_2}{H - hv} \times 100$ |
|---|---|---|
| In model | In nature | |
| 0·0340 | 16,000 | 63 per cent |
| 0·0255 | 12,000 | 63 ,, ,, |
| 0·0164 | 7,700 | 65 ,, ,, |
| 0·00935 | 4,400 | 78 ,, ,, |

It is of interest to observe, in fact, that the introduction of a cut (designated Sherborne Cut in *Fig. 1*) reduces the total rise in water level between micrometers 3 and 2 from the original value of 0·0477 feet (4·57 feet in nature) to 0·0125 feet (1·20 feet in nature) when the discharge is 0·0340 cusec (16,000 cusecs in nature). Thus $H$ was reduced by nearly 75 per cent, whilst the axial length of the channel was shortened by only 36 per cent.

## Experiments on a Widened Channel

The river section so far discussed was moulded in the model with the aid of male templets. Buried in the cement mortar were other templets providing for a wider—and rather deeper—channel.

The widening affected virtually the whole of the channel upstream of Section 20 (*Fig. 1*), and also the stretch between Sections 14 and 3 ; the results are compared in Table 5.

The mean bed level (measured at the deepest points in the cross-sections) was equivalent to O.D. 78·80 feet between G and S with the original moulding, and O.D. 77·10 feet with the widened channel. The corresponding levels between X and A9 were O.D. 66·0 feet and O.D. 65·0 feet respectively.

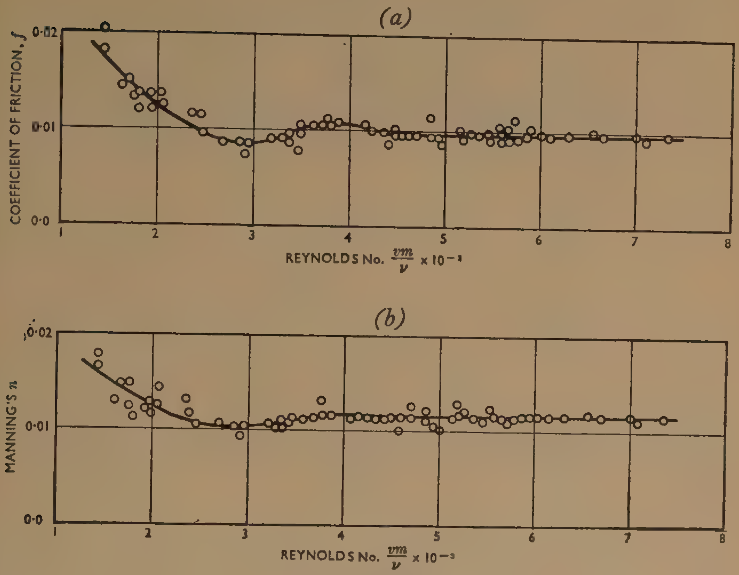
Table 5.—Ratio of properties of sections in widened channel to those in original channel

| Discharge : cusecs | | Between G (Section 46) and S (Section 22) | | Between micrometers 2 and 3 | | Between X (Section 12) and A9 (Section 4A) | |
|---|---|---|---|---|---|---|---|
| In model | In nature | Mean area | Mean $m$ | Mean area | Mean $m$ | Mean area | Mean $m$ |
| 0·0340 | 16,000 | 1·18 | 1·12 | 1·29 | 1·11 | 1·20 | 1·05 |
| 0·0255 | 12,000 | 1·23 | 1·12 | 1·35 | 1·14 | 1·22 | 1·10 |
| 0·0164 | 7,700 | 1·22 | 1·08 | 1·36 | 1·15 | 1·23 | 1·17 |
| 0·00935 | 4,400 | 1·17 | 1·05 | 1·44 | 1·17 | 1·27 | 1·10 |

As previously explained, the face of the narrow channel was made of cement mortar having 1 part by volume of cement to 10 parts of sand ; the wider channel was moulded in a richer mix—1 of cement to 6 of sand. The frictional properties of this richer mix were again found by tests on a straight channel, this time of rectangular section approximating to the average width (3·25 inches) of the wider river model and made with the 1 : 6 mortar. The results of these preliminary experiments are shown in *Figs 3*. Comparison with *Figs 2* shows that in the fully turbulent stage, the value of Manning's $n$ is probably slightly less (by about 2 per cent) for the wider than for the original channel. If this is a real effect it is consistent with the fact that the material of the wider channel contains fewer sand grains. The effective $k$ (height of roughness) is about 10 per cent smaller than that of the narrower channel.

Adopting the values of Manning's $n$ provided by the straight widened channel tests, the proportion of resistance in the widened river model caused by bends and changes of section has been calculated as summarized in Table 6.

*Figs 3*




WIDER CHANNEL

TABLE 6

| Discharge: cusecs | | $\frac{h_2}{H - h_v} \times 100$ | | |
|---|---|---|---|---|
| In model | In nature | Between G and S | Between X and A9 | Between micrometers 2 and 3 |
| 0·0340 | 16,000 | 38 per cent | 72 per cent | 76 per cent |
| 0·0255 | 12,000 | 35 ,, ,, | 72 ,, ,, | 75 ,, ,, |
| 0·0164 | 7,700 | 46 ,, ,, | 74 ,, ,, | 79 ,, ,, |
| 0·00935 | 4,400 | 43 ,, ,, | 79 ,, ,, | 82 ,, ,, |

Introducion of the Sherborne Cut (see *Fig. 1*) in place of the natural course between Sections 4B and 11, was found to have the effects shown in Table 7, and supports the contention that the major resistance of the tortuous channel in this stretch is caused by the bends and changes of section.

The Cut does not reduce the difference in water levels to the full

extent, which might at first sight be expected from the last column of Table 6, for the following reasons :—

(1) There still remains a marked change of curvature between Section 4B and micrometer 3.

(2) As the level falls at micrometer 2 for a given discharge, the velocity there increases and the hydraulic mean depth decreases. Both these effects make for an increase of friction head.

TABLE 7

| Discharge : cusecs | | Rise in water surface level between micrometers 3 and 2—converted to feet in nature | |
|---|---|---|---|
| In model | In nature | Without Sherborne Cut | With Sherborne Cut |
| 0·0340 | 16,000 | 3·5 (9) | 1·0 (4) |
| 0·0255 | 12,000 | 3·0 (1) | 0·7 (4) |
| 0·0164 | 7,700 | 2·0 (5) | 0·5 (9) |
| 0·00935 | 4,400 | 1·3 (7) | 0·4 (2) |

*Note:* The Cut in the model was made rectangular in cross-section with a width equivalent to 120 feet. It was moulded so that its bed joined smoothly on to the existing channel at Sections 4B and 11. The gradient of its bed in the model was 1 : 250, representing 1 : 1,300 in nature. The Cut had the effect of reducing the axial length of the channel between micrometers 2 and 3 by about 36 per cent.

Comparing Tables 3 and 6 and Tables 4 and 6, it is seen that widening the channel causes a reduction of the *proportionate* effect of the bends and changes of cross-section in the region between Douglas Green Weir and Adelphi Weir ; downstream of Adelphi Weir, the opposite is true.

No precise explanation can be offered for this apparent contradiction. Physically, it seems that in one case the widening does not reduce the friction so much as it eases the bends, whilst in the other case, the reverse applies. Table 5, in fact, indicates that the increase of sectional area in the particularly tortuous stretch between micrometers 2 and 3 is much more than in the other stretches ; this would certainly tend towards a larger reduction of friction, while the bend between Sections 6 and 10 is so sharp as to discourage easing by widening alone.

## THE BEHAVIOUR OF THE MODEL COMPARED WITH THE NATURAL RIVER

Observations have been supplied by the Mersey River Board in the form of longitudinal profiles for various river flows as indicated by the Adelphi Weir Recorder. In many cases, the water levels are "wrack readings," that is to say, levels of debris left by the subsiding waters, and

the same degree of accuracy—judged as observations—cannot be attached to them as in controlled laboratory experiments.

It should be realized that the River Irwell flows into the Salford Docks of the Manchester Ship Canal Company and the level of water in these docks is kept approximately constant by manipulating sluice gates (Mode Wheel Sluices) which discharge into the Ship Canal. The model under discussion terminates some distance upstream of Mode Wheel Sluices and the procedure adopted in comparing its behaviour with nature has been to adjust the level of a control weir (actually a block of plasticine) at the tail end of the model so as to reproduce the correct water level at Hunt's Bank under a flow of 12,000 cusecs. The control weir was then left unaltered for the other discharges.

Table 8 shows the comparison between the model and nature.

The levels quoted for the natural river at Adelphi Weir are those registered in the recorder, whereas in the model they are levels measured on the surface of the *open stream* at Section 17. When Adelphi Weir was calibrated at Manchester University by means of a 1 : 50 scale model, it was found that there were appreciable changes of water level over cross-sections near the Weir. For discharges up to 5,000 cusecs, the difference between a midstream level and a level measured in the recorder basin was less than 0·25 foot, but for 12,000 cusecs, the midstream level was higher than the recorder by 0·5 foot. This accounts in part, but only in part, for the discrepancies shown in Table 8 between nature and model at Adelphi Weir. Because of the vertical distortion of scale, the model weir was not a true representation of the actual weir and it was not considered desirable to adopt the compromise of making its section, parallel to the direction of flow, in accordance with the vertical scale, since this would have meant interference with the river sections for some distance in the vicinity of the weir. The weir was, therefore, made with a crest thickness and crest level approximately in agreement with the vertical scale, the upstream and downstream faces being sloped approximately in accordance with the distorted scale.

On the whole, however, it appears that the model gives a reasonable reproduction of natural conditions, despite the facts that (*a*) no artificial roughening was used after the original moulding, (*b*) the Reynolds numbers in nature are about 1,700 times those in the model, and (*c*) even with the maximum flow, the average top-water width in the model was only about 3·75 inches and the average depth of water only about 2·25 inches.

## Effect of Varying the Tailwater Level

It has previously been explained that the control weir at the exit of the model was adjusted so as to give the appropriate water level at Hunt's Bank (or Salford Bridge) for a discharge representing 12,000 cusecs.

It was thought to be of interest, however, to try the effect in the

TABLE 8.—COMPARISON BETWEEN WATER SURFACE LEVELS (FEET O.D.) IN RIVER IRWELL AND ITS MODEL WITH CORRESPONDING DISCHARGES

| Place | Chainage: feet | Discharge: cusecs 16,500 * River | 16,500 * Model | 13,800 * River | 13,800 * Model | 12,000 † River | 12,000 † Model | 10,200 River | 10,200 Model | 7,700 River | 7,700 Model | 4,800 † River | 4,800 † Model |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Site of gauge A9 | −1,725 | — | 80·0 | 77·2 | 79·6 | — | 78·5 | — | 77·5 | — | 76·4 | — | 74·3 |
| Site of micrometer 4 | −1,340 | — | 81·0 | 77·6 | 80·2 | — | 79·3 | — | 78·4 | — | 76·9 | — | 75·1 |
| Salford Bridge | 0 | 82·0 | 82·5 | 79·8 | 81·5 | 80·3 | 80·3 | 78·3 | 78·8 | 75·8 | 77·6 | 75·4 | 75·2 |
| Sherborne Bridge | 2,515 | 85·9 | 84·8 | 83·0 | 84·1 | 84·1 | 82·3 | 79·0 | 80·6 | 78·3 | 78·8 | 76·7 | 76·8 |
| Broughton Bridge | 5,415 | 89·2 | 88·4 | 86·3 | 87·2 | 86·0 | 85·6 | 79·7 ‡ | 84·2 | 81·5 | 81·7 | 79·7 | 79·0 |
| Adelphi Weir Recorder | 8,275 | 91·8 | 92·1 | 88·2 | 90·3 | 86·8 | 88·8 | 85·8 | 87·5 | 84·7 | 85·0 | 83·4 | 83·6 |
| Hough Lane Footbridge | 11,845 | 94·1 | 93·5 | 91·3 | 92·1 | 90·7 | 90·3 | 86·0 ‡ | 89·3 | 86·5 | 87·2 | 85·3 | 85·2 |
| Walness Bridge | 13,550 | {94·3 95·2} | 93·9 | 92·3 | 93·0 | 91·3 | 91·0 | 89·5 | 90·1 | 87·5 | 87·7 | 86·7 | 85·7 |
| Cromwell Bridge | 15,285 | {94·8 96·8} | 94·8 | 93·2 | 93·5 | 91·3 | 91·4 | 90·0 | 90·5 | 89·0 | 88·9 | 87·3 | 86·6 |
| Littleton Rd Bridge | 25,025 | 103·1 | 100·5 | 100·5 | 99·4 | 98·7 | 97·7 | 97·6 | 96·7 | 95·0 | 94·4 | 92·7 | 92·6 |
| Site of micrometer 1 | 28,720 | 107·5 | 105·4 | 104·2 | 104·7 | 103·5 | 103·7 | 102·7 | 102·8 | 101·7 | 101·4 | — | 99·9 |

*Dates of Observations in Natural River*

| | | | | | |
|---|---|---|---|---|---|
| 16,500 cusecs | 20 Sept. 1946 | 12,000 cusecs | 12 Nov. 1947 | 7,700 cusecs | 10 Feb. 1950 |
| 13,800 ,, | 7 Sept. 1950 | 10,200 ,, | 23 Aug. 1950 | 4,800 ,, | 30 Dec 1948 |

* With 16,500 cusecs recorded at Adelphi Weir, the river overflows its banks over very considerable areas. The observations in the model relate to the channel provided with flood banks to prevent overflow; they were made with "16,000 cusecs."

There is also flooding of the river with 13,800 cusecs; the model observations are for 13,800 but with flood banks. Since in the model the whole of the 13,800 cusecs continues to flow to the downstream end, it is not surprising that the levels at A9 and micrometer 4 are higher than in nature.

† There is some doubt as to the discharges of 12,000 and 4,800 cusecs quoted for the river. In the former case, the recorder float is said to have been sticking; in the latter (4,800), the inlet to the recorder became silted. Consequently, the discharges quoted are estimates based upon extrapolation of the recorder graphs.

‡ The figures for Broughton Bridge and Hough Lane Footbridge under 10,200 cusecs must be regarded as suspect: they are inconsistent with those quoted for larger or smaller flows.

model of varying the exit conditions. The results are summarized in Table 9:

Table 9.—WATER LEVELS IN MODEL (MOULDED TO REPRESENT PRESENT-DAY RIVER) WITH VARIOUS EXIT LEVELS

| | Station | Water level (feet O.D.) | | | |
|---|---|---|---|---|---|
| Discharge 4,400 cusecs | Micrometer 4 | 74·0 | 78·0 | 82·0 | 86·0 |
| | " 3 | 75·0 | 78·4 | 82·1 | 86·2 |
| | " 2 | 78·5 | 79·7 | 82·6 | 86·4 |
| | Walness Bridge | 85·4 | 85·5 | 85·7 | 88·2 |
| Discharge 7,700 cusecs | Micrometer 4 | 74·0 | 78·0 | 82·0 | 86·0 |
| | " 3 | 76·0 | 79·0 | 82·5 | 86·3 |
| | " 2 | 81·5 | 81·9 | 83·8 | 87·0 |
| | Walness Bridge | 87·6 | 87·7 | 88·1 | 89·5 |
| Discharge 12,000 cusecs | Micrometer 4 | 74·0 | 78·0 | 82·0 | 86·0 |
| | " 3 | 78·7 | 80·5 | 83·3 | 86·6 |
| | " 2 | 85·0 | 85·2 | 86·2 | 88·6 |
| | Walness Bridge | 90·9 | 91·0 | 91·3 | 92·7 |

Table 9 confirms what would be expected, namely, that as the tail-water level rises, the resulting increment upstream rapidly diminishes. For example, with the heavy discharge of 12,000 cusecs, a rise of 12 feet at micrometer 4 is accompanied by only 1·8 foot rise at Walness Bridge.

## Friction in the River Irwell Itself

Taking the available information for river levels and flows as they stand, consider first the stretch between G and S. The length is 12,440 feet and, if friction only were involved, the corresponding value of Manning's coefficient, which will be called $n_1$, would be given by

$$H - h_v = \frac{n_1{}^2 v^2 l}{(1{\cdot}49)^2 m^{4/3}} = \frac{n_1{}^2 v^2 l}{2{\cdot}22\, m^{4/3}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Three discharges have been considered: 12,000, 7,700, and 4,800 cusecs. Table 8 shows that in these cases, the water levels recorded in the model are sensibly the same as those in nature over the region G to S. It has been assumed, therefore, that at intermediate points the water levels in nature would also be practically the same as those suggested by the model. Correspondingly, the areas of section, velocities, and squares of velocities are very closely equal to those in the model when interpreted in terms of the scale-ratios; for example ($v$ in nature)$^2$ = 96 ($v$ in model)$^2$. The hydraulic mean depths follow a more complex relationship, because of the distorted vertical scale of the model, and have been computed by planimetering the sections of the natural river, drawn to a scale of 1 inch

= 10 feet, and measuring the wetted perimeters of the sections by means of a "map reader." Table 10 summarizes the results.

Now the coefficient $n_1$ estimated in this way absorbs the effect of bends and other sources of loss of energy as well as skin friction. The true value of $n$, defined as that appropriate to skin friction alone, *must be less than* $n_1$, and although its precise value is indeterminate it should evidently be that

TABLE 10.—NATURAL RIVER—G TO S (12,440 FEET)

| Discharge: cusecs | H: feet | H—$h_v$: feet | Mean |  |  | $n_1$ (from equation (4)) |
|---|---|---|---|---|---|---|
| | | | $v^2$: (ft/sec.)$^2$ | $m$: feet | $m^{4/3}$: feet$^{4/3}$ | |
| 12,000 | 7·1 | 7·25 | 55·0 | 10·4 | 22·2 | 0·0224 |
| 7,700 | 7·7 | 7·85 | 42·4 | 9·00 | 17·7 | 0·0242 |
| 4,800 | 7·0 | 7·08 | 29·6 | 7·20 | 14·1 | 0·0246 |

for a straight uniform channel of the same material and having area and perimeter equivalent to the mean section of the given channel but free from all discontinuities. A commonly quoted $n$ for straight and uniform earth channels in "best condition"[4] is 0·017.

It seems reasonable, therefore, to adopt this value and see what conclusion it leads to concerning the relative importance of skin friction and other influences.

The quantity $h_2$ has already been defined as the drop in water-surface-level caused by bends and changes of width and depth. Now:

$$\frac{h_2}{H - h_v} = 100\left[1 - \left(\frac{n}{n_1}\right)^2\right] \text{ per cent} \quad . \quad . \quad . \quad . \quad (5)$$

Inserting $n = 0·017$ and $n_1 = 0·0224$, 0·0242, and 0·0246 respectively, it is found that the results given in Table 11 are not unlikely for the stretch of river between G and S.

TABLE 11.—NATURAL RIVER—G TO S

(Assuming $n = 0·017$, and $n_1$ as in Table 10)

| Discharge: cusecs | Percentage effect of bends and changes of section $\frac{h_2}{H - h_v} \times 100$ |
|---|---|
| 12,000 | 42 |
| 7,700 | 51 |
| 4,800 | 52 |

The percentages computed for the model (Table 3) were 56, 59, and 67 (about), which are, on the average, 1·26 times those for nature, according to Table 11.

Before commenting upon this conclusion, it is interesting to consider the stretch of natural river between Station X and Hunt's Bank. The relevant data are set out in Table 12.

TABLE 12.—NATURAL RIVER—STATION X TO HUNT'S BANK

($l$ = 6,440 feet)

| Discharge: cusecs | $H$: feet | $H-h_v$: feet | Mean | | | $n_1$ (from equation (4)) | $\frac{h_2}{H-h_v} \times 100$ if $n = 0.017$ |
|---|---|---|---|---|---|---|---|
| | | | $v^2$: (ft./sec.)$^2$ | $m$: feet | $m^{4/3}$: feet$^{4/3}$ | | |
| 12,000 | 6·0 | 6·5 | 54·4 | 12·3 | 28·1 | 0·034 | 75 per cent |
| 7,700 | 6·9 | 7·25 | 54·8 | 8·99 | 18·6 | 0·029 | 66 per cent |
| 4,800 | 5·7 | 5·94 | 25·1 | 8·34 | 16·9 | 0·037 | 79 per cent |

The figures appearing in the final column of Table 12 should be compared with the model estimates in Table 3 (X to A9) and Table 4. It will be seen that they are at any rate of the same order, although on the average the estimated skin friction in nature is now less than in the model, whereas for the region G to S, the reverse was found.

A method of deliberately *overestimating* the effect of textural roughness in the natural river would be to assume that the whole resistance between G and S is caused by skin friction and then to work out the skin frictional head in the portion X to Hunt's Bank as equal to $(H - h_v)$ for G to S multiplied by $\frac{6{,}440}{12{,}440}$ and by $\left(\frac{v_{X1}}{v_{GS}}\right)^2 \left(\frac{m_{GS}}{m_{X1}}\right)^{4/3}$, where X1 refers to the stretch from X to Hunt's Bank and GS to the stretch from G to S. On this basis, the skin friction in the stretch between X and Hunt's Bank would be 45, 69, and 44 per cent of $(H - h_v)$, and these would be *overestimated* quantities.

Taking into account the whole of the analysis for both the model and the natural river, it appears that, on the average:

(*a*) between Douglas Green Weir and Adelphi Weir, the textural roughness of the sides and bed accounts for approximately one-half of the total resistance; and

(*b*) between Adelphi Weir and Hunt's Bank, textural roughness is responsible for only about one-third of the total resistance.

In either case, it is seen that any error in reproducing the required roughness in the model is not so serious as might at first be supposed,

provided that the loss of head at the bends and changes of section is simulated to scale.

Using suffix (1) to denote the actual river and suffix (2) the model (horizontal scale 1 : 500 and vertical scale 1 : 96), then the friction head in the model will be $\frac{1}{96}$ of that in nature, under a corresponding discharge, if

$$n_2 = \sqrt{500}\, n_1 \left(\frac{m_2}{m_1}\right)^{2/3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Applying this equation on the supposition that $n_1 = 0{\cdot}017$, it follows that:

| | | | |
|---|---|---|---|
| $n_2$ should be | 0·0122 | for a flow representing | 12,000 cusecs |
| $n_2$ ,, | 0·0131 | ,, ,, ,, | 7,700 ,, |
| $n_2$ ,, | 0·0133 | ,, ,, ,, | 4,800 ,, |

The actual values of $n_2$ (that is, $n$ in the model) lie between 0·0115 and 0·0119 for the depths and velocities concerned.

On the assumption that $n_1 = 0{\cdot}017$, therefore, there is a possibility that the model produces a friction head of $\left(\frac{0{\cdot}0115}{0{\cdot}0122}\right)^2$, or 0·89, of what it should at 12,000 cusecs and $\left(\frac{0{\cdot}0119}{0{\cdot}0133}\right)^2$, or 0·80 of the proper head at 4,800 cusecs.

But if textural roughness accounts for one-half of the whole resistance, the corresponding *total* discrepancies will be 6 and 10 per cent respectively, provided the effect of the bends and changes of section is correctly reproduced. Actually, the errors will not amount to 6 or 10 per cent because a deficiency of friction in the model will tend to flatten the water gradient and to decrease the sectional areas. Thus, the velocities (for a given discharge) will increase and both frictional and bend losses will tend to rise again.

On the whole, therefore, the final result will be errors in the fall of water surface amounting to less than the 6 and 10 per cent at first expected from 11 and 20 per cent errors in friction, whilst if the textural roughness is responsible for only one-third instead of one-half of the whole resistance, the final errors at flows representing 12,000 and 4,800 cusecs may be expected to be less than 4 and 7 per cent respectively.

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*Fig. 4*

*By courtesy of Aberdeen Journals Ltd.*

General View of River Irwell Model

F. G. Willox, B.Sc.(Eng.), in the experiments on the straight channels and in the computation of results.

The Authors desire further to record their appreciation of the approval granted by the Court of the University of Aberdeen to the building and testing of the model in the Engineering Laboratories at Marischal College.

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4. H. W. King, " Handbook of Hydraulics." McGraw-Hill, 1939, see p. 268.


The Paper is accompanied by three sheets of diagrams and two photographs, from which the Figures in the text have been prepared, and by the following five Appendices.

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## APPENDIX I

### Spacing of Sections in Model

$D$ = Distance along channel, measured from Hunt's Bank (feet)

| Section No. | D | Section No. | D |
|---|---|---|---|
| G (or 46) | 46·90 | 25 | 25·78 |
| 45 | 46·30 | 24 | 24·00 |
| 44 | 46·00 | 23 | 23·98 |
| 43 | 45·04 | S (or 22) | 22·03 |
| 42 | 44·22 | | |
| 41 | 42·46 | | |
| 40 | 42·34 | X (or 12) | 13·06 |
| 39 | 40·42 | 11 | 9·50 |
| 38 | 39·96 | 10 | 6·83 |
| 37 | 38·59 | 9 | 6·60 |
| 36 | 38·06 | 8 | 6·32 |
| 35 | 37·03 | 7 | 6·14 |
| 34 | 35·86 | 6 | 5·74 |
| 33 | 34·15 | 5 | 4·26 |
| 32 | 33·58 | 4 | 2·26 |
| 31 | 32·00 | 3 | 1·62 |
| 30 | 31·82 | 2 | 0·40 |
| 29 | 30·03 | 1 | 0·18 |
| 28 | 29·37 | 1A | −0·09 |
| 27 | 27·90 | 2A | −0·52 |
| 26 | 26·70 | 3A | −1·36 |
| | | A9 (or 4A) | −3·45 |

## APPENDIX II

Model Discharge 0·0340 Cusec, Representing 16,000 Cusecs in Nature

| Section No. | Observed water level (converted to feet O.D. in nature) | Hydraulic mean depth (feet in model) | Mean velocity (feet per sec. in model) |
|---|---|---|---|
| G (or 46) | 99·0 | 0·0689 | 0·940 |
| 45 | 99·1 | 0·0739 | 0·700 |
| 44 | 99·0 | 0·0544 | 0·800 |
| 43 | 99·0 | 0·0731 | 0·730 |
| 42 | 99·0 | 0·0747 | 0·619 |
| 41 | 98·9 | 0·0781 | 0·874 |
| 40 | 98·9 | 0·0622 | 0·798 |
| 39 | 98·6 | 0·0649 | 0·740 |
| 38 | 98·4 | 0·0756 | 0·699 |
| 37 | 97·7 | 0·0664 | 0·905 |
| 36 | 97·2 | 0·0672 | 0·844 |
| 35 | 96·2 | 0·0822 | 0·753 |
| 34 | 96·0 | 0·0689 | 0·857 |
| 33 | 95·6 | 0·0739 | 0·857 |
| 32 | 95·2 | 0·0672 | 0·905 |
| 31 | 95·1 | 0·0651 | 0·978 |
| 30 | 95·1 | 0·0774 | 0·764 |
| 29 | 94·6 | 0·0825 | 0·696 |
| 28 | 94·4 | 0·0759 | 0·635 |
| 27 | 94·0 | 0·0676 | 0·814 |
| 26 | 93·9 | 0·0659 | 0·788 |
| 25 | 93·7 | 0·0681 | 1·01 |
| 24 | 93·6 | 0·0814 | 0·740 |
| 23 | 93·6 | 0·0824 | 0·795 |
| S (or 22) | 93·0 | 0·0789 | 0·792 |
| X (or 12) | 89·4 | 0·0679 | 0·915 |
| 11 | 88·2 | 0·0731 | 0·896 |
| 10 | 86·6 | 0·0571 | 1·21 |
| 9 | 86·5 | 0·0726 | 0·867 |
| 8 | 86·2 | 0·0744 | 0·700 |
| 7 | 86·0 | 0·0754 | 0·865 |
| 6 | 85·6 | 0·0646 | 1·12 |
| 5 | 84·1 | 0·0635 | 1·08 |
| 4 | 83·4 | 0·0631 | 0·944 |
| 3 | 83·2 | 0·0701 | 0·906 |
| 2 | 83·0 | 0·0761 | 0·785 |
| 1 | 82·5 | 0·0761 | 0·715 |
| 1A | 82·0 | 0·0611 | 1·04 |
| 2A | 80·9 | 0·0631 | 1·05 |
| 3A | 80·5 | 0·0655 | 1·07 |
| A9 (or 4A) | 80·0 | 0·0711 | 0·960 |

## APPENDIX III

Model Discharge 0·0255 Cusec, Representing 12,000 Cusecs in Nature

| Section No. | Observed water level (converted to feet O.D. in nature) | Hydraulic mean depth (feet in model) | Mean velocity (feet per sec. in model) |
|---|---|---|---|
| G (or 46) | 97·0 | 0·0625 | 0·829 |
| 45 | 97·0 | 0·0665 | 0·620 |
| 44 | 96·5 | 0·0510 | 0·720 |
| 43 | 96·6 | 0·0670 | 0·668 |
| 42 | 96·7 | 0·0700 | 0·548 |
| 41 | 96·1 | 0·0635 | 0·865 |
| 40 | 95·9 | 0·0605 | 0·757 |
| 39 | 95·5 | 0·0630 | 0·693 |
| 38 | 95·5 | 0·0740 | 0·645 |
| 37 | 94·2 | 0·0645 | 0·875 |
| 36 | 94·0 | 0·0605 | 0·825 |
| 35 | 93·2 | 0·0740 | 0·680 |
| 34 | 93·0 | 0·0630 | 0·800 |
| 33 | 92·4 | 0·0657 | 0·815 |
| 32 | 92·1 | 0·0633 | 0·834 |
| 31 | 91·9 | 0·0610 | 0·895 |
| 30 | 91·8 | 0·0670 | 0·780 |
| 29 | 91·3 | 0·0643 | 0·834 |
| 28 | 91·1 | 0·0700 | 0·607 |
| 27 | 91·2 | 0·0647 | 0·760 |
| 26 | 90·7 | 0·0610 | 0·780 |
| 25 | 90·6 | 0·0650 | 0·800 |
| 24 | 90·6 | 0·0708 | 0·734 |
| 23 | 90·6 | 0·0702 | 0·754 |
| S (or 22) | 89·7 | 0·0670 | 0·769 |
| X (or 12) | 86·6 | 0·0575 | 0·810 |
| 11 | 85·1 | 0·0640 | 0·815 |
| 10 | 83·3 | 0·0510 | 1·29 |
| 9 | 83·0 | 0·0665 | 0·805 |
| 8 | 82·8 | 0·0655 | 0·663 |
| 7 | 82·9 | 0·0680 | 0·780 |
| 6 | 83·0 | 0·0595 | 1·01 |
| 5 | 81·9 | 0·0575 | 0·960 |
| 4 | 81·0 | 0·0580 | 0·817 |
| 3 | 80·6 | 0·0655 | 0·776 |
| 2 | 80·5 | 0·0745 | 0·664 |
| 1 | 80·3 | 0·0725 | 0·597 |
| 1A | 79·7 | 0·0587 | 0·857 |
| 2A | 79·3 | 0·0602 | 0·966 |
| 3A | 78·8 | 0·0615 | 0·915 |
| A9 (or 4A) | 78·5 | 0·0670 | 0·794 |

## APPENDIX IV

Model Discharge 0·0164 Cusec, Representing 7,700 Cusecs in Nature

| Section No. | Observed water level (converted to feet O.D. in nature) | Hydraulic mean depth (feet in model) | Mean velocity (feet per sec. in model) |
|---|---|---|---|
| G (or 46) | 93·8 | 0·0550 | 0·738 |
| 45 | 93·5 | 0·0515 | 0·666 |
| 44 | 93·3 | 0·0535 | 0·647 |
| 43 | 93·1 | 0·0580 | 0·629 |
| 42 | 92·9 | 0·0575 | 0·460 |
| 41 | 92·7 | 0·0520 | 0·806 |
| 40 | 92·7 | 0·0565 | 0·647 |
| 39 | 92·4 | 0·0715 | 0·629 |
| 38 | 92·2 | 0·0700 | 0·605 |
| 37 | 91·2 | 0·0498 | 0·752 |
| 36 | 91·0 | 0·0522 | 0·750 |
| 35 | 90·0 | 0·0660 | 0·567 |
| 34 | 89·8 | 0·0550 | 0·676 |
| 33 | 89·7 | 0·0575 | 0·677 |
| 32 | 89·8 | 0·0605 | 0·647 |
| 31 | 89·6 | 0·0580 | 0·686 |
| 30 | 89·6 | 0·0598 | 0·647 |
| 29 | 88·3 | 0·0477 | 0·703 |
| 28 | 88·2 | 0·0670 | 0·463 |
| 27 | 88·3 | 0·0630 | 0·581 |
| 26 | 87·8 | 0·0562 | 0·686 |
| 25 | 87·8 | 0·0577 | 0·686 |
| 24 | 87·6 | 0·0582 | 0·647 |
| 23 | 87·6 | 0·0580 | 0·651 |
| S (or 22) | 86·6 | 0·0580 | 0·664 |
| X (or 12) | 82·8 | 0·0515 | 0·722 |
| 11 | 81·2 | 0·0535 | 0·787 |
| 10 | 80·0 | 0·0410 | 1·12 |
| 9 | 79·8 | 0·0595 | 0·656 |
| 8 | 79·5 | 0·0560 | 0·564 |
| 7 | 79·5 | 0·0600 | 0·695 |
| 6 | 79·3 | 0·0527 | 0·657 |
| 5 | 78·4 | 0·0533 | 0·785 |
| 4 | 78·0 | 0·0530 | 0·631 |
| 3 | 78·0 | 0·0600 | 0·603 |
| 2 | 77·8 | 0·0698 | 0·521 |
| 1 | 77·7 | 0·0672 | 0·432 |
| 1A | 77·5 | 0·0565 | 0·635 |
| 2A | 77·0 | 0·0570 | 0·675 |
| 3A | 76·9 | 0·0580 | 0·675 |
| A9 (or 4A) | 76·4 | 0·0633 | 0·590 |

## APPENDIX V

MODEL DISCHARGE 0·00935 CUSEC, REPRESENTING 4,400 CUSECS IN NATURE

| Section No. | Observed water level (converted to feet O.D. in nature) | Hydraulic mean depth (feet in model) | Mean velocity (feet per sec. in model) |
|---|---|---|---|
| G (or 46) | 91·6 | 0·0487 | 0·519 |
| 45 | 91·4 | 0·0440 | 0·538 |
| 44 | 91·0 | 0·0550 | 0·481 |
| 43 | 90·4 | 0·0510 | 0·517 |
| 42 | 90·0 | 0·0423 | 0·364 |
| 41 | 89·8 | 0·0440 | 0·640 |
| 40 | 89·8 | 0·0665 | 0·530 |
| 39 | 89·7 | 0·0545 | 0·500 |
| 38 | 89·4 | 0·0600 | 0·517 |
| 37 | 89·1 | 0·0445 | 0·538 |
| 36 | 89·1 | 0·0455 | 0·481 |
| 35 | 87·5 | 0·0575 | 0·420 |
| 34 | 87·5 | 0·0485 | 0·517 |
| 33 | 87·2 | 0·0500 | 0·561 |
| 32 | 87·2 | 0·0540 | 0·500 |
| 31 | 87·0 | 0·0535 | 0·500 |
| 30 | 87·0 | 0·0495 | 0·517 |
| 29 | 86·0 | 0·0400 | 0·614 |
| 28 | 85·4 | 0·0550 | 0·418 |
| 27 | 85·6 | 0·0520 | 0·481 |
| 26 | 85·3 | 0·0475 | 0·561 |
| 25 | 85·2 | 0·0525 | 0·500 |
| 24 | 85·2 | 0·0540 | 0·510 |
| 23 | 84·4 | 0·0530 | 0·500 |
| S (or 22) | 84·3 | 0·0525 | 0·479 |
| X (or 12) | 79·6 | 0·0440 | 0·585 |
| 11 | 78·5 | 0·0420 | 0·753 |
| 10 | 78·2 | 0·0380 | 0·842 |
| 9 | 78·2 | 0·0522 | 0·467 |
| 8 | 78·0 | 0·0465 | 0·408 |
| 7 | 77·5 | 0·0540 | 0·498 |
| 6 | 76·8 | 0·0470 | 0·674 |
| 5 | 76·6 | 0·0510 | 0·535 |
| 4 | 75·7 | 0·0455 | 0·492 |
| 3 | 75·4 | 0·0485 | 0·481 |
| 2 | 75·3 | 0·0580 | 0·398 |
| 1 | 75·3 | 0·0602 | 0·328 |
| 1A | 75·2 | 0·0530 | 0·432 |
| 2A | 75·0 | 0·0525 | 0·476 |
| 3A | 74·8 | 0·0535 | 0·474 |
| A9 (or 4A) | 74·2 | 0·0550 | 0·443 |

Paper No. 5923

# "Reconstruction of the Ground Floor of the Royal Edward Cold Store, Avonmouth Docks"

by

**William James Sivewright, M.A., A.M.I.C.E., and Stanley Price Whittington, B.Sc., A.M.I.C.E.**

*(Ordered by the Council to be published with written discussion.)* †

SYNOPSIS

The Paper deals with the remedial work carried out at the Avonmouth Cold Store of the Port of Bristol Authority, following extensive damage to the ground floor of the building by frost heave.

A general description of the cold store is followed by an account of the damage and the phenomena associated with it. Reference is made to the mechanism of frost heave.

The considerations governing the design of the remedial work are discussed and an account is given of the work involved in replacing the damaged ground floor, which was originally laid directly on the ground, by a suspended floor with an air space beneath.

Working conditions during the excavation of the original floor were difficult because of the thawing of the exposed ground, and trouble was later experienced from condensation in the partially completed ground-floor rooms at a stage in the reconstruction when they were exposed simultaneously to high atmospheric temperatures and to refrigeration from the rooms above.

An Appendix to the Paper deals in a general way with the possible effect of reinforced-concrete piles on soil temperatures beneath a cold store having an air space below the cold rooms.

## Introduction

The Royal Edward Cold Store, of the Port of Bristol Authority, Avonmouth Docks, is a reinforced-concrete-framed building of four-storeys, with brick walls to the ground floor, and measures approximately 100 feet by 200 feet in plan ; it was constructed in 1922. (See Figs 1, Plate 1.)

The ground, first, and second floors comprise the Cold Store proper, with a total of 430,000 cubic feet of refrigerated space. Each of these floors is divided into three insulated cold chambers, separated by passageways—referred to as the air-locks—which accommodate lift shafts and communicating stairways.

The northernmost chamber on each floor is cooled by direct expansion

† Correspondence on this Paper should be received at the Institution by the 15th August, 1954, and will be published in Part III of the Proceedings. Contributions should be limited to about 1,200 words.—Sec. I.C.E.

of ammonia through steel pipe grids. The remaining six chambers are cooled by air circulation, the air being refrigerated in brine coolers situated outside the Cold Store and conveyed to and from the rooms through timber ducts. The working temperature in the cold chambers ranges from 14° F. to 20° F. The third (top) floor—the sorting floor—receives meat and other produce ex-ship via an overhead conveyor gallery. This floor, extending over the whole area of the Cold Store, is worked at atmospheric temperature.

The refrigerating machinery, which works on the "wet compression" ammonia cycle, is housed, together with the coolers referred to above and the brine tanks, in a two-storey annexe to the main building.

Eight electric lifts serve the store. Four operate internally, two to each air-lock, and communicate between the sorting floor and each of the storage floors. The four other lifts, situated against the outer walls, communicate only between the sorting floor and ground level outside the building, and are used primarily for refrigerated produce which is to be dispatched by road or rail. There are also three sets of concrete stairways, two sets communicating internally with all floors and the third between the sorting floor and ground level outside the building. The external walls are insulated by a 9-inch thickness of slag wool retained behind a double skin of tongued-and-grooved boarding with bitumen-bonded building paper between the boards. The ground-floor and second-floor ceilings are insulated with baked granulated-cork slab.

The ground beneath the Cold Store consists of about 50 feet of soft brown alluvial silty clay overlying the Red Keuper Marl. Normal ground-water in the vicinity of the store is about 2 feet 6 inches below ground level. The original ground-floor level was 31·42 O.D. (Liverpool); normal water level in the nearby dock being 20·42 O.D.

The whole structure, with the exception of the ground-floor slab as originally constructed, is supported on groups of reinforced-concrete piles under the columns. Reinforced-concrete ties connect the pile caps in both directions, at a level below that of the original ground floor.

This floor, which consisted of 6 inches of unreinforced concrete, was laid independently of the main frame of the building directly on a sub-base consisting of a 12-inch layer of ashes which covered the natural ground. Above the unreinforced-concrete slab were laid two 3-inch layers of granulated cork set in cement mortar. The cork was protected with a 2-inch thickness of granolithic concrete as a wearing surface.

The interior columns were insulated with 3 inches of cork slab for a height of 3 feet above floor level, the cork slab being rendered with cement mortar.

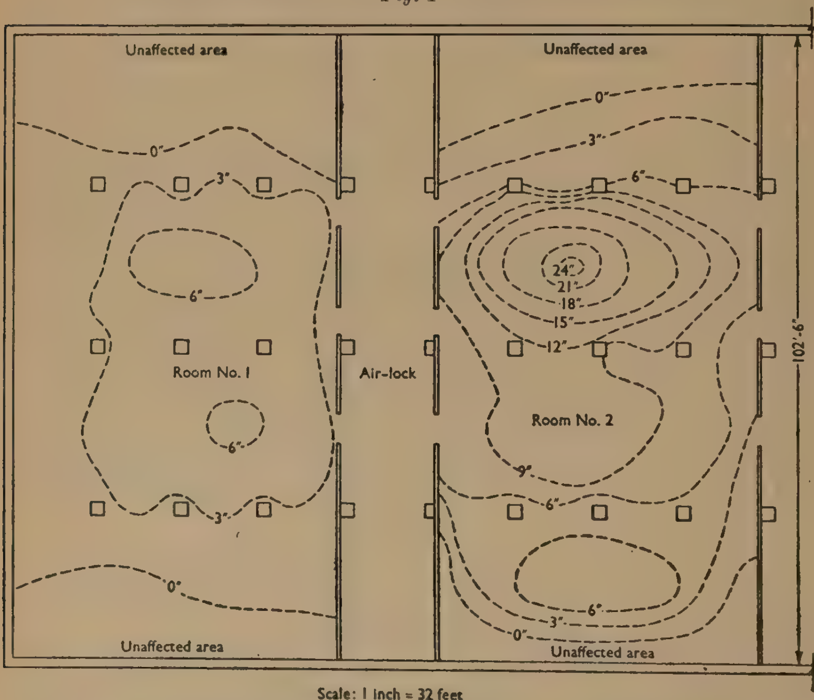
## Extent of Damage

Three years after its construction, damage to the ground-floor slab by frost-heave was already evident. Although unsuccessful attempts

were made from time to time to restore damaged concrete in situ, hogging of the floor increased with the passage of time, until by 1949 the amount of heave had become such that the floor slab had suffered extensive cracking and breaking; the chamber doors jammed, and these effects, combined with the consequent reduction of headroom beneath the ceiling beams, rendered the working of the ground-floor rooms exceedingly difficult.

The chamber worst affected was, as might be expected on account of

*Fig. 4*




CONTOUR PLAN OF HEAVED FLOORS, ROYAL EDWARD COLD STORE

(Note: The heights given to the contours represent the heights of the heaved floor above original floor level, May 1950.)

its being farthest from the boundaries of the building, the middle one, No. 2. Here the maximum heave was 2 feet 1½ inch, reducing the headroom under the ceiling beams to 6 feet 4½ inches, instead of the nominal 8 feet 6 inches. *Fig. 2* illustrates the general condition of the floor surface consequent upon the heave. The lath in the background, standing against a ceiling beam, is marked in feet and half-feet. *Fig. 3* shows the effect of the heave on the timber barriers used to separate different parcels

of meat. The timber cladding to the wall insulation was also buckled in places. *Fig. 4* is a contour plan of the floors of Rooms 1 and 2 immediately before reconstruction. A similar record for Room 3 could not be obtained but from casual observation the contours appeared to be similar to those in Room 1.

A trial pit dug near the point of maximum heave showed that the ground had been frozen to a depth of about 12 feet below the original floor level. The upward expansion of the ground was, therefore, about 18 per cent. Since water expands by about 10 per cent on freezing, with a natural water content of the soil of about 30 per cent it might be expected that simple freezing would produce a heave of only about 3 per cent.

The sides of the pit, as shown in *Fig. 5*, revealed a mass of ice seams, in both horizontal and vertical directions, with thicknesses of up to 1 inch. The layers were separated by blocks of soil which, though frozen solid, appeared to be otherwise undisturbed. The total thickness of horizontal layers as measured in a representative 2-foot depth of pit wall amounted to 5 inches, that is, about 20 per cent of the thickness of ground. This agrees approximately with the 18-per-cent heave mentioned above.

The trial pit was put down at a nearby column, rather than at the point of maximum heave, in order to discover if the heaving of the ground had damaged the pile cap. This was thought particularly necessary since some first-floor beams in the vicinity of the point of maximum heave showed cracks, suggesting the possibility of general distortion of the framework.

There are many recorded cases of frost heave, but the extent of the heave in the Avonmouth Cold Store seems to have been rarely exceeded. There are numbers of cases known of stores in which, with the ground floor founded directly on the ground and monolithic with the framework, hogging of the floor and unequal lifting of the foundations have occurred, with distortion of the framework.

No sign of damage was apparent on the pile cap and supporting piles exposed in the trial pit, nor was any to be seen when the other caps were subsequently exposed during the course of the reconstruction.

It was observed that above the tie-beam exposed in the trial pit, the frozen ground had heaved several inches away from the top of the beam, leaving a cavity which extended over the length of the tie and which was partly filled with delicate ice crystals growing from the sides and top of the cavity.

A second trial pit dug near an outer wall of the cold store showed no trace of ground freezing below floor level.

### *Mechanism of Frost Heave*

The mechanism of frost heave has been discussed by numerous authorities. Cooling and Ward [1] appear to have explained the effect as

[1] L. F. Cooling and W. H. Ward, "Damage to Cold Stores Due to Frost-Heaving." *Proc. Inst. Refrigeration*, vol. XLI (1944), p. 37.

being the result of an initial supercooling of ground-water, followed by freezing, with a consequent drop in vapour pressure. With the reduction of pressure, further ground water is drawn in to fill the vacuum and the process continues in a series of similar cycles, so building up the ice lenses. When the supply of water ceases at any level the ice lenses cease to grow. Normal freezing of the ground then proceeds downwards until further water becomes available and the growth of an ice lens begins again. According to one recent American authority,[2] frost heave has been produced under laboratory conditions with fluids which actually contract upon freezing.

The phenomenon can occur only in ground similar to that under the Royal Edward Cold Store, which is neither so impervious as to prevent the percolation of water nor so porous as to prevent the capillary suction of water promoting the formation of ice layers.

The horizontally stratified structure of the ground, which was revealed during the reconstruction, no doubt contributed to the movement of ground-water.

Thermometers, packed in slag wool, were inserted in one wall and the floor of the main test pit in the positions shown in *Fig. 6*, which also gives the recorded readings. Except when readings were being taken, the pit was insulated from the chamber above by a temporary wooden cover carrying bags of slag wool.

Unfortunately, excavation below the limit of freezing permitted ingress of ground-water to the bottom of the pit, and the water rose about 4 feet inside the pit before it froze, so that the reading on the lowest thermometer soon became unavailable.

## Design and Reconstruction of New Floor

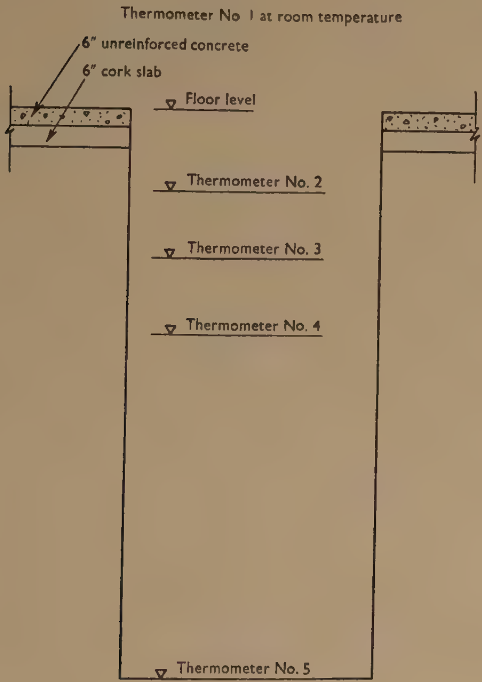
In the consideration of the means to be adopted for overcoming the trouble there was from the outset close co-operation with the contractors for the work, who had experience of dealing with similar problems elsewhere.

Owing to the extent of damage to the floor, it was not practicable to remedy the heave by adopting the method sometimes employed, namely, that of inserting heating pipes or cables to thaw the ground and so permit the floor to descend to its original level.

The store foundations and framework had been originally designed with the possibility in mind of another storey being added. Consideration was therefore given to abandoning the ground floor and replacing it by new chambers on the top of the building. Such a proposal would have meant that the existing sorting floor would have become intermediate between the existing cold floors and the new fourth floor. The latter, exposed as it would be on all external faces to atmospheric temperatures and conditions, would have required heavy insulation. Moreover, it was considered that

[2] E. L. Nelson, "Freezer Room Construction Problems." *J. Nat. Assoc. Prac. Refrig. Engrs. Amer.*, 1952.

*Fig. 6*




| Date | No. 1 | No. 2 | No. 3 | No. 4 | No. 5 |
|---|---|---|---|---|---|
| 29. 9.49 | 19° | First placed | First placed | First placed | |
| 30. 9.49 | 19° | — | — | 26° | |
| 3.10.49 | — | — | — | 27° | |
| 6.10.49. | — | — | — | — | First placed |
| 11.10.49 | 19° | 28° | 29° | 30° | 33° |
| 25.10.49 | 18° | — | — | 30° | — |
| 10.11.49 | 16° | 29° | 29° | 30° | — |
| 12. 5.50 | 16° | 29° | 29° | 29° | — |

*Note:* Readings are given in degrees Fahrenheit.

POSITIONS OF THERMOMETERS IN TRIAL HOLE AND TABLE OF READINGS

though the refrigerating plant was designed to serve a possible fourth floor, such a proposal would put an excessive load on the plant after more than 30 years of operation.

Fortunately, the pile caps over the groups of piles under each column are of considerably greater area in plan than the columns they support. Consequently they were able to provide adequate bearing area for the main members of a suspended floor, the construction of which would not put any more weight on the piles than the ultimate load for which they were designed.

The new ground floor (see Figs 7, Plate 2) was therefore arranged to be carried on a grid of concrete-encased steel beams spanning between pile caps. The designed floor loading is 3 cwt per square foot and the beams consist mostly of 22-inch-by-7-inch rolled steel joists with 12-inch-by-$\frac{3}{4}$-inch welded flange plates. Larger plated sections are provided in certain positions where the loading increases owing to trimming members and the incidence of partitions, walls, etc. On the bottom flange of the main steel floor-beams a reinforced-concrete haunching is provided, the reinforcement of which passes through the web of the beams to safeguard the stability of the haunching. The haunching carries patent precast reinforced-concrete floor sections which are covered with 3 inches of concrete. The surface of this structural sub-floor is coincident with the level of the top flange of the main steel floor-beams. This arrangement allows a uniform 8-inch-thick layer of insulation to be provided over the whole area of the cold rooms, and a 6-inch layer over the air-locks. The insulation consists of two layers of baked granulated-cork slab bedded in cement mortar to the required overall thickness. Over the insulation generally is laid a wearing course of 3 inches of granolithic concrete, the exposed surfaces of which have been treated with carborundum grit and a silico-fluoride-type hardener to prevent dusting.

In the air-locks and in the main trucking ways of the chambers, where the greatest wear from the iron-wheeled meat barrows may be expected, the 1-inch-thick wearing surface is of Loliondo, an East African hardwood, laid in strips in the direction of traffic, and secret-nailed to softwood grounds inserted in the concrete. Maple was the timber originally proposed, but dollar restrictions precluded its use.

The timber can easily be renewed in case of wear, whereas satisfactory repair of worn or damaged concrete is virtually impossible in the low working temperatures.

The cork insulation has been continued up the columns to the underside of the haunches of the ceiling beams and, after rendering with cement mortar, has been protected by tongued-and-grooved boarding.

The ground beneath the new floor has been dug away over the whole area of the building to permit circulation of air between the new ground surface and the underside of the reconstructed floor. Two considerations determined the depth of the excavation. It was thought undesirable to

expose unduly the reinforced-concrete tie-beams connecting the pile caps, and at the same time adequate clearance under the floor beams had to be maintained to permit air circulation and access for inspection. It was decided that these conditions would be satisfied by a clearance of 2 feet beneath the soffit of the floor beams; this resulted in only the top of the tie-beams being exposed to the atmosphere.

Sloping ducts (Fig. 8, Plate 2) constructed under the outer walls of the building communicate between the ventilated space and the outer air at original ground level. The outer ends of the ducts are covered by close wire mesh to prevent the entrance of mice and rats.

Since access to the underside of the floor via the ventilation ducts is difficult, four inspection manholes have been provided in the floor, one at each end of each air-lock. The manholes are closed by thick insulated plugs of cork and teak (Fig. 9, Plate 2).

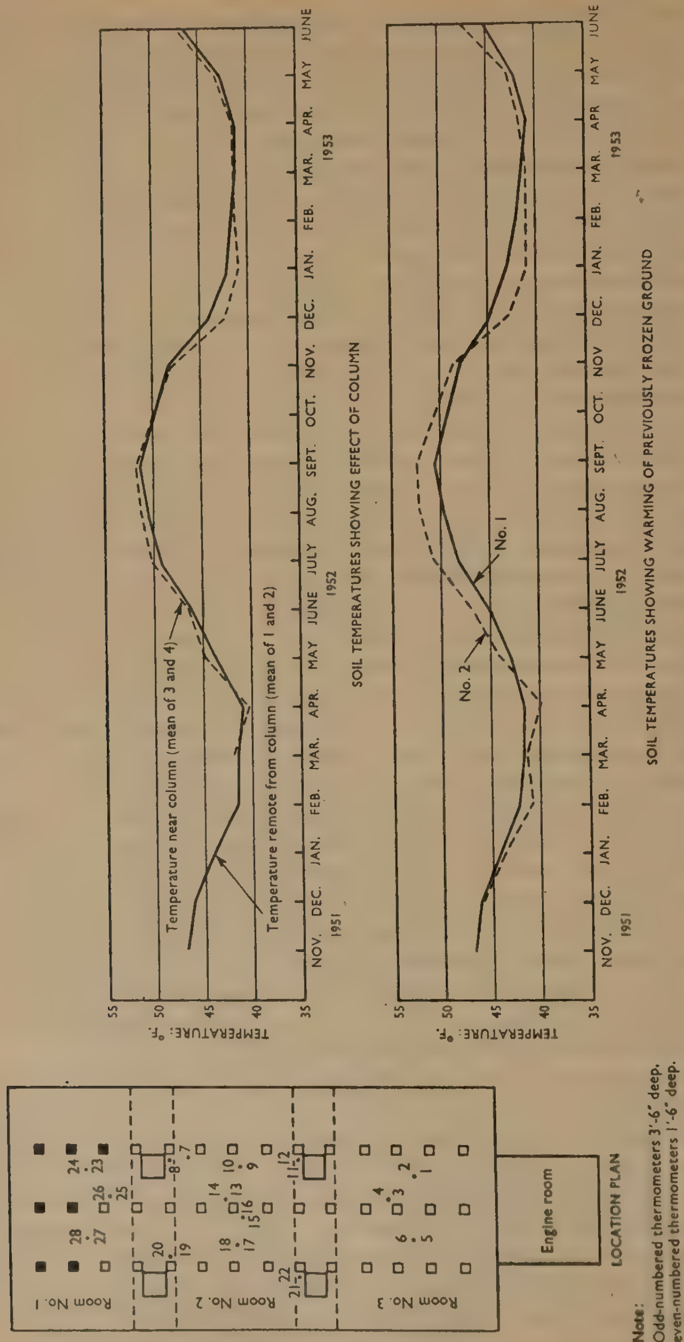
Much thought was given to the desirability of inserting heaters in the columns or pile caps below the floor level, to combat recurrence of freezing of the ground by heat loss upward through the piles. The arguments on which the final decisions were based are summarized in the Appendix. So many imponderable factors were involved that it was not possible to arrive at a calculated decision, but it was finally decided that heaters could be dispensed with and reliance placed on the air space and increased insulation of the columns.

As a check on actual conditions in the ground below the floor, remote-reading thermometers have been inserted at various points. *Figs 13* show typical readings up to the time of the presentation of this Paper. Readings of the thermometers under Room 3 have been selected because this room has been operating the longest since reconstruction.

The work of reconstruction began with the isolation of the ground floor from the remainder of the building by constructing insulated screens round the stair and lift wells. The air ducts into Rooms 2 and 3 were blocked off by means of slag-wool plugs and the ground-floor ceilings were temporarily insulated by means of 6 inches of slag wool carried on asbestos sheets, which were themselves supported by timber bearers wedged between the beams. At the same time, openings large enough to admit lorries were cut through the brick walls, one in the south end of the west wall and one in the west end of the north wall.

At a quite early stage, the temporary insulation proved to be not so effective as had been hoped and difficulty was experienced in one period of hot weather in keeping hard the frozen meat stored on the floors above Rooms 2 and 3. The slag wool was found to be very wet, and its insulating properties impaired. The measures taken to combat the trouble included the provision of timber fillets along the edges of the panels of temporary insulation in an attempt to restrict the circulation of air within the slag wool; the raising of the meat off the floors of the rooms above; the adaptation of the air ducts in those rooms to deliver a proportion of

*Figs 13*




RECORDS OF BURIED THERMOMETERS

air below the meat; and the workings of the refrigerating machinery at its greatest possible output. Having been originally rated for four floors, it proved capable of just holding its own with two floors in operation under the arduous conditions prevailing during the reconstruction.

When the ground floor had been opened up, the direct-expansion pipe-grids were removed from Room 1 and the air ducts from the other two rooms, and this was followed by the stripping of insulation from the columns and walls. It had originally been intended to strip only the bottom of the wall insulation up to the new floor level, but it was found that the slag wool was sodden and the lower 3 or 4 feet of the timber studding badly decayed. The whole of the insulation was, therefore, taken down, eventually to be replaced by new. Before the replacement, the inside faces of the brick walls were coated with bitumen, whilst all the new studding was treated with two coats of wood preservative.

The next stage was the breaking-up of the old floor by pneumatic breakers and its loading into lorries which were run into the building via the temporary openings. The exhaust fumes from the compressor and tools were extracted by means of fans set in openings, one being cut in the east wall of each chamber.

The whole of the necessary excavation beneath the floor—about 3,000 cubic yards—was taken out by hand to the required level, pneumatic tools being used to break up the ground in the middles of the rooms, since although the top foot or so thawed rapidly on exposure, this was not so at lower depths. In the early stages, lorries entered the chambers to pick up spoil, but when ground conditions deteriorated they remained outside the building and the spoil was barrowed out on plank runways over the tops of the tie-beams.

Before the work began, the use of a low-headroom scraper and dumper had been considered for the excavation, but had been turned down on account of the high hire charges in relation to the uncertainty of continuous working and the difficulty of running over the tops of the tie-beams. The wisdom of this decision was confirmed by the extent of frozen ground which could not have been dealt with by scraper.

The thawing of the upper layers of ground resulted in a layer of wet slurry which made working conditions difficult. In anticipation of this a herringbone pattern of 3-inch land drains had been ordered, draining to a manhole outside the building. The levels of the existing outside drains were such that the land drains could not be laid under the tie-beams, which were passed by means of open catch-pits as shown in Fig. 10, Plate 2.

After the excavation had been carried down to the desired level, probing showed that pockets of ground remained frozen to a depth of several feet. It was evident that the natural thawing of these pockets would be prolonged, and it was considered that they should not be allowed to remain in existence to serve as a possible " source of infection " from which further freezing of the ground might possibly begin after the reconstruction had

been completed. A pipe, perforated at its lower end, was driven into the ground and a second pipe was lowered into it, fed by steam through hoses from the boiler of a crane standing outside the building. Steam was passed in, and the outer pipe driven in farther as necessary until lack of resistance to a probe showed that all ice had been melted in the vicinity of the pipe, which was then withdrawn and the process repeated at points at about 2-foot centres.

Many of the tie-beams towards the middle of the rooms were found to have hogged upwards under the pressure of the heaving ground by as much as 1 foot on a span of 20 feet, with serious cracking of the concrete. Such beams were stripped completely of their concrete, the main bars straightened and links restored, and then re-concreted.

The available drawings did not indicate whether the lift wells and the bottom flights of the reinforced-concrete stairs were carried on piles or directly on the ground. It was found that the latter was the case. There was also at one side of each air-lock a reinforced-concrete partition resting on the ground floor.

The partitions were propped for as long as possible from the tie-beams beneath, but there were prolonged periods during excavation and during the rebuilding of the tie-beams and construction of the new floor when the partitions had to be left suspended from the columns and ceiling beams with which they were monolithic. The partitions withstood this treatment without damage.

*Reconstruction of Lift Wells*

The interior lift wells offered a more serious problem. Since the rest of the cold store was in use throughout the reconstruction, the lifts had to remain in operation without interruption. At first the lift guides, balance weights, etc., continued to extend to the ground floor, although the lifts stopped at the first floor.

In the case of these interior lifts, thawing of the ground, following excavation, resulted in marked settlement of the brick wells which rapidly developed cracks. This distortion threatened the proper working of the lifts, so it was decided to attempt cementation of the ground beneath the pits to ensure the safety of the lift structures until the guides, etc., could be shortened.

There were no signs of settlement under the two lifts on the east side of the building and the adjacent ground had not been frozen except to a very shallow depth. The wells of these lifts, however, were built internally against the main wall of the store. Because the excavation for the new floor was to be carried down below the concrete foundation slab supporting the lift wells, it was thought desirable to extend the cementation process to the ground under these lifts. Before the cementation process was applied, a girdle wall was constructed on three sides of each well to constrain the ground beneath the foundation slab.

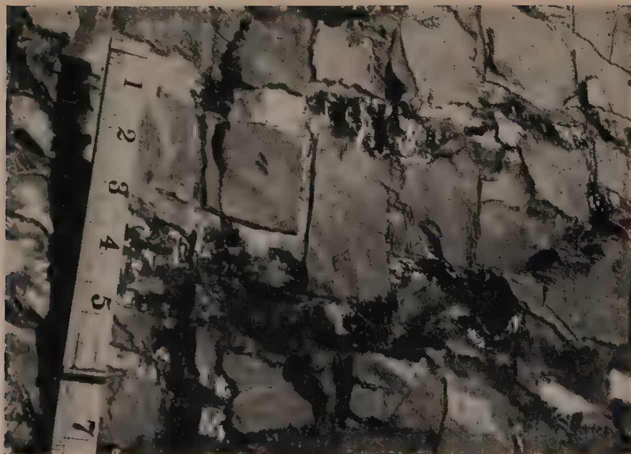
*Fig. 2*

GROUND FLOOR OF COLD STORE (NO. 2 ROOM) SHOWING FLOOR HEAVE
(Main trial pit was sited beneath sacks in right-hand background)

*Fig. 3*

EFFECT OF HEAVE ON TIMBER PARTITIONS

*Fig. 5*



Ice Formation in Main Trial Pit, Room No. 2

*Fig. 12*

Completed Chamber after Reconstruction

Since the remaining exterior lifts were wholly outside the building and the ground beneath had not been subjected to freezing, no treatment was considered necessary.

After the lift gear in the interior lifts had been shortened, the pits were demolished and the ground beneath excavated. It was found that the cementation had been by no means complete. The grout had spread outwards in thin horizontal sheets, with layers of unconsolidated ground between, having followed either the laminations produced by the formation of the ice layers or, what is more likely, the planes between the strata of alluvium originally laid down under tidal conditions. So far as could be seen, the cementation under the exterior lift wells, within the constraint of the girdle walls, was much more complete.

Considerable difficulty was encountered in arriving at a satisfactory design for the reconstruction of the lift pits which had been demolished.

The main floor steelwork had already been fabricated and delivered to the site and, since this was so, the additional weight which would have been imposed barred any consideration of providing a suspended well with a ventilated space beneath in common with the floor generally.

Moreover, in arriving at a suitable design, attention had to be paid to the possibly excessive loads to which the pit would be subjected through the remote chance of the lift overrunning its limit switches or a similar failure. This indicated the desirability of the pit being structurally isolated from the main floor to limit the extent of damage which might result from such an occurrence. This reasoning led to the acceptance of the risk of promoting further ground freezing and to founding the pits on the ground, at the same time maintaining the pits independent of the floor. The junction of the pit walls and the floor are sealed with a continuous bent copper strip to prevent ingress of wet or damp (Fig. 11, Plate 2). It would have been possible to provide a piled foundation for each pit and still maintain a ventilated space beneath the pit foundations. However, piling in the restricted space available would only have been possible using special plant for forming in-situ piles, and the proposal was rejected in view of the excessive cost and the delay which would have been caused to the completion of the work.

To reduce the risk of promoting ground freezing, the ground beneath the pits was excavated to a depth below the limit of the frozen ground (about 5 feet). During this excavation only small quantities of ice were found. The holes were filled with tightly rammed stones on which the pits were founded. The reconstructed pits incorporate spring buffers to take the blow of the lift cages or of the balance weights in the event of the lift over-running in either direction. Any settlement of the lift pits caused by such an occurrence, or any ground subsidence, would not damage the floor or the lift guides.

No attempt was made, other than by temporary propping, to preserve the bottom flights of the concrete stairways which had already suffered

some damage by way of cracked stringers as a result of the frost heave. The damaged sections were taken down and reconstructed.

After the excavation had been completed, the drains were laid and the whole area beneath the floor covered with a 1-foot thickness of ashes. The latter are intended to provide an easily drained blanket which will not only serve to insulate the natural ground from heat loss but will also provide a reasonably clean surface for anyone crawling under the beams on routine visits of inspection and maintenance of ground thermometers. Commencing with Room 3, the main steelwork was placed in position and the concrete haunches constructed; the precast flooring was then installed and concreted in, and the granolithic and timber wearing surfaces were laid. The restoration of the insulation, doors, air-ducts, dunnage barriers, electric lighting, etc., completed the reconstruction.

As each of the first two rooms (3 and 2) was nearing completion, considerable condensation occurred, because of the high summer outdoor temperatures (70°–80°), in conjunction with the low temperatures prevailing inside the rooms through cooling from the floor above. This effect was barely evident in Room 1, since the room above had an insulated floor.

The damp atmosphere, coupled with the unavoidable lack of ventilation, promoted the growth of moulds, particularly on the timberwork in Room 3, which had to be washed off with a strong solution of copper sulphate.

Tarpaulins had to be hung to protect the wet granolithic surfacing, as it was being laid, from being marked by water drops falling from the ceiling and beams.

It was not possible to protect the whole of the floor, and concern was felt at the amount of moisture likely to be absorbed by the cork-slab insulation before the granolithic surfacing was laid. Test samples showed the moisture content to range from 3 to 14 per cent of the dry weight, and some impairment of the insulating properties was feared.

The room doors were kept shut as much as possible to prevent circulation of air from outside, and the temperature was kept down to about 36° F., which was thought to be the lowest compatible with the proper setting of the granolithic surfacing. Trays of calcium chloride were placed about the floor to absorb as much moisture as possible.

Subsequently, after the rooms had been put into commission, further test samples were cut from the cork insulation after removing the granolithic. The samples gave no noticeable moisture content and appeared to be in a perfectly dry condition. It is thought that part of the moisture originally contained in the cork had been absorbed by the hydration of the cement in the cement-mortar jointing and the granolithic surfacing, and that the rest had been absorbed into the room's atmosphere in the process of cooling the room down to working conditions.

The reconstruction work began in May 1950. Room 3 and the adjoining

air-lock were completed in September 1951 and brought into operation. Unfortunately, shortage of reinforcement for the precast flooring sections had already held up their completion in July, and, from the completion of Room 3 until the end of the year, progress became slower and slower until the work was very nearly at a standstill. Work was resumed in earnest in March 1952. Room 2 and the second air-lock were completed in July 1952 and the whole of the work in September 1952. *Fig. 12* shows a completed chamber.

The work forming the subject of this Paper cost approximately £58,000. Unit costs are not available, since although certain items, such as the steelwork and flooring, were carried out for lump sums or on measurement, a great part of the work, including the whole of the excavation and the various works of demolition, was done on a time-and-materials basis.

While the reconstruction of the ground floor was proceeding, opportunity was taken to modernize the cold-store lifts. In addition, the insulation on the main ammonia pipe-lines was renewed; the doors to all the chambers were overhauled, improvements were carried out on the insulation of the air-locks, and baffle screens were constructed in the second-floor air-locks to prevent circulation of warm air brought down from the sorting floor by the lifts. In addition, minor alterations were made to the refrigerating machinery to improve its efficiency.

## Acknowledgements

The Paper is presented by kind permission of Mr N. A. Matheson, M.I.C.E., A.M.I.Mech.E., Engineer-in-Chief, Port of Bristol Authority. The Authors are indebted to Mr F. P. Phillips, B.Sc. (Eng.), A.M.I.Mech.E., A.M.I.E.E., Chief Assistant Engineer (Electrical) Port of Bristol Authority, for the subject matter of the Appendix.

The main Contractors were W. A. Taylor, Ltd., of Mitcham, Surrey, whose previous experience in similar remedial work proved of value in the successful carrying out of the reconstruction.

The Paper is accompanied by four photographs and seven sheets of drawings, from which the half-tone page plates, the Figures in the text, and folding Plates 1 and 2 have been prepared, and by the following Appendix.

## APPENDIX

### A Consideration of the Effect of Reinforced-Concrete Piles on Soil Temperatures beneath a Cold Store having an Air Space below the Cold Rooms

The theoretical and qualitative reasoning which follows resulted from a suggestion that the reinforced columns would be a cause of local soil freezing because their conductivity—and especially that of the reinforcing steel—would allow heat to be extracted from their immediate surroundings at an excessive rate. It was further suggested

that electric or other heaters would be necessary, either embedded in the pile caps or placed near them. Such heaters would have been undesirable because :

(*a*) They would consume energy.
(*b*) They would heat the cold rooms and result in greater consumption of energy in the refrigerating machinery.

After due consideration it was decided that the fear of lower temperatures near the piles was unjustified ; the temperatures near the piles might well be higher than in the surrounding soil. Heaters were omitted and records of 28 buried remote-indicating electric thermometers have already been kept long enough to show that the soil temperature is reasonably uniform over a horizontal plane (see *Figs 13*).

In what follows, purely hypothetical values have been selected to demonstrate extreme theoretical conditions. They could not, in the design stage, be based on tests.

*Figs 14* (*a*) and (*b*) indicate two sets of conditions, the building column, pile cap, and group of piles being shown, for simplicity, as a uniform column. *Fig. 14* (*a*) illustrates the case of heat flowing out of the surface soil to the column, that is, where the latter tends to promote freezing. *Fig. 14* (*b*) shows that of the heat flowing into the surface soil from the column, that is, where the latter tends to prevent freezing.

Considering again *Fig. 14* (*a*), the arrows indicate direction of heat flow. The broken-line curve shows temperatures along a vertical line so far from a column that the effect of the latter can be ignored. The rise in temperature from the cold room to the body of earth is indicated, together with the irregular temperature gradient brought about by the insulation and the ventilated air space. The full-line curve indicates the temperatures along a vertical line coincident with the core of the column. The minimum temperature of the column core, at room level, must be higher than room temperature and the difference is shown as 5° F. Except near the level of the room floor, the full line is to the left of the broken line. Now the soil temperature immediately adjacent to the column must lie between the dotted line and the full, and the soil will, therefore, *lose heat to the column* as shown by the thick arrows.

Now considering *Fig. 14* (*b*), the column in the room is shown insulated, resulting in an assumed rise in temperature from the room to the core of the column of 15° F. The broken and full lines have the same meanings as in *Fig. 14* (*a*). The full line is to the right of the broken soil-line. The soil near the column will, therefore, *gain heat from the column*, as shown by the thick arrows.

Clearly the matter of heat gain or heat loss can be influenced in a great variety of ways, each of which could be illustrated by adjusting the shapes and/or slopes of the full- and broken-line curves. Insufficient data existed for the exact determination of what would happen after the cold-store reconstruction, but it was apparent that the assumption that the danger spot would of necessity be adjacent to a column was not justified. The reverse might well be the case. The effect of the reinforcing steel is to increase the mean conductivity of the columns, but has no other special significance.

The effect of additional floor insulation is to move the point X on the dotted curves to the right, and tends to increase the soil temperature, thus making the column appear *relatively* colder. The column surface would, in fact, be warmer. The effect of moving X to the left, that is, towards the condition of no floor insulation, makes the column appear *relatively* warmer. This was the condition before reconstruction at the Avonmouth Royal Edward Cold Store, where freezing, as indicated by the extent of frost heave, appeared less at the columns than in the soil between them.

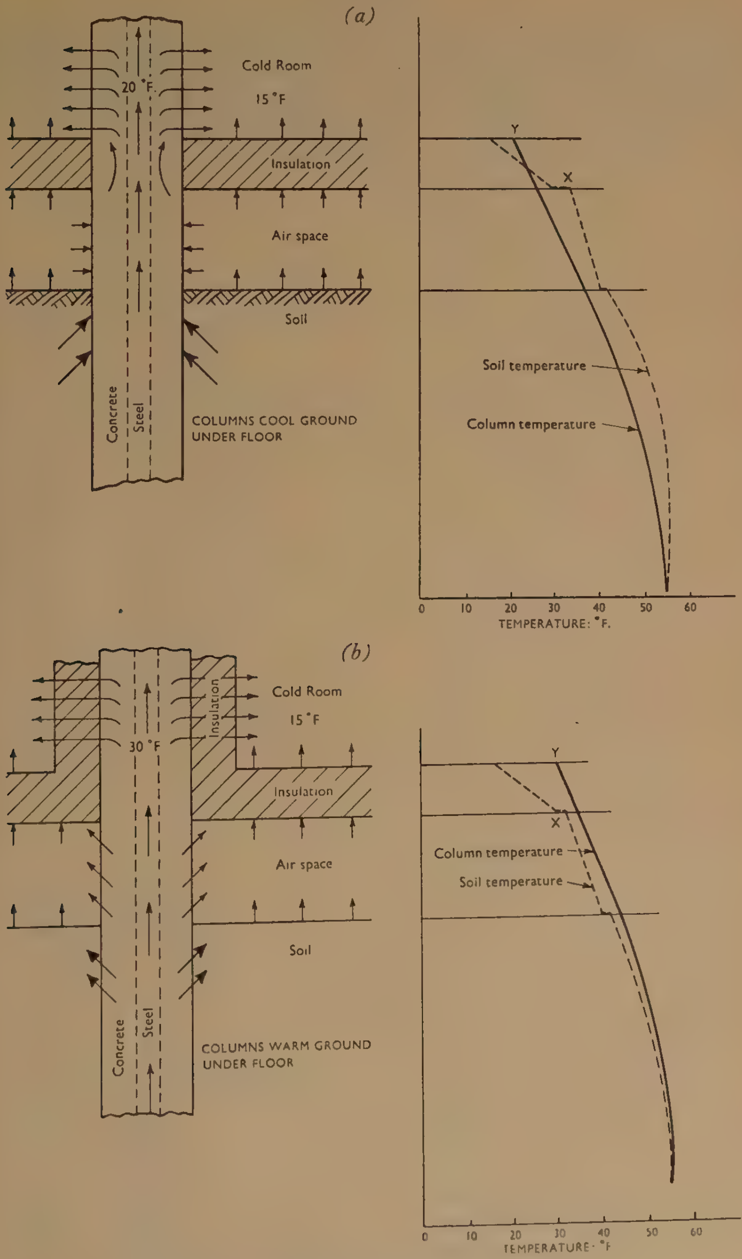
The effect of additional insulation on the columns where they pass through the rooms is to move the point Y on the full-line curves to the right and to make the columns warmer relative to the adjacent soil.

The soil temperature near the columns will be represented by some point between the full and broken lines at ground level. Measures directed to moving either or both of the lines to the right are beneficial.

Possible precautions against freezing of the soil are :

(1) More floor insulation.
(2) More insulation on columns in rooms.
(3) More or warmer ventilation of the air space. (Plans were made for forced ventilation and/or air heating but it was thought that such measures would be unnecessary. Temperatures recorded so far confirm this (see *Fig. 13*), though of course it is likely to be several years before stable conditions are established.)

*Figs 14*

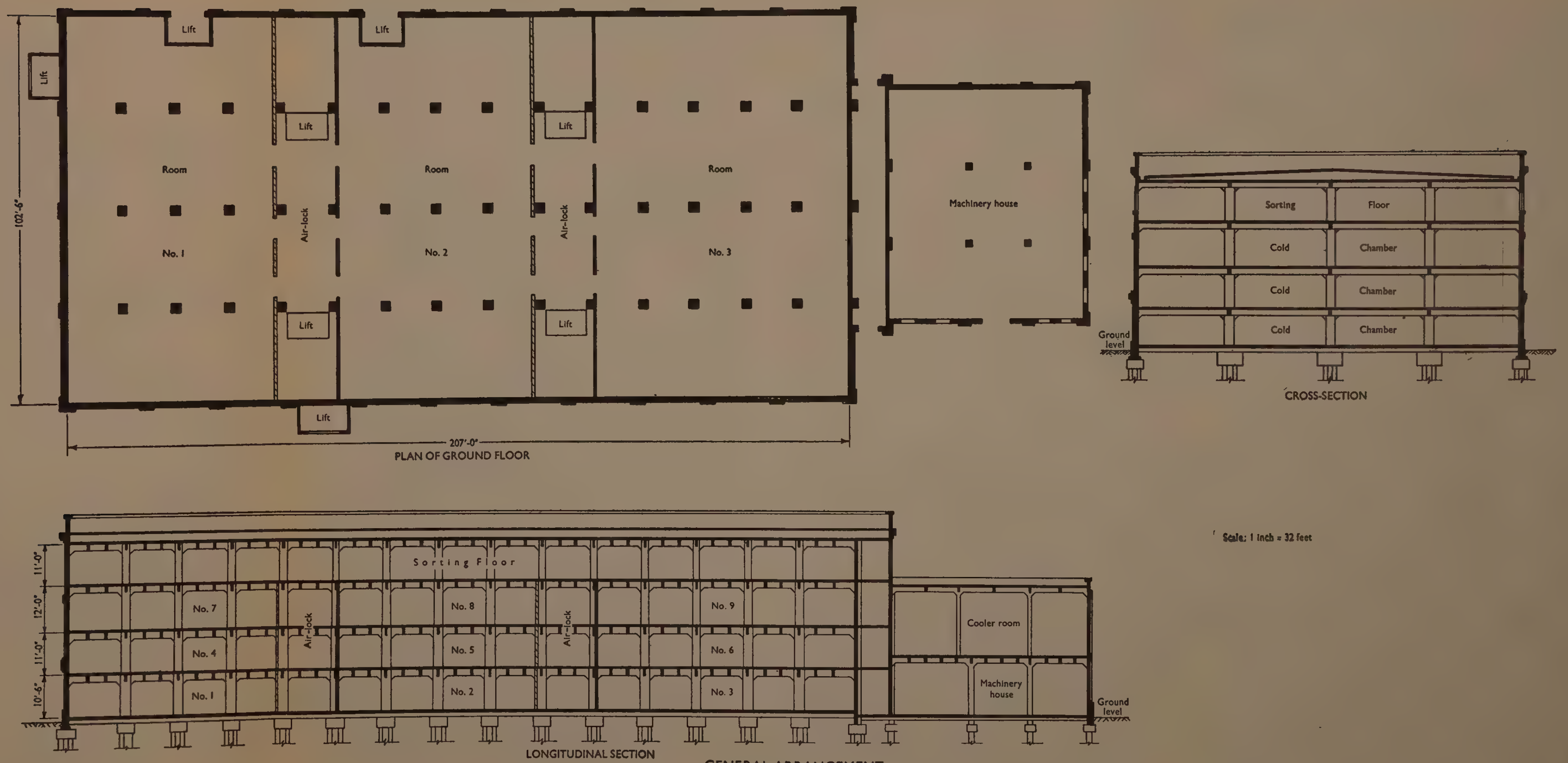



Typical Heat Flow in and out of Columns

The location of the coldest spot in the soil is influenced by the relationship between the above three factors. They are all beneficial. As demonstrated the columns could lower the surface soil temperature if conditions allowed them to conduct more heat to the cold rooms than they supplied from the reservoir of heat formed by the deep soil into which they were driven. Alternatively, they could increase the surface soil temperature if they brought up more heat than they lost to the cold rooms. At the reconstructed cold store their influence appears to be negligible.

# RECONSTRUCTION OF THE GROUND FLOOR OF THE ROYAL EDWARD COLD STORE, AVONMOUTH DOCKS

Figs 1

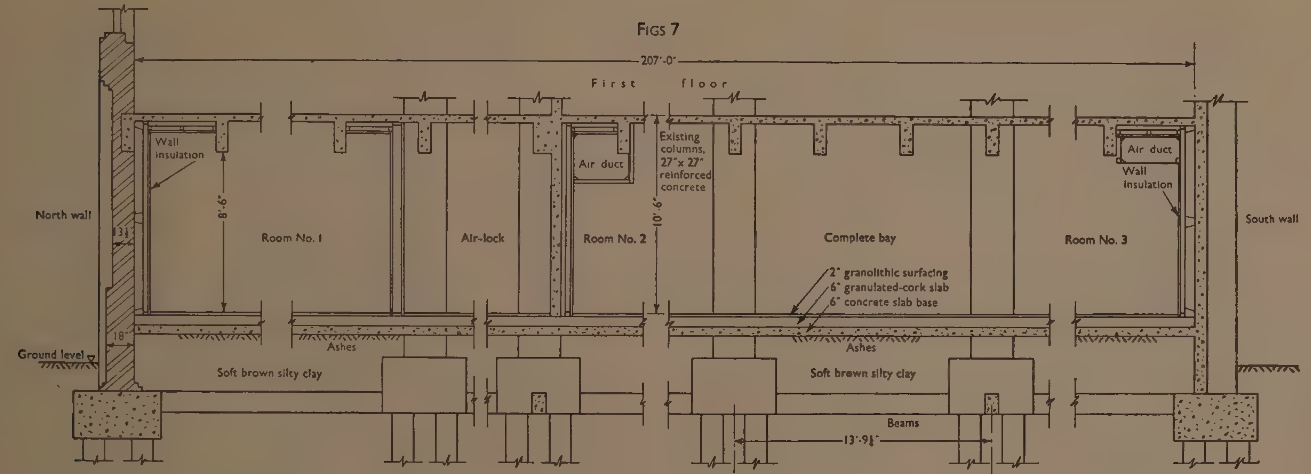



GENERAL ARRANGEMENT

PLATE 2
COLD STORE RECONSTRUCTION,
AVONMOUTH DOCKS

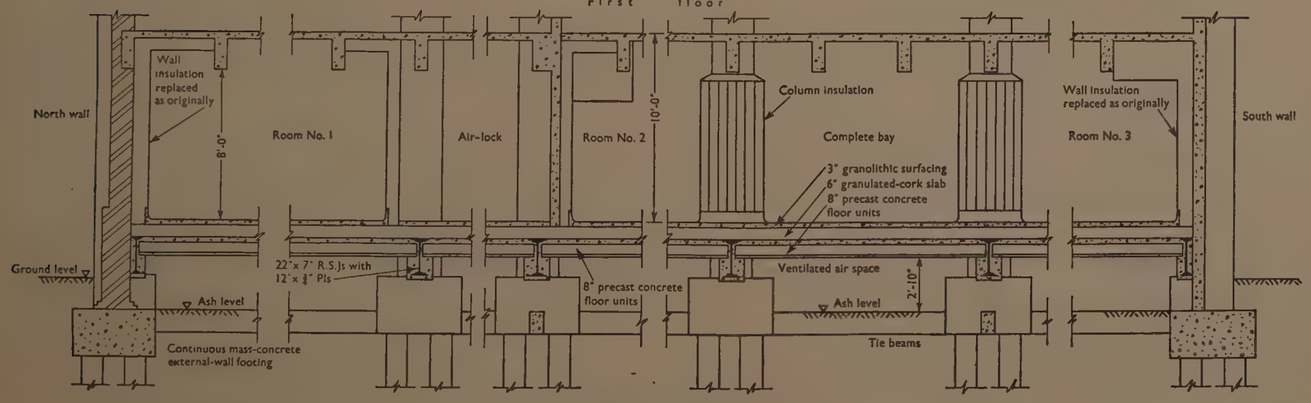
# RECONSTRUCTION OF THE GROUND FLOOR OF THE ROYAL EDWARD COLD STORE, AVONMOUTH DOCKS

FIGS 7




ORIGINAL CONSTRUCTION - 1922

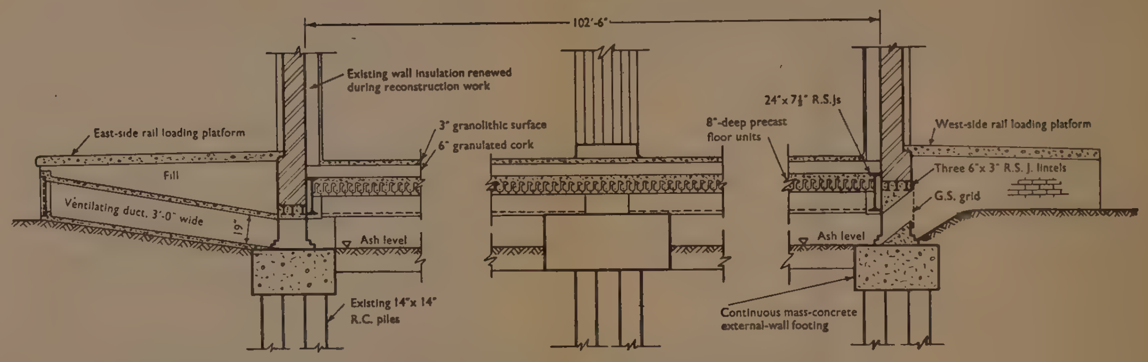
Scale: ⅛ inch = 1 foot




RECONSTRUCTION - 1952

LONGITUDINAL SECTIONS OF GROUND FLOOR

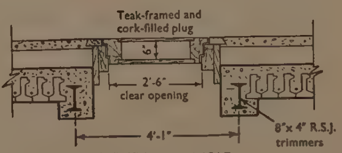
FIG. 8




CROSS-SECTION OF RECONSTRUCTED FLOOR

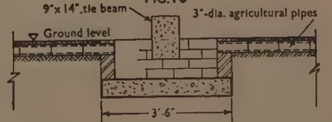
Scale: ¼ inch = 1 foot

FIG. 9



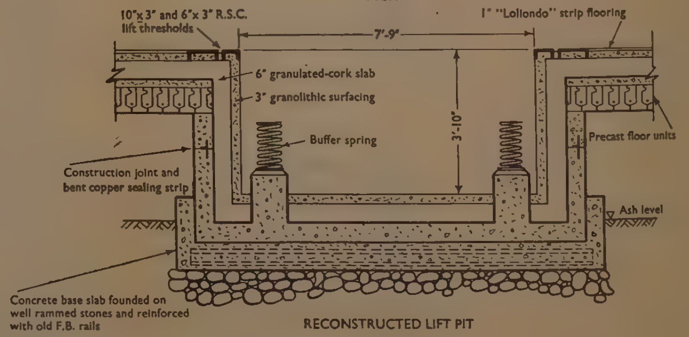

INSPECTION MANHOLE

FIG. 10




CATCH PIT

FIG. 11




RECONSTRUCTED LIFT PIT

DETAILS OF RECONSTRUCTION, GROUND FLOOR

Scale: ½ inch = 1 foot

Paper No. 5930

# "Model-Scale Relations for Open Channels with Non-uniform Flow"

by

**Richard Dawson Watkins, B.E., and Arthur Brebner, B.Sc., Ph.D., A.M.I.C.E.**

(*Ordered by the Council to be published with written discussion*) †

## SYNOPSIS

The Paper deals with some considerations concerning the inter-relation between scales or reproduction in hydraulic models featuring both uniform and non-uniform channel-flow, and in particular the necessary difference in gradient between model and prototype channels, necessitated by taking into account the relative effect of momentum changes and boundary shear or friction losses.

The theory is also developed to enable the model-measurements to be used to determine the degree of accuracy of reproduction, especially of depths of flow.

Finally, the Paper deals with the considerations involved in the choice of scales and gradients for a model of an overflow weir and ski-jump spillway channel. The results obtained are compared with flow profiles calculated by analytical means from a theory developed in the Paper.

## INTRODUCTION

CONSIDERABLE attention has been given to the problem of surface roughness in hydraulic scale models. This is of great importance where the roughness is responsible for major energy changes; the extreme case is that of a straight open channel of constant bed-slope, cross-sectional shape, and boundary roughness, along which flow has uniform velocity and depth. Consequently, the only energy changes are losses due to boundary shear effects.

However, in many cases other causes are dominant; for example, in open channels where there are abrupt changes in direction and shape of cross-section, and where major energy transformations are directly related to inertia effects, which can be expressed in terms of the Froude parameters $\left(\frac{V^2}{Lg}\right)$.

† Correspondence on this Paper should be recieved at the Institution by the 15th August, 1954, and will be published in Part III of the Proceedings. Contributions should be limited to about 1,200 words.—SEC. I.C.E.

In this Paper, a general theory has been derived which may be applied to any particular problem of open-channel model-analysis, to provide necessary adjustments to obtain desired similitude where boundary shear and inertia effects occur in differing degrees. The theory is also developed to enable the actual similarity of reproduction to be estimated when model-adjustment has not been possible.

For instance, it may be desirable to reproduce, to certain scales, conditions at a particular section of a channel for a specific flow, where both surface roughness and inertia effects influence flow conditions: for such a case, it will be shown that, for most practicable models, this may be achieved by suitable adjustment; however, at any other section or for any other flow, reproduction will not necessarily be to the same scales. Nevertheless, the actual scales may readily be calculated for any section under any flow conditions, and the value of the model thereby enhanced.

As a particular example, for the purpose of illustration, the method has been applied to the case of a model-spillway-channel in which it was desired to reproduce, to certain scales, conditions at the entry to a ski-jump situated at the end of that channel.

## Notation

$l$ denotes length along the channel from a given datum section (feet).
$z$ ,, height above a given datum of channel bed at a given section (feet).
$y$ ,, depth of flow at a given section (feet).
$b_o$ ,, a fixed width-dimension at a given section (feet).
$b$ ,, mean width of flow at a given section (feet) = function $(y, b_o)$.
$A$ ,, area of flow at a given section (square feet) = function $(y, b_o)$.
$P$ ,, wetted perimeter at a given section (feet) = function $(y, b_o)$.
$R$ ,, mean hydraulic depth at a given section (feet) $= \frac{A}{P}$ = function $(y, b_o)$.
$Q$ ,, flow along the channel (cusecs).
$v$ ,, mean velocity of flow at a given section (feet per second) $= \frac{Q}{A}$
$i$ $= -\frac{dz}{dl}$ = sine (angle of slope of channel bed).
$g$ denotes acceleration due to gravity (ft/sec.$^2$).
$C$ ,, Chezy coefficient for boundary shear (ft$^{\frac{1}{2}}$/sec.)

$$= \frac{1\cdot486}{n} \cdot R^{\frac{1}{6}}, \text{ in the Manning formula;}$$

$$= \left(\frac{2g}{f}\right)^{\frac{1}{2}}, \text{ in the Darcy formula.}$$

$n$ ,, roughness coefficient in the Manning formula.

$f$ denotes Darcy friction coefficient.

$\rho$ ,, density of fluid $\left(\frac{\text{lb.}}{g}/\text{ft}^3\right)$.

$\mu$ ,, dynamic viscosity (lb.-sec./ft $^2$).

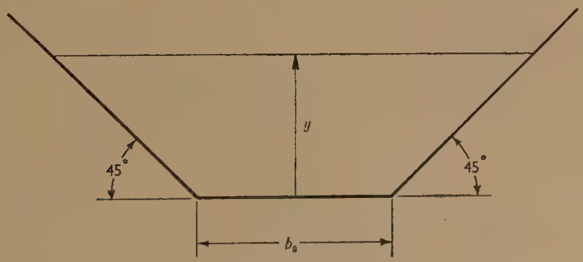
$\nu = \frac{\mu}{\rho}$ = kinematic viscosity (ft$^2$/sec.).

$k$ denotes magnitude of roughness projection on channel surface (feet).

It is assumed that channel cross-sections are in planes normal to the channel bed (and therefore normal to the direction of mean velocity), and depths of flow are measured in this plane. However, for convenience of graphical representation, the Figures in the text show depths measured vertically.

In the following theory, it is accepted that for most shapes of channel cross-sections the areas ($A$) and hydraulic mean depths (R), corresponding

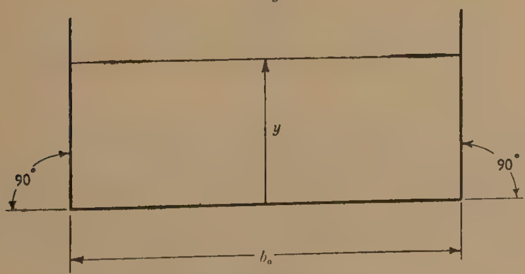
*Fig. 1*




TRAPEZOIDAL CHANNEL

For range of depths $\frac{y}{b_o} = 0{\cdot}2$ to $0{\cdot}4$, $a = 1{\cdot}22$, and $c = 0{\cdot}78$.

*Fig. 2*




RECTANGULAR CHANNEL

For range of depths $\frac{y}{b_o} = 0{\cdot}2$ to $0{\cdot}4$, $a = 1{\cdot}00$, and $c = 0{\cdot}64$.

to a particular range of depths ($y$), may be expressed as approximately monomial functions of $y$ and $b_o$:

$$A = \text{function } (y_1 b_o) \doteqdot \text{constant} \times y^a b_o^{2-a}$$
$$R = \text{function } (y_1 b_o) \doteqdot \text{constant} \times y^c b_o^{1-c}$$

where $a$ and $c$ denote the average values of the indices obtained from plotting log $A$ versus log $y$ and log $R$ versus log $y$ for the particular range of $y$ under consideration. For examples, see *Figs 1* and *2*.

## Scale Relationships

The following scales are assumed to hold between model and prototype, where the subscripts $m$ and $p$ apply respectively :—

*Basic Scales*

| | |
|---|---|
| Bed-level scale : | $z_p/z_m = \alpha$ |
| Longitudinal and width scales : | $l_p/l_m = b_{op}/b_{om} = \beta$ |
| Flow scale : | $Q_p/Q_m = \gamma$ |
| Depth scale : | $y_p/y_m = \delta$ |

*Approximate Derived Scales*

| | |
|---|---|
| Cross-sectional-area scale : | $A_p/A_m \doteqdot \delta^a \cdot \beta^{2-a}$ |
| Hydraulic-mean-depth scale : | $R_p/R_m \doteqdot \delta^c \cdot \beta^{1-c}$ |
| Velocity scale : | $v_p/v_m \doteqdot (\gamma/\beta^2) \cdot (\beta/\delta)^a$ |

## Theory

The equation of flow for the element of length $\Delta l$ of the channel (see *Fig. 3*), may be written approximately :

$$z_2 + y_2 + \frac{v_2^2}{2g} = z_1 + y_1 + \frac{v_1^2}{2g} - \sum_1^2 \frac{K}{g} \cdot v_s \cdot \Delta v_s - \left(\frac{f \cdot v^2}{2g \cdot R}\right)_{Av} \cdot \Delta l$$

or

$$\Delta E_z + \Delta E_y + \Delta E_v + \Delta E_s + \Delta E_f = 0 \quad . \quad . \quad . \quad (1)$$

where,

$$\Delta E_z = z_2 - z_1$$
$$= \text{change in bed level over length } \Delta l.$$

$$\Delta E_y = y_2 - y_1$$
$$= \text{change in depth over length } \Delta l.$$

$$\Delta E_v = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} = \frac{Q^2}{2g} \cdot \left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)$$
$$= \text{change in kinetic head over length } \Delta l.$$

$$\Delta E_s = \sum_1^2 \frac{K}{g} \cdot v_s \cdot \Delta v_s = \frac{Q^2}{g} \cdot \sum_1^2 K \cdot \frac{(-\Delta A_s)}{A_s^3}$$
$$= \text{surge head over length } \Delta l.$$

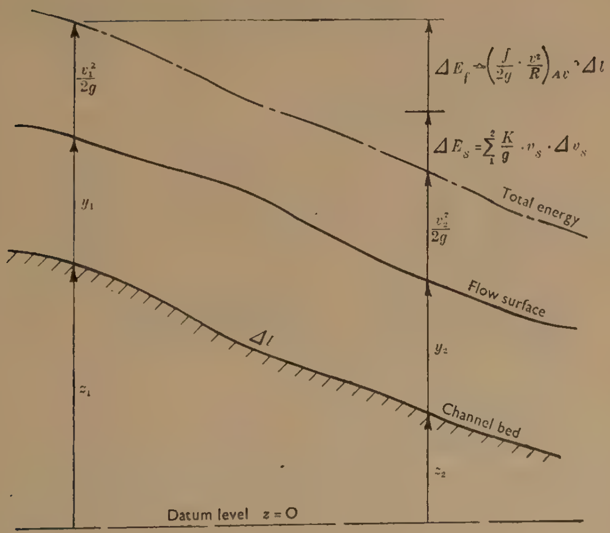
This quantity includes kinetic head changes not covered by the value calculated on the basis of mean velocities of flow. These are due to variations of flow momentum influenced by the geometry of the channel and the velocity distributions across channel sections.

$$\Delta E_f \simeq \left(\frac{f \cdot v^2}{2g \cdot R}\right)_{Av} \cdot \Delta l = Q^2\left(\frac{f \cdot 1}{2g \cdot A^2 \cdot R}\right)_{Av} \cdot \Delta l$$

$$\doteq \frac{Q^2 n^2}{1{\cdot}486^2} \cdot \left(\frac{1}{A^2 \cdot R^{\frac{4}{3}}}\right)_{Av} \cdot \Delta l$$

= loss of head calculated from average boundary shear over length $\Delta l$.

*Fig. 3*




In the general case of model-reproduction of equivalent prototype effects, scale values for the geometry of the model, $\alpha$ and $\beta$, and the flow $\gamma$ have constant selected values. Scale values for depths $\delta$ may have varying values, depending on the hydraulic characteristics of model and prototype. Thus the scale relations of depths at adjacent sections of flow in model and prototype respectively, may be expressed as :—

$$\frac{y_{1p}}{y_{1m}} = \delta_1 \, ; \; \frac{y_{2p}}{y_{2m}} = \delta_2 = \delta_1 + \Delta\delta$$

Terms of equation (1) for the prototype may be written as functions of corresponding model values and scales :—

$$(\Delta E_z)_p = (z_2 - z_1)_p = \alpha \,.\, (z_2 - z_1)_m$$
$$= \alpha \,.\, (\Delta E_z)_m$$

$$(\Delta E_y)_p = (y_2 - y_1)_p = \delta_2 \,.\, y_{2m} - \delta_1 \,.\, y_{1m} = (\delta_1 + \Delta\delta) \,.\, y_{2m} - \delta_1 y_{1m}$$
$$= \delta_1 \,.\, (\Delta E_y)_m + \delta_1 \,.\, (E_{y2})_m \cdot \left\{\frac{\Delta\delta}{\delta_1}\right\}$$

$$(\Delta E_v)_p = \left(\frac{Q^2}{2g} \cdot \left\{\frac{1}{A_2^2} - \frac{1}{A_1^2}\right\}\right)_p$$
$$= \frac{\gamma^2}{\beta^4} \cdot \left(\frac{\beta}{\delta_1}\right)^{2a} \cdot \left(\frac{Q^2}{2g}\right)_m \cdot \left\{\frac{1}{\left(1 + \frac{\Delta\delta}{\delta_1}\right)^{2a} \,.\, A_{2m}^2} - \frac{1}{A_1{}^2{}_m}\right\}$$
$$\bumpeq \frac{\gamma^2}{\beta^4} \cdot \left(\frac{\beta}{\delta_1}\right)^{2a} \cdot \left(\Delta E_v\right)_m - 2a \cdot \frac{\gamma^2}{\beta^4} \cdot \left(\frac{\beta}{\delta_1}\right)^{2a} \cdot \left(E_{v2}\right)_m \cdot \left\{\frac{\Delta\delta}{\delta_1}\right\}$$

$$(\Delta E_s)_p = \left(Q^2 \cdot \Sigma\frac{K}{g} \cdot \frac{(-\Delta A_s)}{A_s^3}\right)_p \bumpeq$$
$$\bumpeq \frac{\gamma^2}{\beta^4} \cdot \left(\frac{\beta}{\delta_1}\right)^{2a} \cdot \left(Q^2 \cdot \Sigma\frac{K}{g} \cdot \frac{(-\Delta A_s)}{A_s^3}\right)_m \cdot \left\{\frac{1}{\left(1 + \frac{\Delta\delta}{2\delta_1}\right)^{2a}}\right\}$$
$$\bumpeq \frac{\gamma^2}{\beta^4} \cdot \left(\frac{\beta}{\delta_1}\right)^{2a} \left(\Delta E_s\right)_m - a\frac{\gamma^2}{\beta^4} \cdot \left(\frac{\beta}{\delta_1}\right)^{2a} \cdot \left(\Delta E_s\right)_m \cdot \left\{\frac{\Delta\delta}{\delta_1}\right\}$$

$$(\Delta E_f)_p \bumpeq \left(\frac{Q^2 n_p{}^2}{1{\cdot}486^2} \cdot \frac{\Delta l}{(A^2 \,.\, R^{\frac{4}{3}})_{Av}}\right)_p$$
$$\bumpeq \frac{\gamma^2}{\beta^4} \cdot \left(\frac{\beta}{\delta_1}\right)^{2a\frac{4}{3}+c} \left(\frac{n_p}{\beta^{\frac{1}{6}} \,.\, n_m}\right)^2 \cdot \left(\frac{Q^2 n_m{}^2}{1{\cdot}486^2} \cdot \frac{\Delta l}{(A^2 \,.\, R^{\frac{4}{3}})_{Av}}\right)_m$$
$$\times \left\{\frac{1}{\left(1 + \frac{\Delta\delta}{2\delta_1}\right)^{2a+\frac{4}{3}c}}\right\}$$
$$\bumpeq \frac{\gamma^2}{\beta^4} \cdot \left(\frac{\beta}{\delta_1}\right)^{2a+\frac{4}{3}c} \left(\frac{n_p}{\beta^{\frac{1}{6}} \,.\, n_m}\right)^2 \cdot (\Delta E_f)_m$$
$$- \frac{2a + \frac{4}{3}c}{2} \cdot \frac{\gamma^2}{\beta^4} \cdot \left(\frac{\beta}{\delta_1}\right)^{2a+\frac{4}{3}c} \cdot \left(\frac{n_p}{\beta^{\frac{1}{6}} \,.\, n_m}\right)^2 \cdot (\Delta E_f)_m \cdot \left\{\frac{\Delta\delta}{\delta_1}\right\}$$

By substituting into equation (1) for the prototype :

$$0 = (\Delta E_z)_p + (\Delta E_y)_p + (\Delta E_v)_p + (\Delta E_s)_p + (\Delta E_f)_p$$

it may be found that:

$$\left\{\frac{\Delta\delta}{\delta_1}\right\} \simeq \frac{L\cdot\left(\dfrac{\beta}{\delta_1}\right)\cdot(\Delta E_z)_m + (\Delta E_y)_m + M\cdot\left(\dfrac{\beta}{\delta_1}\right)^s\cdot(\Delta E_v+\Delta E_s)_m + N\cdot\left(\dfrac{\beta}{\delta_1}\right)^t\cdot(\Delta E_f)_m}{-(E_{y2})_m + (s-1)\cdot M\cdot\left(\dfrac{\beta}{\delta_1}\right)^s\cdot\left(E_{v2}+\dfrac{\Delta E_s}{2}\right)_m + \dfrac{(t-1)}{2}\cdot N\cdot\left(\dfrac{\beta}{\delta_1}\right)^t\cdot(\Delta E_f)_m} \quad (2)$$

Alternatively by expressing terms for the model in equation (1) as functions of corresponding prototype values and scales, it may be found that:

$$\left\{\frac{\Delta\delta}{\delta_1}\right\} \simeq \frac{\dfrac{1}{L}\cdot\left(\dfrac{\delta_1}{\beta}\right)\cdot(\Delta E_z)_p + (\Delta E_y)_p + \dfrac{1}{M}\cdot\left(\dfrac{\delta_1}{\beta}\right)^s\cdot(\Delta E_v+\Delta E_s)_p + \dfrac{1}{N}\cdot\left(\dfrac{\delta_1}{\beta}\right)^t\cdot(\Delta E_f)_p}{(E_{y2})_p - (s-1)\cdot\dfrac{1}{M}\cdot\left(\dfrac{\delta_1}{\beta}\right)^s\cdot\left(E_{v2}+\dfrac{\Delta E_s}{2}\right)_p - \dfrac{(t-1)}{2}\cdot\dfrac{1}{N}\cdot\left(\dfrac{\delta_1}{\beta}\right)^t\cdot(\Delta E_f)_p} \quad (3)$$

where, $$L = \frac{\alpha}{\beta}; \quad M = \frac{\gamma^2}{\beta^5}; \quad N = \left(\frac{n_p}{\beta^{\frac{1}{6}}\cdot n_m}\right)^2\cdot\left(\frac{\gamma^2}{\beta^5}\right);$$

$$s = 1 + 2a; \quad \text{and} \quad t = 1 + 2a + \tfrac{4}{3}c$$

From equation (2), for a particular model with selected scales, $\delta$ may be checked by evaluating $\Delta\delta$ in terms of values measured from the model; this may be done by calculating successive values of $\delta_2$ $(=\delta_1+\Delta\delta)$ commencing from a section where $\delta_1$ may be known with reasonable accuracy.

From equation (3), for a particular model with selected values, probable values of $\delta$ may be calculated in terms of prototype values estimated by analytical means.

In this theory, it is presumed that turbulent conditions will apply in both model and prototype, and that the head losses in model and prototype are related to constant values of the Manning roughness coefficients $n_m$ and $n_p$ respectively. It has been found for the cases of uniform flow in smooth straight open channels that the boundary shear coefficient $f$ may be derived from the equation[1]:

[1] The references are given on p. 215.

$$= 0{\cdot}0234\left(\frac{vR}{\nu}\right)^{-0\cdot15} = 0{\cdot}0234\left(\frac{Q \,.\, R}{A \,.\, \nu}\right)^{-0\cdot15}$$

$$= 0{\cdot}0234\,\nu^{0\cdot15}\left(\frac{P}{Q}\right)^{0\cdot15},$$

from which the equivalent Manning roughness coefficient,

$$n = 1{\cdot}486 \,.\, R^{\frac{1}{6}} \cdot \left(\frac{f}{2g}\right)^{\frac{1}{2}}$$

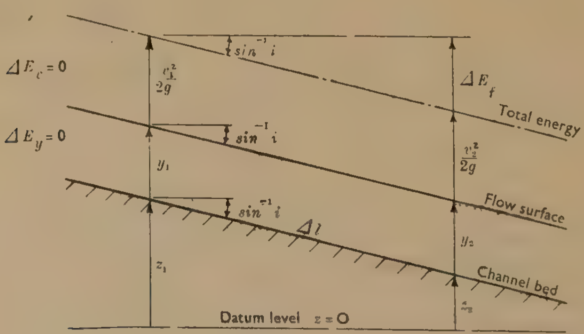
$$= 0{\cdot}0122 \,.\, R^{\frac{1}{6}} \cdot \left(\frac{P}{Q}\right)^{0\cdot075} \text{ approximately for water at } 15^{\circ}\text{C}$$

It will be seen that $n$ for a smooth channel is a function of flow conditions, and hence if the prototype channel is rough so that $n_p$ may be assumed to have a constant value, and the model channel is smooth so that $n_m$ will vary according to flow conditions, exact similarity is impossible. However, it is generally possible to assign an average value to $n_m$ for a particular value of flow and the degrees of similarity may be estimated according to the theory.

## Particular Applications of Theory

### *Uniform Flow*

*Fig. 4*




$$\Delta E_z + \Delta E_f = 0\,;\ \text{or, } (z_2 - z_1) + \frac{f}{2g} \cdot \frac{v^2}{R} \cdot \Delta l = 0$$

and
$$v^2 = \frac{2g}{f} \cdot R \cdot \frac{(-\Delta z)}{\Delta l} = \frac{2g}{f} \cdot R \,.\, i$$

$$v = \left(\frac{2g}{f}\right)^{\frac{1}{2}} \cdot (R \,.\, i)^{\frac{1}{2}} = C(R \,.\, i)^{\frac{1}{2}},$$

which may be recognized as the Chezy formula for uniform flow, where the

Chezy coefficient $C = \left(\frac{2g}{f}\right)^{\frac{1}{2}} = \frac{1{\cdot}486}{n} \cdot R^{\frac{1}{6}}$ in the Manning formula. From equation (3) to obtain a constant value of the depth scale : $\delta = \delta_1, \Delta\delta = 0$ and

$$(\Delta E_z)_p + \frac{L}{N} \cdot \left(\frac{\delta_1}{\beta}\right)^{t-1} (\Delta E_f)_p = 0 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Equation (1) for the prototype is :

$$(\Delta E_z)_p + (\Delta E_f)_p = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For exact similarity between model and prototype the coefficients in equation (4) of terms corresponding to those in equation (5), must be unity :

$$\frac{L}{N} = \left(\frac{\beta}{\delta_1}\right)^{t-1} = \left(\frac{\alpha}{\beta}\right) \cdot \left(\frac{\beta^5}{\gamma^2}\right) \cdot \left(\frac{\beta^{\frac{1}{6}} \cdot n_m}{n_p}\right)^2$$

$$\alpha = \beta \cdot \left(\frac{\gamma^2}{\beta^5}\right) \cdot \left(\frac{n_p}{\beta^{\frac{1}{6}} \cdot n_m}\right)^2 \cdot \left(\frac{\beta}{\delta_1}\right)^{t-1} \quad . \quad . \quad . \quad . \quad (6)$$

$$\gamma^2 = \beta^5 \cdot \left(\frac{\alpha}{\beta}\right) \cdot \left(\frac{\beta^{\frac{1}{6}} \cdot n_m}{n_p}\right)^2 \cdot \left(\frac{\delta_1}{\beta}\right)^{t-1} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$n_m{}^2 = \left(\frac{n_p}{\beta^{\frac{1}{6}}}\right)^2 \cdot \left(\frac{\beta}{\alpha}\right) \cdot \left(\frac{\gamma^2}{\beta^5}\right) \cdot \left(\frac{\beta}{\delta_1}\right)^{t-1} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

From the above equations it will be seen that, in order to obtain similarity, adjustment of model-scale values and channel roughness may be used. Normally, the linear-scale value $\beta$ is fixed by space considerations for the model, and the roughness coefficient $n_m$ is fixed by the limited range of model-material. Consequently, adjustment is usually limited to either the flow scale $\gamma$, which may also be limited in certain cases to the capacity of available water supply, or the bed-level scale $\alpha$, which generally can have a fairly wide range of values without inconveniencing model construction.

Example.

For a model to simulate uniform flow in an open channel, let the linear scale $\beta$ be fixed, and let the required depth scale $\delta = \beta$, that is, $\frac{\delta}{\beta} = 1$.

(1) Let the velocity scale $\frac{v_p}{v_m} = \delta^{\frac{1}{2}} = \beta^{\frac{1}{2}}$; then the flow scale ($\gamma = \beta^{\frac{5}{2}}$).

The required conditions of similarity are obtained by adjusting the bed-level scale $\alpha$ or the model roughness coefficient $n_m$ :

$$\alpha = \beta \cdot \left(\frac{\gamma^2}{\beta^5}\right) \cdot \left(\frac{n_p}{\beta^{\frac{1}{6}} \cdot n_m}\right)^2 \cdot \left(\frac{\beta}{\delta_1}\right)^{t-1}$$

$$\frac{\beta}{\alpha} = \left(\frac{\beta^{\frac{1}{6}} \cdot n_m}{n_p}\right)^2$$

or

$$n_m = \left(\frac{n_p}{\beta^{\frac{1}{6}}}\right) \cdot \left(\frac{\beta}{\alpha}\right)^{\frac{1}{2}}$$

Table 1 shows the necessary adjustment of model-bed slope for various linear scales $\beta$ and ratios of model and prototype roughness coefficients $\frac{n_m}{n_p}$.

TABLE 1

| | $\frac{n_m}{n_p}$ | 1·00 | 0·95 | 0·90 | 0·85 | 0·80 | 0·75 | 0·70 | 0·65 | 0·60 | 0·55 | 0·50 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{\beta}{\alpha}$ | $\beta = 24$ | 2·88 | 2·58 | 2·25 | 1·64 | 1·45 | 1·28 | 1·11 | 0·96 | 0·82 | 0·69 | 0·57 |
| | $\beta = 60$ | 3·92 | 3·53 | 3·17 | 2·83 | 2·51 | 2·20 | 1·90 | 1·65 | 1·41 | 1·18 | 0·98 |
| | $\beta = 100$ | 4·64 | 4·16 | 3·75 | 3·35 | 2·96 | 2·60 | 2·26 | 1·96 | 1·67 | 1·40 | 1·13 |

For illustration, when the linear scale $\beta = 60$ and prototype roughness coefficient $n_p = 0{\cdot}0135$, for a channel in smooth-finished concrete, and the smoothest available material for a model is Perspex or glass (for which $n_m = 0{\cdot}009$ approx.*),

then $\qquad \frac{\beta}{\alpha} = 1{\cdot}74$, for $n_m = 0{\cdot}009$ ;

that is, the model slope has to be 74 per cent greater than prototype,

or, $\qquad n_m = 0{\cdot}0068$, for $\alpha = \beta$,

which is impracticable for the particular case.

(2) Let the bed-level scale $\alpha = \beta$. The required conditions of similarity are obtained by adjusting either the flow scale $\gamma$ or the model roughness coefficient $n_m$ :—

$$\gamma = \beta^{\frac{5}{2}} \cdot \left\{\left(\frac{\alpha}{\beta}\right) \cdot \left(\frac{\beta^{\frac{1}{6}} \cdot n_m}{n_p}\right)^2 \cdot \left(\frac{\delta_1}{\beta}\right)^{t-1}\right\}^{\frac{1}{2}}$$

$$\frac{\beta^{\frac{5}{2}}}{\gamma} = \left(\frac{n_p}{\beta^{\frac{1}{6}} \cdot n_m}\right)$$

* As has been previously mentioned, for a hydraulically smooth material such as Perspex or glass, the Manning roughness coefficient $n$ is a function of Reynolds number. The approximate value $n_m = 0{\cdot}009$ represents an estimation for anticipated conditions in the model.

or,

$$n_m = \left(\frac{n_p}{\beta^{\frac{1}{6}}}\right) \cdot \left(\frac{\gamma}{\beta^{\frac{5}{2}}}\right)$$

Table 2 shows the necessary adjustment of model flow for various linear scales $\beta$ and ratios of model and prototype roughness coefficients $\dfrac{n_m}{n_p}$.

TABLE 2

| | $\dfrac{n_m}{n_p}$ | 1·00 | 0·95 | 0·90 | 0·85 | 0·80 | 0·75 | 0·70 | 0·65 | 0·60 | 0·55 | 0·50 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\dfrac{\beta^{\frac{5}{2}}}{\gamma}$ | $\beta = 24$ | 0·59 | 0·62 | 0·67 | 0·78 | 0·83 | 0·88 | 0·95 | 1·02 | 1·10 | 1·20 | 1·33 |
| | $\beta = 60$ | 0·51 | 0·53 | 0·56 | 0·60 | 0·63 | 0·68 | 0·72 | 0·78 | 0·84 | 0·92 | 1·01 |
| | $\beta = 100$ | 0·47 | 0·49 | 0·52 | 0·55 | 0·58 | 0·62 | 0·67 | 0·71 | 0·78 | 0·85 | 0·94 |

For illustration, when $\beta = 60$, $n_p = 0{\cdot}0135$, and $n_m = 0{\cdot}009$,

then $$\frac{\beta^{\frac{5}{2}}}{\gamma} = 0{\cdot}75 \text{ for } n_m = 0{\cdot}009,$$

that is, the required model flow has to be 25 per cent less than the value required if similarity is to be based on the Froude parameter, where $\gamma = \beta^{\frac{5}{2}}$.

Alternatively; $n_m = 0{\cdot}0068$, for $\gamma = \beta^{\frac{5}{2}}$, which is impracticable in this particular case.

*Non-Uniform Flow*

It is most important to remember that uniform flow, with the only loss being that due to boundary shear effects and without any acceleration of flow, is a limit which is approached only in long straight channels; generally the depth of flow will vary in the direction of flow, giving rise to varying velocities, and thus, as indicated in *Fig. 3*, the energy line, water surface and channel bed do not have identical slopes. It will be shown that adjustment of model-scale values for certain flow conditions may be less dependent on the boundary roughness coefficients $n_p$ and $n_m$ as the head loss due to boundary shear becomes less significant compared to kinetic head changes.

For similarity between model and prototype such that the depth scale has a constant value, $\delta = \delta_1$, $\Delta\delta = 0$, then in equation (3):

$$0 = \frac{1}{L} \cdot \left(\frac{\delta_1}{\beta}\right) \cdot (\Delta E_z)_p + (\Delta E_y)_p + \frac{1}{M}\left(\frac{\delta_1}{\beta}\right)^s \cdot (\Delta E_v + \Delta E_s)_p + \frac{1}{N} \cdot \left(\frac{\delta_1}{\beta}\right)^t \cdot (\Delta E_f)_p \quad \cdots \cdots \quad (9)$$

Equation (1) for the prototype is :—

$$0 = (\Delta E_z)_p + (\Delta E_y)_p + (\Delta E_v + \Delta E_s)_p + (\Delta E_f)_p \quad . \quad . \quad (10)$$

For exact similarity between model and prototype the coefficients in equation (9) of corresponding terms to those in equation (10) must be unity :

$$L = \frac{\alpha}{\beta} = \frac{\delta_1}{\beta}$$

$$M = \frac{\gamma^2}{\beta^5} = \left(\frac{\delta_1}{\beta}\right)^s$$

$$N = \left(\frac{n_p}{\beta^{\frac{1}{6}} \, . \, n_m}\right)^2 \cdot \frac{\gamma^2}{\beta^5} = \left(\frac{\delta_1}{\beta}\right)^t$$

from which :

$$\delta = \alpha = \beta \, . \left(\frac{n_p}{\beta^{\frac{1}{6}} \, . \, n_m}\right)^{\frac{2}{t-s}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

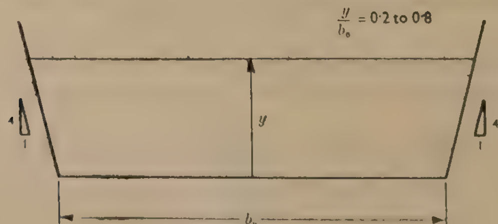
$$\gamma^2 = \beta^5 \, . \left(\frac{n_p}{\beta^{\frac{1}{6}} \, . \, n_m}\right)^{\frac{s}{t-s}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$n_m^2 = \left(\frac{n_p}{\beta^{\frac{1}{6}}}\right)^2 \cdot \left(\frac{\beta}{\alpha}\right)^{t-s} = \left(\frac{n_p}{\beta^{\frac{1}{6}}}\right)^2 \cdot \left(\frac{\beta^5}{\gamma^2}\right)^{2 \cdot \frac{(t-s)}{s}} \quad . \quad . \quad (13)$$

It will be noted that in comparison with the scale selections possible in the case of uniform flow to obtain exact similarity, the scale selections for similarity in non-uniform flow are fixed after selecting only two of the scale values. Usually $\beta$ and $n_m$ are limited by practical considerations ; therefore the other scale values for exact similarity will be directly dependent upon these two values. It will also be seen that the scale relationships are dependent on the shape of the channel cross-sections, which is not necessary in the case of uniform flow, except when the depth scale $\delta_1$ may have a value other than $\beta$.

For illustration, consider the case of a channel with trapezoidal section, as shown in *Fig. 5*.

*Fig. 5*



For such a channel it can be shown that:

$$A = b_0^2 \cdot \left(\frac{y}{b_0} + \frac{y^2}{4b_0^2}\right) \simeq \text{constant} \times y^a \,.\, b_0^{2-a}\,; \quad a = 1{\cdot}09$$

$$R = b_0 \cdot \frac{\left(\dfrac{y}{b_0} + \dfrac{y^2}{4b_0^2}\right)}{\left(1 + \dfrac{\sqrt{17}\,.\,y}{2\,b_0}\right)} \simeq \text{constant} \times y^c \,.\, b_0^{1-c}\,; \quad c = 0{\cdot}66$$

The selected model had a linear scale $\beta = 60$.

The roughness coefficients for models and prototype were respectively $n_m = 0{\cdot}009$ approximately and $n_p = 0{\cdot}0135$. For exact similarity, the required slope and flow scales are:

$$\frac{\beta}{\alpha} = \left(\frac{\beta^{\frac{1}{6}} \,.\, n_m}{n_p}\right)^{2/(t-s)}\,; \quad \frac{\beta^{\frac{5}{2}}}{\gamma} = \left(\frac{\beta^{\frac{1}{6}} \,.\, n_m}{n_p}\right)^{s/2(t-s)}$$

where $s \quad = 1 + 2a \qquad\qquad = 3{\cdot}18$

$t \quad = 1 + 2a + \frac{4}{3} \cdot c = 4{\cdot}06$

$t - s = 0{\cdot}88$:

thus, $$\frac{\beta}{\alpha} = 1{\cdot}88 \text{ and } \frac{\beta^{\frac{5}{2}}}{\gamma} = 2{\cdot}72\,;$$

that is, to obtain complete similarity, model slopes have to be 88 per cent greater than the prototype, and the model flow 172 per cent greater than the flow required by Froude-parameter consideration based solely on the linear-scale value $\beta$, namely $\gamma = \beta^{\frac{5}{2}}$.

## Selection of Scales when Models have Exaggerated Vertical to Horizontal Dimensions Relative to the Prototype

In this case it may be shown for many shapes of channel cross-sections that:

$$\frac{A_p}{A_m} \simeq \frac{\text{constant }(p) \times (y_p)^{ap} \,.\, (b_{op})^{2-ap}}{\text{constant }(m) \times (y_p/\delta)^{am}(b_{op}/\beta)^{2-am}}$$

$$= \delta^a \,.\, \beta^{2-a}\,\{B \,.\, (y_p/b_{op})^b\}$$

where $a = a_m$

$b = a_p - a_m$

$$B = \frac{\text{constant }(p)}{\text{constant }(m)}$$

$$\frac{R_p}{R_m} \simeq \frac{\text{constant }(p) \times (y_p)^{cp} \,.\, (b_{op})^{1-cp}}{\text{constant }(m) \times (y_p/\delta)^{cm} \,.\, (b_{op}/\beta)^{1-cm}}$$

$$= \delta^c \,.\, \beta^{1-c} \,.\, \{D \,.\, (y_p/b_{op})^d\}$$

where $c = c_m$

$$d = c_p - c_m$$

$$D = \frac{\text{constant } (p)}{\text{constant } (m)}$$

By modifying equation (9) correspondingly it may be deduced that for exact similarity between model and prototype :

$$\delta = \alpha = \beta \,.\, \left[ \left( \frac{n_p}{\beta^{\frac{1}{6}} \,.\, n_m} \right) \Big/ \left\{ D \,.\, \left( \frac{y_p}{b_{op}} \right)^d \right\}^{\frac{2}{3}} \right]^{2/(t-s)}$$

$$\gamma^2 = \beta^5 \,.\, \left[ \left( \frac{\delta}{\beta} \right)^s \cdot \left\{ B \,.\, \left( \frac{y_p}{b_{op}} \right)^b \right\}^2 \right]$$

$$n_m = n_p \,.\, \left[ \frac{1}{\beta^{\frac{1}{6}}} \cdot \left( \frac{\beta}{\delta} \right)^{(t-s)/2} \Big/ \left\{ D \cdot \left( \frac{y_p}{b_{op}} \right)^d \right\}^{\frac{2}{3}} \right]$$

Thus unless $(y_p/b_{op})$ has a constant value, exact similarity is impossible. However, when $(y_p/b_{op})$ varies between very close limits, as in the case of broad rectangular-shaped channels, constant similarity of reproduction may be assumed throughout the model.

*Example**

For an open channel of rectangular section ($y_p/b_{op} \simeq 1/160$) it is required that a model with selected scales $\beta = 720$ and $\alpha = 72$ should have depth scales $\delta = 72$. What roughness is required in the model ?

For a rectangular channel with $y_p/b_{op} = 1/160$ and $\beta/\delta \simeq 10$ :

$$\begin{aligned} a = a_m &= 1{\cdot}0\,; & c = c_m &= 0{\cdot}89\,; \\ a_p &= 1{\cdot}0\,; & c_p &= 0{\cdot}99\,; \\ b &= 0\,; & d &= 0{\cdot}10\,; \\ & & D &= 1{\cdot}42\,; \\ B &= 1{\cdot}0\,; & t - s &= \tfrac{4}{3}c = 1{\cdot}19. \end{aligned}$$

$$\begin{aligned} \frac{n_m}{n_p} &= \left[ \frac{1}{\beta^{\frac{1}{6}}} \cdot \left( \frac{\beta}{\delta} \right)^{(t-s)/2} \Big/ \left\{ D \,.\, \left( \frac{y_p}{b_{op}} \right)^d \right\}^{\frac{2}{3}} \right] \\ &= \left[ \frac{1}{\beta^{\frac{1}{6}}} \cdot \left( \frac{\beta}{\delta} \right)^{0.594} \Big/ 0{\cdot}903 \right] \\ &= 1{\cdot}45. \end{aligned}$$

That is to say, the model roughness coefficient must be 45 per cent greater than the prototype value.

* Comparison should be made with the channels described in reference 2 and the exaggerated roughness required in the hydraulic model described in reference 3.

### Method of adjusting Scales to produce Approximate Similarity at Particular Sections of Model and Prototype for Particular Flow Conditions, based on the Magnitude of Boundary Shear Head Loss

Let the required similarity be such that at a control section, that is, one at which the scales of reproduction are known within close limits, the depth scale $\delta_1 = \beta$, and let the flow scale $\gamma = \beta^{\frac{5}{2}}$ and the model and prototype coefficients $n_m$ and $n_p$ respectively be known.

It is required that at another particular section the depth scale should be the same as at the control section: $\delta_2 = \delta_1 = \beta$, $\Delta\delta = 0$. Then in equation (3):

$$0 = \frac{1}{L} \cdot (\Delta E_z)_p + (\Delta E_y)_p + (\Delta E_v + \Delta E_s)_p + \frac{1}{N} \cdot (\Delta E_f)_p \quad (14)$$

From equation (1) for the prototype:

$$(\Delta E_y)_p + (\Delta E_v + \Delta E_s)_p = -(\Delta E_z)_p - (\Delta E_f)_p \quad . \quad (15)$$

Substituting into equation (14),

$$\frac{1}{L} = \frac{\beta}{\alpha} = 1 + N' \cdot \frac{(\Delta E_f)_p}{(\Delta E_z)_p} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where

$$N' = 1 - \frac{1}{N}$$

$$\frac{1}{N} = \left(\frac{\beta^{\frac{1}{6}} \cdot n_m}{n_p}\right)^2 \cdot \frac{\beta^5}{\gamma^2} = \left(\frac{\beta^{\frac{1}{6}} \cdot n_m}{n_p}\right)^2$$

Thus, for the required similarity at a particular section for a particular flow condition, the model-bed-level scale may be adjusted to satisfy equation (16), which is a function of the probable boundary shear losses for a particular flow, and the change in bed level between the control section and the particular section in the prototype. Strictly, equation (16) applies to the case when the two sections are at the ends of a small element of length $\Delta l$ of the channel. However, for practical use the formula may be extended to the case of sections more widely separated without appreciable sacrifice of accuracy.

For the case of channels of uniform slope $i$:

$$(\Delta E_z)_p = \left(\frac{dz}{dl} \cdot \Delta l\right)_p = -(i \cdot \Delta l)_p$$

$$(\Delta E_f)_p = \left\{\left(\frac{f}{2g} \cdot \frac{v^2}{R}\right)_{Av} \Delta l\right\}_p = \left\{\frac{Q^2 n^2}{1{\cdot}486^2} \cdot \frac{\Delta l}{(A^2 R^{\frac{4}{3}})_{Av}}\right\}_p$$

and equation (16) may be written as:

$$\frac{\beta}{\alpha} = 1 - N'' \left\{\frac{Q^2}{(A^2 R^{\frac{4}{3}})_{Av}}\right\}_p \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where

$$N'' = \left(\frac{n^2}{1{\cdot}486^2 . i}\right)_p . \left(1 - \frac{1}{N}\right)$$

$$\frac{1}{N} \doteq \left(\frac{\beta^{\frac{1}{6}} . n_m}{n_p}\right)^2$$

It will be seen that similarity at the particular section could be closely approximate only for one specific flow Q. For any flow the approximate value of the depth scales for any section may be derived from equation (2) or equation (3). For the case under discussion, $\frac{\delta_1}{\beta} = 1$, $\frac{\gamma^2}{\beta^5} = 1$, equation (2) may be written as :

$$\left\{\frac{\Delta\delta}{\delta_1}\right\} \doteq \frac{(L-1)\,(\Delta E_z)_m + (N-1)\,.\,(\Delta E_f)_m}{-(E_{y2})_m + (s-1)\,.\,(E_{v2} + \Delta E_s)_m + (t-1)\,.\,(\Delta E_f)_m} \quad . \quad (18)$$

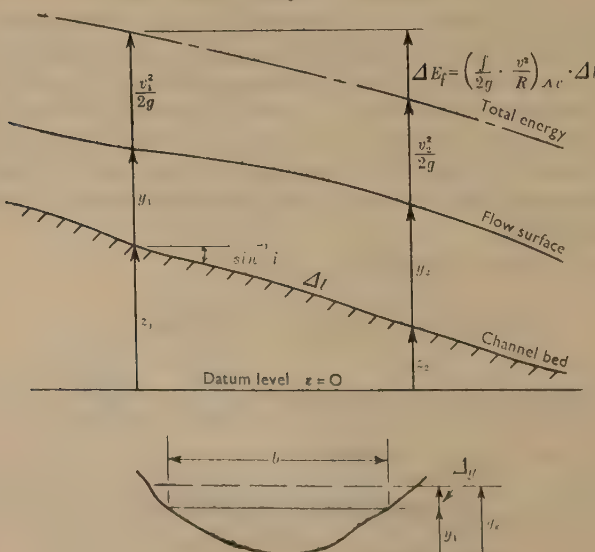
where $(E_z)_m$, $(E_y)_m$, $(E_v)_m$, are measurable from the model and where $(\Delta E_f)_m$ is the calculated boundary shear loss ; thus, surge head may be estimated by :

$$(\Delta E_s)_m \doteq -(\Delta E_z + \Delta E_y + \Delta E_v + \Delta E_f)_m$$

Therefore it is possible to derive depth scales for *any section* for *any flow* and the complete prototype flow profile may be plotted from the measured model depths multiplied by their individual scales.

## Method of Calculation of Flow Profile for Non-uniform Flow with No Shock Losses

*Figs 6*



The energy equation may be written as :

$$0 = (z_2 - z_1) + (y_2 - y_1) + \left(\frac{v_2^2}{2g} - \frac{v_1^2}{2g}\right) + \left(\frac{f}{2g}\cdot\frac{v^2}{R}\right)_{Av}\cdot\Delta l$$

$$= -i\,.\,\Delta l + \Delta y + \Delta\left(\frac{v^2}{2g}\right) + \left(\frac{f}{2g}\cdot\frac{v^2}{R}\right)_{Av}.\,\Delta l$$

In this equation,

$$\Delta\left(\frac{v^2}{2g}\right) = \frac{Q^2}{2g}\left(\frac{1}{(A_1 + \Delta A)^2} - \frac{1}{A_1^2}\right)$$

$$\doteqdot \frac{Q^2}{g}\cdot\frac{(-\Delta A)}{A_1^3} \doteqdot -\left(\frac{Q^2}{g}\Big/\left(\frac{A^3}{b}\right)_{Av}\right)\Delta y$$

$$\left(\frac{f}{2g}\cdot\frac{v^2}{R}\right)_{Av} = \left(\frac{f}{2g}\cdot\frac{Q^2}{A^2\,.\,R}\right)_{Av} = \left(\frac{Q^2\,.\,n^2}{1{\cdot}486^2}\cdot\frac{1}{(A^2\,.\,R^{4/3})_{Av}}\right)$$

Substituting these values into the energy equation,

$$0 = -i\,.\,\Delta l + \Delta y - \left(\frac{Q^2}{g}\Big/\left(\frac{A^3}{b}\right)_{Av}\right).\,\Delta y + \left(\frac{f}{2g}\cdot\frac{Q^2}{A^2\,.\,R}\right)\cdot\Delta l$$

From this latter expression the slope of the water surface at a particular section is given by :

$$\frac{dy}{dl} = \frac{i - \left(\dfrac{f}{2g}\cdot\dfrac{Q^2}{A^2\,.\,R}\right)}{1 - \left(\dfrac{Q^2}{g}\Big/\left(\dfrac{A^3}{b}\right)\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

From this equation or the previous equations it is clearly seen that :—

$$\Delta l = \frac{(y_2 - y_1) + \dfrac{Q^2}{2g}\cdot\left(\dfrac{1}{A_2^2} - \dfrac{1}{A_1^2}\right)}{i - \left(\dfrac{f}{2g}\cdot\dfrac{Q^2}{A^2\,.\,R}\right)_{Av}} \quad . \quad . \quad . \quad (20)$$

$$\doteqdot \frac{\Delta y\,.\left\{1 - \dfrac{Q^2}{g}\Big/\left(\dfrac{A^3}{b}\right)_{Av}\right\}}{i - \left(\dfrac{f}{2g}\cdot\dfrac{Q^2}{A^2\,.\,R}\right)}$$

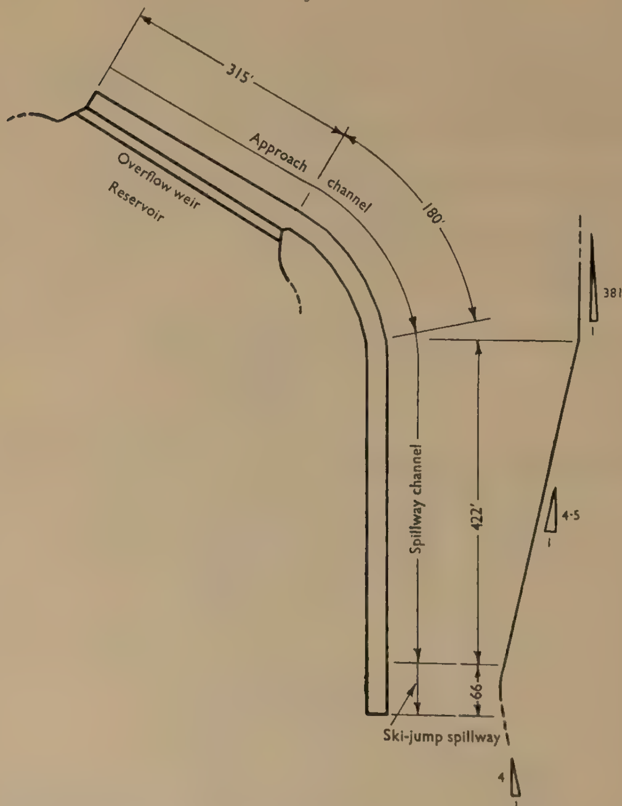
Since $A$ and $R$ are functions of the depth of flow $y$ and the channel cross-section, the distance $\Delta l$ between sections with depths $y_1$ and $y_2\,(= y_1 + \Delta y)$ may be calculated for a given flow $Q$.

Thus, commencing from a control section at which the depth $y_1$ is known, the distances between successive sections with differing depths may be calculated over a given length of channel, and the water-surface profile plotted.

Such a method of solution, which is, in effect, successive arithmetical intergration, is illustrated in the following analytical calculations for a specific channel.

### Model Analysis for Ski-Jump Spillway Channel

*Fig. 7.*



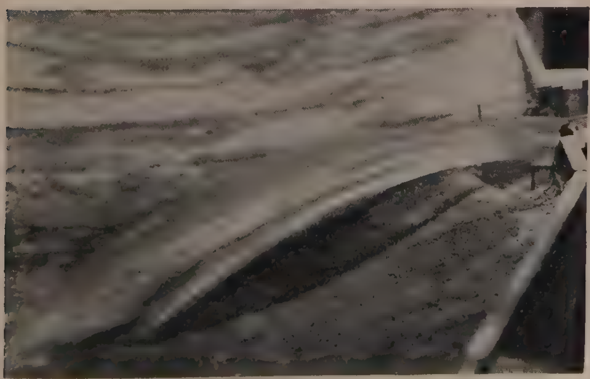

Diagrammatic Representation of Proposed Flood Overflow with Ski-Jump Spillway

See also *Figs 8* and *9*, opposite.

*Fig. 8*

General Lay-out of Overflow Weir and Ski-Jump Spillway Channel to a Model Scale of 1 : 60

*Fig. 9*



Model Ski-Jump Discharging a Prototype Flow of 10,000 Cusecs

*Prototype: Model-Scale Relationships*

Linear scale : $\beta = 60$ ;

Flow scale : $\gamma = \beta^{\frac{5}{2}} = 27{,}890$ ;

Depth scale : $\delta \simeq \beta$ ;

Velocity scale : $v_p/v_m \simeq \beta^{\frac{1}{2}} = 7{\cdot}75$.

Bed-level scale :

For spillway channel : $\alpha = \beta/1{\cdot}10 = 54{\cdot}5$

For other portions of model : $\alpha = \beta = 60$

*Fig. 7* represents diagrammatically a proposed hydraulic structure for flood overflows with final discharge from a ski-jump spillway. Since a major interest was the trajectory of the ski-jump jet for maximum flood conditions, it was considered desirable that the velocity scales at the end of the spillway channel should closely approximate to a value $v_p/v_m = \beta^{\frac{1}{2}}$ and depth-scales $\delta = \beta$, so that similarity for the trajectory would be a function of Froude parameters based on the linear scale $\beta$.

Owing to the geometry of the proposed structure from the overflow weir to the approach of the spillway channel, energy changes were assumed to be attributable entirely to inertia effects and shock losses which could be regarded as functions of Froude parameters $v^2/L\,.\,g$, so that the scale relations could be :

$$\text{Depth scale } \delta = \text{bed-level scale } \alpha = \text{linear scale } \beta.$$

$$\text{Velocity scale } v_p/v_m = \beta^{\frac{1}{2}}$$

$$\text{Flow scale } \gamma = \beta^{\frac{5}{2}}$$

However, on the relatively straight spillway channel it was considered that boundary shear effects would have a greater influence on the energy of flow.

Adjustment was then made to the slope of the straight portion of the channel for the particular flow of 10,000 cusecs, the anticipated maximum flood flow in the prototype. It was assumed that the depth at the upstream section would have a close approximation to the value $\delta_1 = \beta$ for the reasons already mentioned, and slope adjustment was made to produce an approximate depth scale $\delta_2 = \beta$ at the downstream section by using equation (3) with values for the prototype derived from analytical calculations with equation (20), neglecting shock losses.

The Manning roughness coefficient for the prototype was presumed to be $n_p = 0{\cdot}0135$ (for smooth finished concrete.) The model channel was constructed in Perspex which may be regarded as hydraulically smooth and consequently the equivalent roughness coefficient varies with the flow conditions. However, for a given flow an average value of the equivalent roughness coefficient was calculated for the flow conditions at the median

section along the channel, and the depth-scale $\delta$ relationships could then be estimated.

For the purpose of comparing corresponding model and prototype quantities, all model units of quantity are multiplied by functions of a linear-scale multiplier of value $\beta = 60$. Thus all model quantities will have magnitudes of the same order as the corresponding prototype values; for example:

$$y = \beta \, . \, y_m; \quad A = \beta^2 \, . \, A_m$$
$$P = \beta \, . \, P_m; \quad Q = \beta^{\frac{5}{2}} \, . \, Q_m$$
$$R = \beta \, . \, R_m; \quad v = \beta^{\frac{1}{2}} \, . \, v_m$$

## Analytical and Experimental Results for Straight Portion of Spillway Channel

### (1) *Properties of Cross-Section*

*Fig. 10*

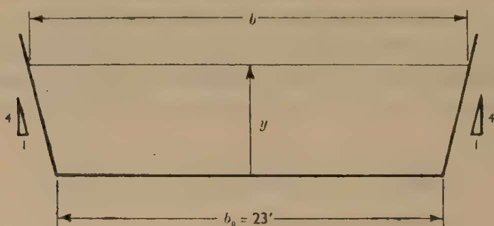



Table 3 lists the properties of the cross-section shown in *Fig. 10;* it may be found, by logarithmic plotting, that:

$$A \doteqdot \text{constant} \times y^a \, . \, b_o^{2-a}, \text{ where } a = 1{\cdot}09$$
$$R \doteqdot \text{constant} \times y^c \, . \, b_o^{1-c}, \text{ where } c = 0{\cdot}66$$

### (2) *Analytical Calculations for Prototype Channel neglecting Shock Losses*

From equation (20):—

$$\Delta l = \frac{(y_2 - y_1) + \dfrac{Q^2}{2g} \cdot \left(\dfrac{1}{A_2{}^2} - \dfrac{1}{A_1{}^2}\right)}{i - \dfrac{Q^2 \, . \, n^2}{1{\cdot}486^2} \cdot \dfrac{1}{(A^2 \, . \, R^{\frac{4}{3}})_{Av}}}$$

expressed in terms of prototype values, where:

$$n = n_p = 0{\cdot}0135 \text{ foot units;}$$

and $\qquad i = i_p = 0{\cdot}217$

TABLE 3.

| $y$ Feet | $b$ Feet | $A$ Feet $^2$ | $P$ Feet | $R = A/P$ Feet | $\frac{1}{A^2}$ | $\frac{1}{A^2 . R^{\frac{4}{3}}}$ |
|---|---|---|---|---|---|---|
| 16·3 | 31·2 | 441·3 | 56·6 | 7·80 | $5{\cdot}13 \times 10^{-6}$ | $0{\cdot}33 \times 10^{-6}$ |
| 16·0 | 31·0 | 432·0 | 56·0 | 7·73 | 5·34 | 0·35 |
| 14·0 | 30·0 | 371·1 | 51·9 | 7·15 | 7·24 | 0·52 |
| 12·0 | 29·0 | 312·0 | 47·8 | 6·54 | 10·3 | 0·88 |
| 10·0 | 28.0 | 255·0 | 43·6 | 5·85 | 15·4 | 1·46 |
| 9·0 | 27·5 | 227·2 | 41·5 | 5·47 | 19·4 | 2·00 |
| 8·0 | 27·0 | 200·0 | 39·5 | 5·07 | 25·0 | 2·68 |
| 7·0 | 26·5 | 173·3 | 37·4 | 4·64 | 33·4 | 4·33 |
| 6·4 | 26·2 | 157·4 | 36·2 | 4·36 | 40·6 | 5·66 |
| 6·0 | 26·0 | 147·0 | 35·4 | 4·16 | 46·3 | 6·92 |
| 5·8 | 25·9 | 141·8 | 34·9 | 4·06 | 49·7 | 7·66 |
| 5·6 | 25·8 | 136·6 | 34·5 | 3·97 | 53·7 | 8·58 |
| 5·4 | 25·70 | 131·5 | 34·1 | 3·86 | 57·8 | 9·57 |
| 5·3 | 25·65 | 128·9 | 33·9 | 3·80 | 60·2 | 10·2 |
| 5·2 | 25·60 | 126·4 | 33·7 | 3·75 | 63·3 | 10·8 |

For the particular case of flow :

$$Q \doteq 10{,}000 \text{ cusecs}$$

$$\frac{Q^2}{2g} \doteq 1{\cdot}553 \times 10^6$$

and

$$\frac{Q^2 . n^2}{1{\cdot}486^2} = 8{\cdot}25 \times 10^3$$

The depth of flow at the upstream section was initially calculated assuming uniform flow in the approach channel of slope $i_A = 1/381$ approximately :

$$(A^2 . R^{\frac{4}{3}})_1 \doteq \frac{Q^2 . n^2}{1{\cdot}486^2} \cdot \Big/ i_A = 3{\cdot}14 \times 10^6 \,; \quad y_1 \doteq 16{\cdot}3 \text{ feet}$$

for a flow $Q = 10{,}000$ cusecs. Analytical calculations were then made for the flow down the spillway channel, and from these the required slope adjustment $\beta/\alpha$ was estimated. In addition, calculations were made for comparison with experimental results and since, as mentioned previously, flow conditions in the approach channel may be regarded as a function of Froude parameters, the measured model-value of $y_1$ with scale $\delta_1 = \beta$ was accepted as more precise than the assumed initial value. The measured model-depth had an equivalent prototype value $y = 13{\cdot}1$ feet. These calculations are contained in Table 4 and the flow profile is shown in *Fig. 11.*

TABLE 4.—PROTOTYPE ANALYTICAL CALCULATIONS

| $Q = 10{,}000$ cusecs, $n_p = 0{\cdot}0135$ | | | | (a) $y_1 \doteq 16{\cdot}3$ | (b) $y_1 \doteq 13{\cdot}1$ |
|---|---|---|---|---|---|
| $y$ feet | $\frac{Q^2}{2g} \cdot \frac{1}{A^2}$ feet | $(y_2 - y_1) + \frac{Q^2}{2g} \cdot \left(\frac{1}{A_2{}^2} - \frac{1}{A_1{}^2}\right)$ | $i - \frac{Q^2 n^2}{1{\cdot}486^2} \cdot \frac{1}{(A^2 \cdot R^{\frac{4}{3}})}$ | $(\Delta l)$; $l$ feet | $(\Delta l)$; $l$ feet |
| 16·3 (a) | 7·96 | | | 0·0 | |
| | | 0·03 | 0·214 | ( 0·1) | |
| 16·0 | 8·29 | | | 0·1 | |
| | | 0·93 | 0·213 | ( 4·4) | |
| 14·0 | 11·22 | | | 4·5 | |
| | | 2·78 | 0·211 | (13·2) | |
| 13·1 (b) | 13·1 | | | | 0·0 |
| | | 1·8 | 0·209 | | (8·6) |
| 12·0 | 16·0 | | | 17·7 | 8·6 |
| | | 5·9 | 0·207 | (28·5) | (28·5) |
| 10·0 | 23·9 | | | 46·2 | 37·1 |
| | | 5·2 | 0·203 | (25·6) | (25·6) |
| 9·0 | 30·1 | | | 71·8 | 62·7 |
| | | 7·7 | 0·198 | (38·9) | (38·9) |
| 8·0 | 38·8 | | | 110·7 | 101·6 |
| | | 12·0 | 0·188 | (63·8) | (63·8) |
| 7·0 | 51·8 | | | 174·5 | 165·4 |
| | | 10·5 | 0·176 | (59·7) | (59·7) |
| 6·4 | 62·9 | | | 234·2 | 225·1 |
| | | 8·5 | 0·165 | (51·5) | (51·5) |
| 6·0 | 71·8 | | | 285·7 | 276·6 |
| | | 5·1 | 0·157 | (32·5) | (32·5) |
| 5·8 | 77·1 | | | 318·2 | 309·1 |
| | | 6·0 | 0·150 | (40·0) | (40·0) |
| 5·6 | 83·3 | | | 358·2 | 349·1 |
| | | 6·2 | 0·142 | (43·7) | (43·7) |
| 5·4 | 89·7 | | | 401·9 | 392·8 |
| 5·32 (a) | 92·6 | | | 422·0 (a) | |
| | | 3·6 | 0·135 | (26·7) | (26·7) |
| 5·3 | 93·4 | | | 428·6 | 419·5 |
| 5·29 (b) | | | | | 422·0 (b) |
| | | 4·7 | 0·130 | | (36·2) |
| 5·2 | 98·2 | | | | 455·7 |

(3) *Adjustment of Model-Slope on Basis of Initial Analytical Calculations for Prototype*

From equation (16):

$$\frac{\beta}{\alpha} = 1 + N' \cdot \frac{(\Delta E_f)_p}{(\Delta E_z)_p}$$

where $$N' = \left(1 - \frac{1}{N}\right) \text{ and } \frac{1}{N} = \left(\frac{\beta^{\frac{1}{6}} \cdot n_m}{n_p}\right)^2$$

For the model-channel in Perspex, where the friction constant is a function of the Reynolds number, it has been shown that:

$$f_m = 0{\cdot}234\, \nu^{0\cdot15} \cdot \left(\frac{P_m}{Q_m}\right)^{0\cdot15}$$

Expressing this in units of prototype magnitude for water at 15°C:

$$f_m = 0{\cdot}00431 \left(\frac{P}{\beta} \cdot \frac{\beta^{\frac{5}{2}}}{Q}\right)^{0\cdot15}$$

$$= 0{\cdot}00431 \left(\frac{P}{Q}\right)^{0\cdot15} \cdot \beta^{0\cdot225}$$

$$= 0{\cdot}01084 \left(\frac{P}{Q}\right)^{0\cdot15}, \text{ for } \beta = 60$$

The equivalent Manning roughness coefficient:

$$n_m \simeq \left(\frac{f_m}{2g}\right)^{\frac{1}{2}} \times \frac{1{\cdot}486 R^{\frac{1}{6}}}{\beta^{\frac{1}{6}}}$$

From Table 4, column ($a$), for the median section of flow ($l = 211$ feet)

$Q = 10{,}000$ cusecs, $y \simeq 6{\cdot}7$ feet, $P \simeq 36{\cdot}8$ feet, $R \simeq 4{\cdot}5$ feet:

$$f_m \simeq 0{\cdot}0047$$

$$\beta^{\frac{1}{6}} \cdot n_m \simeq 0{\cdot}0163; \quad n_m \simeq 0{\cdot}0082, \text{ for } \beta = 60.$$

$$\frac{1}{N} = \left(\frac{\beta^{\frac{1}{6}} \cdot n_m}{n_p}\right)^2 = 1{\cdot}46, \text{ for } n_p = 0{\cdot}0135$$

and $$N' = -0{\cdot}46.$$

From Table 4, column (a),

$$(\Delta E_z)_p = -i \cdot \Delta l = -0{\cdot}217 \times 422 = -91{\cdot}5 \text{ feet}$$

$$(\Delta E_y)_p = 5{\cdot}32 - 16{\cdot}3 = -11{\cdot}0 \text{ feet}$$

$$(\Delta E_v)_p = 92{\cdot}6 - 7{\cdot}96 = +84{\cdot}6 \text{ feet}$$

$$(\Delta E_f)_p = -(\Delta E_z + \Delta E_y + \Delta E_v)_p = +17{\cdot}9 \text{ feet}$$

The required slope adjustment is,

$$\frac{\beta}{\alpha} = 1 + N' \cdot \frac{(\Delta E_f)_p}{(\Delta E_z)_p} \simeq 1{\cdot}10$$

(4) *Analytical Calculations for Model-Channel neglecting Shock Losses*

Expressed in units of prototype magnitude, for the particular case of flow :

$$Q = 10{,}000 \text{ cusecs}$$

$$\frac{Q^2}{2g} = 1{\cdot}553 \times 10^6$$

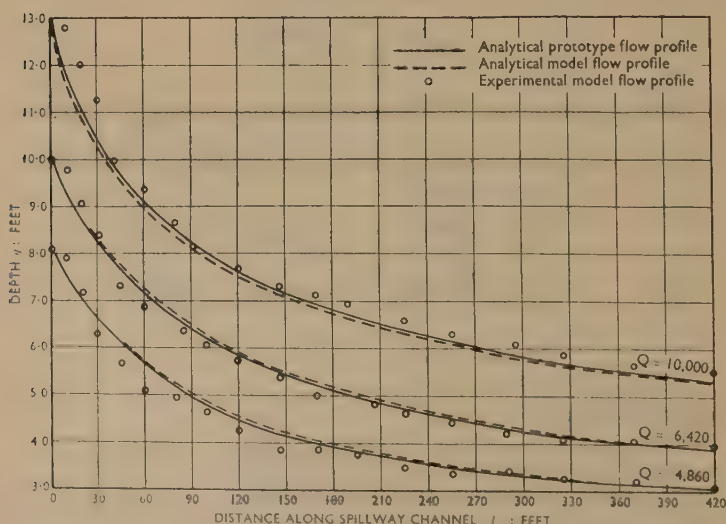
$$i = i_m = \left(\frac{\beta}{\alpha}\right) . i_p = 0{\cdot}239$$

$$f = f_m \simeq 0{\cdot}01084\left(\frac{P}{Q}\right)^{0{\cdot}15}$$

The initial depth of flow was as measured from the model : $y_1 = 13{\cdot}1$ feet.

These calculations are contained in Table 5 and the flow profile is shown in *Fig. 11.*

*Fig. 11*




ANALYTICAL AND EXPERIMENTAL FLOW PROFILES FOR TRAPEZOIDAL SPILLWAY CHANNEL

TABLE 5.—MODEL ANALYTICAL CALCULATIONS

$Q = 10{,}000$ cusecs ; $f_m = 0{\cdot}0108\left(\frac{P}{Q}\right)0{\cdot}15$ ; $i_m = 0{\cdot}239$

| $y$ feet | $\frac{Q^2}{2g}\cdot\frac{1}{A^2}$ feet | $(y_2 - y_1) + \frac{Q^2}{2g}\cdot\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)$ | $i - \frac{f}{2g}\cdot\frac{Q^2}{A^2\cdot R}$ | $(\Delta l)$ ; $l$ feet |
|---|---|---|---|---|
| 13·1 | 13·1 | | | 0·0 |
| | | 1·8 | 0·228 | (7·9) |
| 12·0 | 16·0 | | | 7·9 |
| | | 5·9 | 0·223 | (26·4) |
| 10·0 | 23·9 | | | 34·3 |
| | | 5·2 | 0·216 | (24·1) |
| 9·0 | 30·1 | | | 58·4 |
| | | 7·7 | 0·208 | (37·0) |
| 8·0 | 38·8 | | | 95·4 |
| | | 12·0 | 0·195 | (61·6) |
| 7·0 | 51·8 | | | 157·0 |
| | | 10·5 | 0·179 | (58·7) |
| 6·4 | 62·9 | | | 215·7 |
| | | 8·5 | 0·165 | (51·5) |
| 6·0 | 71·8 | | | 267·2 |
| | | 5·1 | 0·155 | (32·9) |
| 5·8 | 77·1 | | | 300·1 |
| | | 6·0 | 0·146 | (41·1) |
| 5·6 | 83·3 | | | 341·2 |
| | | 6·2 | 0·137 | (45·3) |
| 5·4 | 89·7 | | | 386·5 |
| | | 3·6 | 0·128 | (28·1) |
| 5·3 | 93·4 | | | 414·6 |
| 5·28 | | | | 422·0 |
| | | 4·7 | 0·122 | (38·6) |
| 5·2 | 98·2 | | | 453·2 |

(5) *Model Spillway Channel—Experimental Results*

TABLE 6

$$Q = 10{,}000 \text{ cusecs}\,;\ \frac{Q^2}{2g} = 1{\cdot}552 \times 10^6\,;\ i = i_m = 0{\cdot}239$$

| $(\Delta l)$; $l$ | $y = E_y$ | $\frac{Q^2}{2g}\cdot\frac{1}{A^2} = E_v$ | $\left(\frac{f}{2g}\cdot\frac{Q^2}{A^2}\cdot\frac{1}{R}\right)$ | $(\Delta E_f)$; $E_f$ | $-i\cdot\Delta l = E_z$ | $-\left[\begin{matrix}E_z + E_y\\ + E_v + E_f\end{matrix}\right]_1^2 = E_s$ |
|---|---|---|---|---|---|---|
| 0 | 13·11 | 13·1 | | 0·0 | 0·0 | 0·0 |
| (10) | | (13·5) | ( $9{\cdot}6 \times 10^{-3}$) | (0·10) | | |
| 10 | 12·80 | 13·8 | | 0·1 | − 2·4 | 0·9 |
| (10) | | (14·8) | ($10{\cdot}8 \times 10^{-3}$) | (0·11) | | |
| 20 | 12·05 | 15·8 | | 0·2 | − 4·8 | 2·9 |
| (10) | | (17·2) | ($12{\cdot}7 \times 10^{-3}$) | (0·13) | | |
| 30 | 11·26 | 18·5 | | 0·3 | − 7·2 | 3·3 |
| (15) | | (21·6) | ($16{\cdot}9 \times 10^{-3}$) | (0·25) | | |
| 45 | 9·87 | 24·7 | | 0·6 | − 10·7 | 1·7 |
| (15) | | (26·3) | ($22{\cdot}1 \times 10^{-3}$) | (0·33) | | |
| 60 | 9·33 | 27·8 | | 0·9 | − 14·3 | 4·5 |
| (20) | | (30·4) | ($26{\cdot}8 \times 10^{-3}$) | (0·54) | | |
| 80 | 8·64 | 33·0 | | 1·5 | − 19·1 | 2·2 |
| (20) | | (35·2) | ($31{\cdot}7 \times 10^{-3}$) | (0·63) | | |
| 100 | 8·16 | 37·3 | | 2·1 | − 23·9 | 2·5 |
| (20) | | (39·6) | ($36{\cdot}8 \times 10^{-3}$) | (0·74) | | |
| 120 | 7·68 | 41·8 | | 2·8 | − 28·6 | 2·5 |
| (25) | | (44·5) | ($43{\cdot}2 \times 10^{-3}$) | (1·08) | | |
| 145 | 7·30 | 47·2 | | 3·9 | − 34·6 | 2·4 |
| (25) | | (48·5) | ($48{\cdot}5 \times 10^{-3}$) | (1·21) | | |
| 170 | 7·14 | 49·7 | | 5·1 | − 41·6 | 5·9 |
| (25) | | (51·4) | ($51{\cdot}9 \times 10^{-3}$) | (1·30) | | |
| 195 | 6·92 | 53·1 | | 6·4 | − 46·6 | 6·4 |
| (30) | | (56·0) | ($57{\cdot}6 \times 10^{-3}$) | (1·73) | | |
| 225 | 6·60 | 58·8 | | 8·2 | − 53·7 | 6·3 |
| (30) | | (61·6) | ($65{\cdot}9 \times 10^{-3}$) | (1·98) | | |
| 255 | 6·33 | 64·3 | | 10·1 | − 60·9 | 6·4 |
| (35) | | (66·8) | ($72{\cdot}8 \times 10^{-3}$) | (2·55) | | |
| 290 | 6·11 | 69·2 | | 12·7 | − 69·2 | 7·4 |
| (35) | | (72·1) | ($82{\cdot}2 \times 10^{-3}$) | (2·88) | | |
| 325 | 5·89 | 74·9 | | 15·6 | − 77·6 | 9·4 |
| (45) | | (79·2) | ($91{\cdot}8 \times 10^{-3}$) | (4·13) | | |
| 370 | 5·64 | 83·4 | | 19·7 | − 88·4 | 5·9 |
| (50) | | (83·7) | ($98{\cdot}7 \times 10^{-3}$) | (4·94) | | |
| 420 | 5·58 | 83·9 | | 24·6 | − 100·2 | 12·3 |

The calculations of energy conditions at experimentally measured flow sections are shown in Table 6. It should be noted that the calculations for the final section may be obtained with reasonably close approximation by evaluating $\Delta E_f$ for the whole length of channel from the properties of the median section ($l = 210$). (See Table 7.)

TABLE 7

| $(\Delta l)$; $l$ | $E_y$ | $E_v$ | $\left(\frac{f}{2g} \cdot \frac{Q^2}{A^2} \cdot \frac{1}{R}\right)$ | $(\Delta E_f)$; $E_f$ | $E_z$ | $E_s$ |
|---|---|---|---|---|---|---|
| (a) $Q = 10{,}000$ cusecs ; $\frac{Q^2}{2g} = 1{\cdot}552 \times 10^6$ ; $i = i_m = 0{\cdot}239$ | | | | | | |
| 0<br>(420)<br>420 | 13·11<br>(6·76)<br>5·58 | 13·1<br>(56·0)<br>83·9 | $(57{\cdot}6 \times 10^{-3})$ | 0·0<br>(24·2)<br>24·2*<br>*Error<br>−0·4<br>= 24·6 | 0·0<br>−100·2 | 0·0<br>12·7 |
| (b) $Q = 6{,}420$ cusecs ; $\frac{Q^2}{2g} = 0{\cdot}643 \times 10^6$ ; $i = i_m = 0{\cdot}239$ | | | | | | |
| 0<br>(420)<br>420 | 10·00<br>(4·75)<br>3·99 | 9·9<br>(48·8)<br>67·3 | $(69{\cdot}3 \times 10^{-3})$ | 0·0<br>(29·1)<br>29·1 | 0·0<br>−100·2 | 0·0<br>19·7 |
| (c) $Q = 4{,}860$ cusecs ; $\frac{Q^2}{2g} = 0{\cdot}366 \times 10^6$ ; $i = i_m = 0{\cdot}239$ | | | | | | |
| 0<br>(420)<br>420 | 8·07<br>(3·68)<br>3·08 | 9·0<br>(47·6)<br>69·9 | $(84{\cdot}1 \times 10^{-3})$ | 0·0<br>(35·4)<br>35·4 | 0·0<br>−100·2 | 0·0<br>8·9 |

(6) *Depth Scales for Downstream Section* ($l = 420$ *feet*)

$$\left\{\frac{\Delta\delta}{\delta_1}\right\} \doteqdot \frac{(L-1)(\Delta E_z)_m + (N-1)\,.\,(\Delta E_f)_m}{-(E_{y2})_m + (s-1)\left(E_{v2} + \frac{\Delta E_s}{2}\right)_m + \frac{(t-1)}{2}\,.\,N\,.\,(\Delta E_f)_m}$$

$$L = \left(\frac{\alpha}{\beta}\right) = 0{\cdot}91\ ;\ (L-1) = -0{\cdot}09$$

$$N = \left(\frac{n_p}{\beta^{\frac{1}{6}}\,.\,n_m}\right)^2 ;\ n_p = 0{\cdot}0135\ ;\ \beta = 60.$$

$$(s-1) = 2a = 2 \times 1{\cdot}09 = 2{\cdot}18$$

$$\frac{(t-1)}{2} = \frac{2a + \frac{4}{3}c}{2} = \frac{(2 \times 1{\cdot}09) + (\frac{4}{3} \times 0{\cdot}66)}{2} = 1{\cdot}53$$

(*a*) *Particular Flow* $Q = 10{,}000$ *cusecs*

For the median section of flow, $y \simeq 6{\cdot}76$ feet :

$$f = 4{\cdot}68 \times 10^{-3}\,;\ \ R = 4{\cdot}53 \text{ feet.}$$

$$(\beta^{\frac{1}{6}} \,.\, n_m) \simeq 1{\cdot}486 \,.\, R^{\frac{1}{6}} \,.\, \left(\frac{f}{2g}\right)^{\frac{1}{2}} = 0{\cdot}0163\,;\ \ n_m \simeq 0{\cdot}0082$$

$$N \simeq \left(\frac{0{\cdot}0135}{0{\cdot}0163}\right)^2 = 0{\cdot}69\,;\ \ (N-1) = -0{\cdot}31$$

$$\left\{\frac{\Delta\delta}{\delta_1}\right\} \simeq \frac{-\{0{\cdot}09 \times (-100{\cdot}2)\} - \{0{\cdot}31 \times (+24{\cdot}6)\}}{-\{+5{\cdot}6\} + \left\{2{\cdot}18 \times \left(+83{\cdot}9 + \dfrac{12{\cdot}3}{2}\right)\right\} + \{1{\cdot}53 \times 0{\cdot}69 \times (+24{\cdot}6)\}}$$

$$= +0{\cdot}007$$

$$\delta_2 = \delta_1 + \Delta\delta \simeq 60{\cdot}4$$

(*b*) *Particular Flow* $Q \doteqdot 6{,}420$ *cusecs*

For the median section of flow, $y \simeq 4{\cdot}75$ feet :

$$f = 4{\cdot}92 \times 10^{-3}\,;\ \ R = 3{\cdot}46 \text{ feet.}$$

$$(\beta^{\frac{1}{6}} \,.\, n_m) \simeq 1{\cdot}486 \,.\, R^{\frac{1}{6}} \,.\, \left(\frac{f}{2g}\right)^{\frac{1}{2}} = 0{\cdot}0160\,;\ \ n_m = 0{\cdot}0080$$

$$N \simeq \left(\frac{0{\cdot}0135}{0{\cdot}0160}\right)^2 = 0{\cdot}71\,;\ \ (N-1) = -0{\cdot}29$$

$$\left\{\frac{\Delta\delta}{\delta_1}\right\} \simeq \frac{-\{0{\cdot}09 \times (-100{\cdot}2)\} - \{0{\cdot}29 \times (+29{\cdot}1)\}}{-\{+4{\cdot}0\} + \left\{2{\cdot}18 \times \left(+67{\cdot}3 + \dfrac{19{\cdot}7}{2}\right)\right\} + \{1{\cdot}53 \times 0{\cdot}71 \times (+29{\cdot}1)\}}$$

$$= +0{\cdot}003$$

$$\delta_2 = \delta_1 + \Delta\delta \simeq 60{\cdot}2$$

(*c*) *Particular Flow* $Q = 4{,}860$ *cusecs*

For the median section of flow, $y \simeq 3{\cdot}68$ feet :

$$f = 5{\cdot}07 \times 10^{-3}\,;\ \ R = 2{\cdot}87 \text{ feet}$$

$$(\beta^{\frac{1}{6}} \,.\, n_m) \simeq 1{\cdot}486 \,.\, R^{\frac{1}{6}} \,.\, \left(\frac{f}{2g}\right)^{\frac{1}{2}} = 0{\cdot}0157\,;\ \ n_m \simeq 0{\cdot}0079$$

$$N \simeq \left(\frac{0{\cdot}0135}{0{\cdot}0157}\right)^2 = 0{\cdot}74\,;\ \ (N-1) = -0{\cdot}26$$

$$\left\{\frac{\Delta\delta}{\delta_1}\right\} \simeq \frac{-\{0{\cdot}09 \times (-100{\cdot}2)\} - \{0{\cdot}26 \times (+35{\cdot}4)\}}{-\{3{\cdot}1\} + \left\{2{\cdot}18 \times \left(+69{\cdot}9 + \dfrac{7{\cdot}9}{2}\right)\right\} + \{1{\cdot}53 \times 0{\cdot}74 \times (+35{\cdot}4)\}}$$

$$= -0{\cdot}001$$

$$\delta_2 = \delta_1 + \Delta\delta \simeq 59{\cdot}9$$

## Conclusions

It may be presumed in all cases of fluid flow that energy losses which may be described as caused by boundary shear have their mechanism linked with the forces of viscous shear through the fluid body. This is dependent on the conditions of flow, the properties of the fluid, and the nature of the wetted surface of the channel: namely, this energy loss may be regarded as a function of Reynolds parameters $v.L/\nu$ and roughness parameters $R/k$, where $k$ denotes the magnitude of roughness projections on the channel surface. In model-analysis, for exact similarity with a prototype for all conditions of flow, it would be necessary that the model should have the same values for corresponding parameters of the prototype. Frequently this is impossible because of other energy transformations which also require—for similarity—the reproduction of the same Froude parameters $v^2/L.g$ for the model and for the prototype.

The influence of the geometry of the channel, causing changes in velocities from section to section, for non-uniform flow along that channel, may be expected to result in energy transfer causing surges to develop along the flow; these surges would probably be accompanied by shock losses through sudden changes in the momentum of flow, and therefore both the surge energy and the shock losses may be regarded as functions of Froude parameters $v^2/L.g$.

In the design and operation of hydraulic models, the major difficulty lies in determining what is the influence on the degree of reproduction of the parameters $v.L/r$, $R/k$, and $v^2/L.g$, and an estimation of the effect of at least two of them is necessary before estimating the similarity of flow conditions between model and prototype. Usually the complexity of the flow pattern, as a function of the Froude parameters $v^2/L.g$, is the justification of using hydraulic model analysis instead of theoretical calculations, and therefore the problem is usually resolved into obtaining the best available estimate of the influence of parameters $v.L/r$ and $R/k$.

In the previous theory the estimate of boundary shear losses per unit length of channel has been expressed as $f.v^2/2g.R$. This has been accepted from the theory of uniform flow on the assumption that for non-uniform flow the velocity distribution is the same across a given section as across a section of the same shape and area for uniform flow of the same

quantity. It has been assumed that the Manning formula may give as precise an estimate of boundary shear as any other formula at present in use. However, the fundamental development may be employed to derive scale relationships for any other formula which may be selected.

The depth-scale-relationship theory has three main uses :—

(1) Scales may be selected to achieve similarity of a particular flow condition with reasonable accuracy :

For example, in the particular case of the ski-jump spillway channel, where the main requirement is to obtain a certain scale of similarity at a particular section for a particular flow, a choice may be made for the following selections :—

(*a*) For $\alpha = \beta$ ; $\gamma = \beta^{\frac{5}{2}}$ ; and *at all sections* $\delta = \beta$ very closely.

From equations (11) and (12) : $\left(\dfrac{n_p}{\beta^{\frac{1}{6}} . n_m}\right) = 1$

$$\beta = \left(\frac{n_p}{n_m}\right)^6 = 23 ;$$

for $n_p = 0{\cdot}0135$ and $n_m = 0{\cdot}008$.

(*b*) For a practicable linear scale, $\beta = 60$, and *at all sections* $\delta = \delta_1$ very closely.

From equations (11) and (12) : $\dfrac{\beta}{\delta} = \dfrac{\beta}{\alpha} = \left(\dfrac{\beta^{\frac{1}{6}} . n_m}{n_p}\right)^{2/(t-s)}$

$$= 1{\cdot}44$$

$$\frac{\beta^{\frac{5}{2}}}{\gamma} = \left(\frac{\beta^{\frac{1}{6}} . n_m}{n_p}\right)^{s/2(t-s)}$$

$$= 1{\cdot}33$$

for $\beta = 60$ ; $s = 3{\cdot}18$ ; $t = 4{\cdot}06$ ; $n_p = 0{\cdot}0135$ ; and $n_m = 0{\cdot}008$.

Alternatively, if it is desired that the depth scale may have a value suitable for subsequent calculations and the linear scale $\beta \simeq 60$, let $(\delta = \delta_1 = 40)$ :

$$\frac{\beta}{\delta} = \frac{\beta}{\alpha} = \left(\alpha^{\frac{1}{6}} . \frac{n_m}{n_p}\right)^{6/\{3(t-s) - 1\}}$$

$$= 1{\cdot}40 ; \ \beta = 56 ;$$

$$\frac{\beta^{\frac{5}{2}}}{\gamma} = \left(\alpha^{\frac{1}{6}} . \frac{n_m}{n_p}\right)^{3s/2\{3(t-s) - 1\}}$$

$$= 1{\cdot}31.$$

(*c*) For a practicable linear scale, $\beta = 60$, and flow scale, $\gamma = \beta^{\frac{5}{2}}$, and *at end section*, $\delta_2 = \delta_1 = \beta$ very closely; for a *particular flow*, $Q_p = 10{,}000$ cusecs.

From equation (16): $$\frac{\beta}{\alpha} = 1 + N' \cdot \frac{(\Delta E_f)_p}{(\Delta E_z)_p}$$
$$= 1{\cdot}08 \simeq 1{\cdot}10;$$

for $\beta = 60$; $n_p = 0{\cdot}0135$; $n_m = 0{\cdot}008$;

$$N = 1 - \frac{(\beta^{\frac{1}{6}} \, . \, n_m)^2}{n_p} = -\,0{\cdot}38; \text{ and } \frac{(\Delta E_f)_p}{(\Delta E_z)_p} = -\,0{\cdot}20.$$

*Note:* If the problem had been one of uniform flow, then $(\Delta E_f)_p/(\Delta E_z)_p = -1$, and $\dfrac{\beta}{\alpha} = 1{\cdot}38 \simeq 1{\cdot}40$. This shows the discrepancy which would occur if slope adjustment was made on the basis of uniform flow for the particular case of non-uniform flow.

As may be deduced from equation (16), the necessary slope adjustment decreases proportionately as the friction head diminishes, that is as the flow becomes more closely a function of Froude parameters only.

(2) For a model with selected scales, the actual values of depth scales may be checked at any section for any flow.

Referring to the calculations for the ski-jump spillway channel, it will be seen that the depth scales for the end section have been checked for a number of particular flows. In similar fashion, for a given flow the depth scale may be calculated for any section, and therefore the flow profile for the prototype may be plotted from correctly scaled model-depths. This means that more confidence may be placed in using model-records for designing corresponding hydraulic structures.

(3) For a model with selected scales, the accuracy of reproduction may be checked:

(*a*) Estimation of possible range of depth scales for possible errors in estimation of boundary shear losses in model.

For the particular illustration of the model for a ski-jump spillway channel it has been shown that modification was made to the channel slope for a required scale of similarity for flow at the downstream section. Calculations for boundary shear losses $E_f$ and surge energy $E_s$ are given in Table 6 (p. 208). A possible range of error from the

accepted probable value of boundary shear loss may be deduced from this and the influence on the required depth scale derived :—

For a flow of 10,000 cusecs :

with ($\Delta E_s = 0$), $\Delta E_f = 36{\cdot}9$

$$\frac{\Delta\delta}{\delta_1} \simeq -0{\cdot}054.$$

with ($\Delta E_f = 0$), $\Delta E_s = 36{\cdot}9$

$$\frac{\Delta\delta}{\delta_1} \simeq +0{\cdot}041$$

The possible range of values of $\delta_2$ may be :

$\delta_2 \simeq 56{\cdot}8$ to $62{\cdot}5$ ; probable $\delta_2 \simeq 60{\cdot}4$

(*b*) Estimation of probable range of depth scales for probable range of roughness coefficients for prototype :

If the Manning roughness coefficient for the prototype can only be estimated as having a value within some probable range, an estimation may then be made of the probable range of depth-scale values $\delta_2$.

To obtain a probable range of values of Manning roughness coefficients $n_p$ corresponding to probable sizes of roughness projections $k$, approximate correlation may be made between a quasi-rational equation [4] :

$$C = 37{\cdot}6 + 32{\cdot}1 \log_{10}\left(\frac{R}{k}\right)$$

and the empirical Manning formula :

$$C = \frac{1{\cdot}486}{n} \,.\, R^{\frac{1}{6}} \text{ or } n = \frac{1{\cdot}486}{C} \,.\, R^{\frac{1}{6}}$$

For the prototype ski-jump spillway channel in smooth-finished concrete, it may be estimated that the roughness projections have sizes over a range of 0·01 inch to 0·05 inch :

$k \simeq 0{\cdot}001$ to $0{\cdot}004$ feet ; average $k \simeq 0{\cdot}0025$ feet

For a flow of 10,000 cusecs, at the median section of the channel, $R \simeq 4{\cdot}40$ feet. Therefore the probable range of values of $n_p$ may be :

$n_p \simeq 0{\cdot}0125$ to $0{\cdot}0140$ ; average $n_p \simeq 0{\cdot}0135$

and from the values in Table 6 :

$\frac{\Delta\delta}{\delta_1} \simeq -0{\cdot}005$ to $+0{\cdot}012$ ; average $\frac{\Delta\delta}{\delta_1} \simeq +0{\cdot}007$

$\delta_2 \simeq 59{\cdot}7$ to $60{\cdot}7$ ; average $\delta_2 \simeq 60{\cdot}4$

It must be emphasized that the methods outlined in this Paper can give values whose accuracy depends mainly on how closely the term for boundary shear losses represents actual conditions in either prototype or model.

## Acknowledgements

The Authors are much indebted to Professor Jack Allen, D.Sc., F.R.S.E., M.I.C.E., of the Jackson Chair of Engineering at Aberdeen University for much helpful criticism and many useful suggestions. The Authors were assisted in the experimental tests by Mr A. F. Chalmers, B.Sc.

The 1 : 60-scale model, to which reference is made in the Paper, was sponsored by the North of Scotland Hydro-Electric Board and Sir William Halcrow & Partners for the study of certain features of a hydro-electric project. The Authors are grateful for the opportunity to make observations on this apparatus and to use them in their present study of model scale relations.

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The Paper is accompanied by two photographs and nine drawings, from which the half-tone page plate and the Figures in the text have been prepared.

Paper No. 5932

# "The Prediction of Venturi-Meter Coefficients and Their Variation with Roughness and Age"

**by**

**Stanley Peerman Hutton, M.Eng., A.M.I.C.E.**

*(Ordered by the Council to be published with written discussion)*†

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SYNOPSIS

The elementary theory of the Venturi tube is recounted in order to analyse the influence of various factors on the Venturi-meter coefficient $C$.

It is shown that these influences fall into two main classes :—

(1) "External" effects caused by upstream flow conditions and installation.
(2) "Internal" effects depending on the geometry and surface finish of the meter.

In the case of meters installed downstream of a long straight length of piping, the particular effects of roughness of the upstream piping and of the meter itself are considered.

A theory is developed which gives good agreement with code values for the effect of various sizes of rough pipe upstream of the meter. It is shown that the code values are consistent with pipe-flow theory and correspond to a roughness equivalent to cast iron. This theory may be useful for calculations involving other roughnesses and also for extrapolating beyond the range of pipe sizes (2 to 8 inches diameter) quoted by the codes. It is also shown, as in the codes, that for meters with $m \leqslant 0{\cdot}3$, changes in $C$ caused by external roughness are likely to be negligible in practice.

However, the changes caused by "internal" variations of surface roughness, which so far have not been covered by the codes, are not always avoidable and may amount to 1 or 2 per cent. It is shown that variations of $C$ with Venturi-meter size are most probably caused by changes in the effective hydraulic roughness ratio $\epsilon_v/d$ of the Venturi-meter material. With this knowledge, a semi-empirical method is developed to allow for variations in Venturi-meter size and roughness when predicting $C$ above the constancy limit. This is shown to give good agreement with experimental data for the "Herschel" type of meter.

Assuming that $C$ varies with roughness in the way suggested, a prediction is made of the variation of $C$ with time caused by surface roughening. This is shown to be in good qualitative agreement with experiment and demonstrates that a 1-per-cent decrease of $C$ in 1 or 2 years is quite possible.

More systematic experimental data are required before the general applicability of the method can be established, but it seems to explain many well-known phenomena and to offer the possibility of allowing for meter-roughness.

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## Introduction

It is well known that water pipes will, in certain circumstances, gradually lose their carrying capacity as their surface roughness increases with time

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† Correspondence on this Paper should be received at the Institution by the 15th August, 1954, and will be published in Part III of the Proceedings. Contributions should be limited to about 1,200 words.—Sec. I.C.E.

because of chemical reactions between water and pipe material,[1] or the formation of weed.[2]

The original object of this research was to investigate the possible variation of Venturi-meter coefficients caused by such effects. In so doing it became apparent that little systematic analysis had so far been made on the theoretical side, and therefore that a Paper on this subject might be of general interest and help to produce better understanding of some of the existing empirical rules for predicting Venturi-meter coefficients.

In fact, a possible method of predicting coefficients above the constancy limit has been evolved which may be of general use in allowing for the effects of various kinds of surface roughness, as well as other factors influencing $C$.

## Historical Notes

It is interesting to make a brief review of the progress made in using and predicting the performance of Venturi meters since the postulation of the operating principle by Venturi [3] in 1797. His interest seemed to be mainly academic and it was left to Clemens Herschel [4] to develop an instrument for measuring water flow, the same form of which is used to this day. In his original Paper, Herschel admitted that the meter had originally been developed by Bourdon [5] for measuring the flow of air in mines. However, Herschel can claim to be the first to use such an instrument with water and to utilize the pressure difference between main and throat for measuring purposes. Bourdon and, at first, Herschel had tried to use the suction at the throat to measure the flow. The conical- or Herschel-type Venturi meter (*Fig. 1*) has been standardized in France,[6] Britain,[7] and the United States of America,[8] but the experimental work on Venturi-meters—as compared with that on flow nozzles and orifice plates—is not yet complete and, as a result, some of the standards are at present only tentative.

Originally the theory of the Venturi meter was based on the simple one-dimensional theory of Bernoulli, which for incompressible fluids gives the formula :

$$Q = C \cdot \frac{A_2}{\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]^{\frac{1}{2}}} \cdot \sqrt{\frac{2g}{w}(P_1 - P_2)} \qquad . \qquad . \qquad (1)$$

the notation being as listed in Appendix I.

The variations from one-dimensional theory caused by changes in shape and the effects of friction, non-uniformity of velocity, and pressure and energy distributions are all included in the particular value used for the coefficient $C$.

One of the first attempts to find the factors governing $C$ and therefore to predict its value was by Gibson [9] (1915) who showed that, among

[1] The references are given on p. 234.

other things, $C$ depended upon friction losses in the cone and the type of velocity distribution at the pressure-measuring stations. The latter was really a function of the Reynolds number, although Gibson referred to it in terms of pipe velocity.

Another interesting Paper, by Pardoe [10] (1919), also attempted to predict the values of $C$, and in doing so calculated the losses in the conical contraction between the main and throat stations. However, he made no allowance for the effect of varying velocity distribution across the main and throat whereas Gibson showed that it was extremely important.

Smith [11] (1923) showed the dependence of $C$ upon viscosity for Venturi meters used for oil-flow measurement. These meters were working in the laminar-flow region where $C$ is a function of the Reynolds number only, and therefore of viscosity, pipe size, and fluid velocity.

The various factors involved, apart from surface roughness, have been more recently analysed and set out very clearly in the American Society of Mechanical Engineers' (A.S.M.E.) flow-measurement publication.[8] No attempt has been made to predict these factors, but extensive recourse has been had to Continental and American test data for producing generalized empirical values for practical use. An excellent summary of this work and the present position was given by Jorissen in 1951.[12]

In the following sections an attempt is made to predict the influence of various factors, with particular reference to roughness, this being one of the variables that has been neglected to date.

## Elementary Theory

With reference to stations 1 and 2 in *Fig. 1* and the notation in Appendix I,* the following relations can be obtained (see ref. 13) for the flow through a Venturi meter, allowing for variations in velocity and therefore kinetic energy across the sections 1 and 2 : †

$$Q = \frac{A_2}{\left[\alpha_2 + K_L - \alpha_1\left(\frac{A_2}{A_1}\right)^2\right]^{\frac{1}{2}}} \cdot \sqrt{\frac{2g}{w} \cdot (P_1 - P_2)} \quad . \quad . \quad (2)$$

where
$$\left.\begin{aligned} \alpha_1 &= 2\int_0^1 \left(\frac{v_1}{V_1}\right)^3 \cdot \frac{r_1}{R_1} \cdot d\left(\frac{r_1}{R_1}\right) \quad . \quad . \quad . \quad (3) \\ \alpha_2 &= 2\int_0^1 \left(\frac{v_2}{V_2}\right)^3 \cdot \frac{r_2}{R_2} \cdot d\left(\frac{r_2}{R_2}\right) \quad . \quad . \quad . \quad (4) \end{aligned}\right\} \text{kinetic energy coefficients to allow for uneven velocity distribution.}$$

* This is substantially the same notation as used in ref. 12.

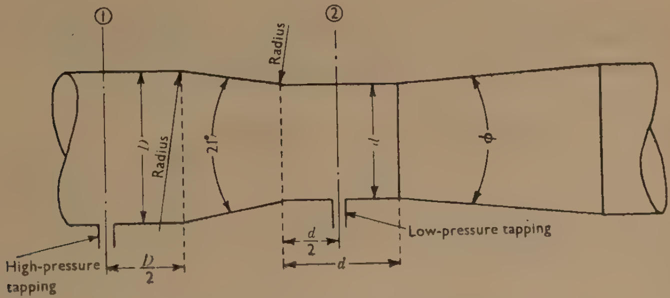
† Similar relations will also apply to the I.S.A. type Venturi meter (see *Fig. 2*)

and the nozzle loss coefficient (loss caused by friction and form resistance),

$$K_L = \frac{2g \cdot \Delta h_L}{V_2^2}.$$

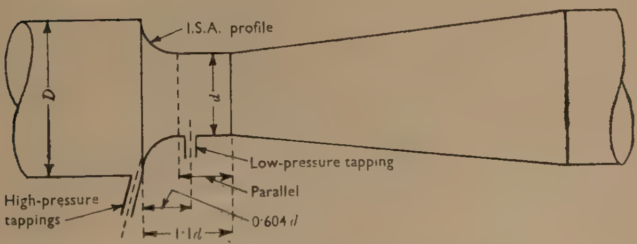
It must be remembered that, in deriving equation (2), $P_1$ and $P_2$ have been assumed to be independent of pipe radius. This only applies if

*Fig. 1*




"HERSCHEL" OR CONICAL VENTURI TUBE

*Fig. 2*




I.S.A. NOZZLE-TYPE VENTURI TUBE

stations 1 and 2 are far from any sudden change of wall curvature, so that the streamlines are parallel to the tube axis. This is usually the case in present practice for the standard conical type of tube, because both sets of pressure tappings are half the local diameter from any such change (see *Fig. 1*).

Comparing equations (1) and (2) and writing $\frac{A_2}{A_1} = m$, the expression for $C$ is:

$$C = \left(\frac{1 - m^2}{\alpha_2 + K_L - \alpha_1 m^2}\right)^{\frac{1}{2}} \qquad . \quad . \quad . \quad . \quad . \quad (5)$$

from which it is seen that $C$ is a function of :—

(1) *External factors* $\left\{ \alpha_1 \right.$ —Entry velocity distribution, dependent on installation, pipe roughness, etc.

(2) *Internal factors* $\left\{ \begin{array}{l} m \text{ —Venturi geometry: contraction ratio.} \\ K_L \text{ —Venturi entry-nozzle losses (shape, roughness, etc.).} \\ \alpha_2 \text{ —Venturi throat-velocity distribution (nozzle shape and } m\text{).} \end{array} \right.$

The relative importance of these factors is discussed in Appendix II, and in the following sections a more detailed analysis will be made to see how practical variations in $\alpha_1$, $m$, $K_L$, and $\alpha_2$ may affect $C$.

*Variations in C caused by "External" Effects* ($\alpha_1$)

The coefficient $\alpha_1$ depends on the velocity distribution at inlet to the meter, which is governed by the flow conditions upstream in the main pipe. In practice, the effect of upstream bends, valves, transitions, etc., upon the velocity distribution will have to be allowed for, and this is usually best done empirically.[13, 14] The correction factor for bends, transitions, etc., may sometimes be generalized and put into a simple form for practical use.[15] Sometimes the errors caused by such fittings are as much as 5 per cent, if the contraction ratio $m$ is large.

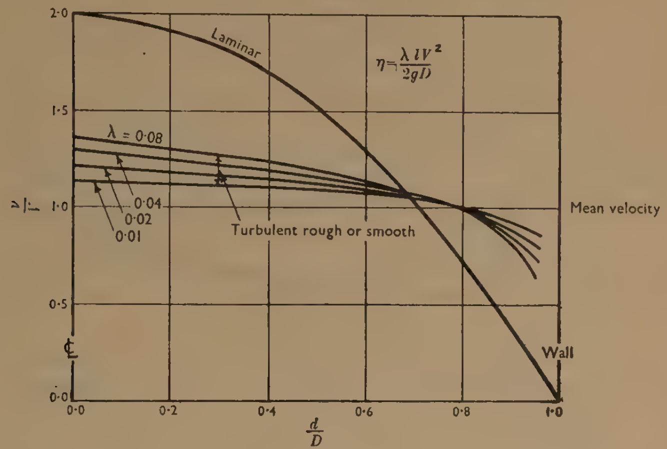
However, for the purposes of this analysis, only the ideal case will be considered where there is a long length of straight pipe upstream of the Venturi meter. This is the configuration necessary for calibration purposes in the laboratory and is also desirable in practice if high accuracy is required. With such a straight pipe it is known [16] that the turbulent velocity distribution and hence $\alpha_1$ are functions of the friction coefficient $\lambda$. Typical velocity distributions are shown in *Fig. 3* for various values of $\lambda$. The corresponding variations of $\alpha_1$ with $\lambda$ are discussed in detail in Appendix 3 and plotted in *Fig. 4*. It is seen that $\alpha_1$ increases with $\lambda$ in an approximately linear fashion for small values of $\lambda$.

Because $\lambda$ is a function of either $R_D$ or $\epsilon/D$ for the "smooth" and "rough" turbulent flow regimes respectively, alternative scales of $R_D$ and $\epsilon/D$ have been added to *Fig. 4*. For the normal range of Reynolds numbers and roughness ratios encountered in practice, $\lambda$ lies between about 0·01 and 0·4 (see Moody [19]) with corresponding values of $\alpha_1$ ranging from 1·03 to 1·11.

As $\epsilon/D$ increases, $\alpha_1$ increases, which from equation (5) means that $C$ increases. Thus, *roughening the upstream piping increases C*, at first sight a rather paradoxical finding but nevertheless borne out in practice. Although it should be well known it is doubtful that the real reason for $C$ increasing with roughness is widely understood. This is partly because the effect of $\epsilon/D$, as such, is not mentioned in the flow measurement codes. In the codes, for a particular value of $m$, the effect of roughness is allowed for

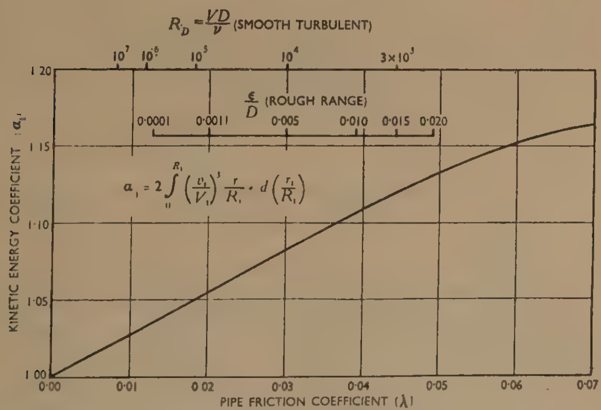
empirically by a factor depending on $D$ only, and neither the actual roughness $\epsilon$ nor the relative roughness $\epsilon/D$ are mentioned. Some of the codes give the correction for what they call a "normally rough pipe," but only

*Fig. 3*




Variation of Velocity Distribution in a Circular Pipe with $\lambda$

*Fig. 4*

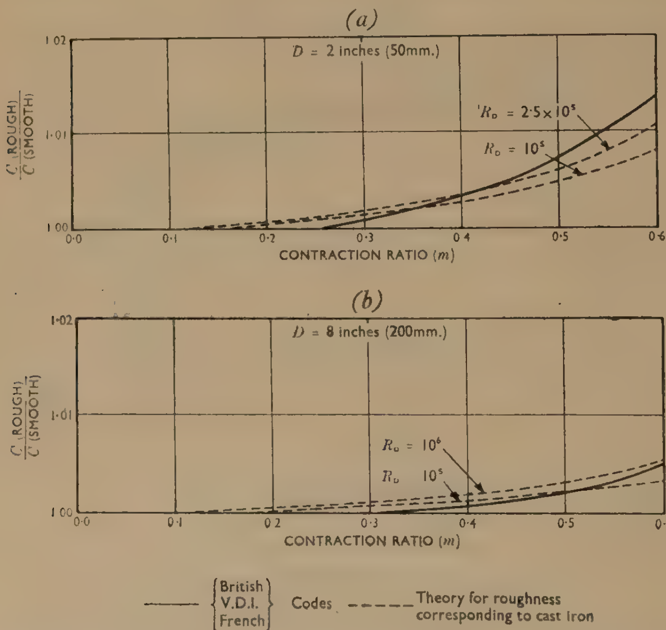



Variation of Energy Factor $\alpha_1$ with $\lambda$ for Turbulent Flow

the British Standard mentions the actual degree of roughness. The A.S.M.E. Code mentions the qualitative effect of roughness upstream but makes no attempt to allow for it quantitatively. The V.D.I.[18] and French Codes give empirical charts for allowing for roughness of upstream piping, which depend on $m$ and $D$, but it is not stated what defines a rough pipe.

The British Standard gives the same value in its correction chart but states that they apply for normal commercial clean cast-iron or mild-steel pipe, but not to nodulated or corroded ones. On this basis, using the method outlined in Appendix III, the expected increases of $C$ have been calculated, corresponding to a change from smooth to cast-iron pipe.

*Figs 5*




Comparison of Code and Theoretical Increase of $C$ caused by Rough Piping Upstream of Meter (2-inch- and 8-inch-diameter pipes)

These calculated increases for typical Reynolds numbers are compared with the code values of 2-inch- and 8-inch-bore pipes in *Figs 5* (*a*) and (*b*). It is seen that the theoretical values for the conditions quoted in Appendix III give fair agreement with those recommended by the British, French, and German Codes. It therefore appears that the code figures are consistent with the simple theory outlined in Appendix III and that their so-called rough-pipe values correspond to the order of roughness of cast iron.

Bearing in mind that, in normal practice, the roughness ratio is not likely to exceed that of a 2-inch-diameter cast-iron pipe, it is seen, both theoretically and from the codes (*Fig. 4*), that changes of $C$ caused by upstream roughness can be made negligible by keeping $m$ less than 0·3.

It is not worthwhile making any more detailed comparisons, because

practical variations of the Reynolds number considerably affect the corrections, as seen in *Figs 5*. However, it does seem that the theory outlined might be used to predict roughness changes for pipes outside the range of sizes and material given by the codes.

*Variations in C caused by "Internal" Effects*

Of the so-called "internal" factors the only one which is directly under the control of the manufacturer is the contraction ratio, $m$. From equation (13) (Appendix II), and a consideration of the normal values of $\alpha_1$ and $C$ obtained in practice, it will be found that changes in $C$ caused by variations in $m$ should be very small [20] (because $1 - \alpha_1 C^2 \doteqdot 0$). This explains why it is usually possible to manufacture Venturi meters to within such close limits on $m$ that associated errors in $C$ are negligible.

There is little experimental evidence regarding the value of $\alpha_2$ because the throat velocity distribution in a Venturi meter is seldom measured. Coleman [21] (1907) is one of the few to have made such measurements for conical meters and his velocity distributions give $\alpha_2 = 0{\cdot}996$ and $0{\cdot}990$ for meters measuring 10 inches by 3·6 inches and 6 inches by 3 inches respectively. More recently, Purdy [22] made measurements in the throat of an 18-inch-by-6-inch water-tunnel which was similar to a Venturi meter, and found that $\alpha_2 = 1{\cdot}002$. Foster's velocity profile [23] measured in the throat of a 50-inch-by-28-inch Venturi meter gives $\alpha_2 = 1{\cdot}017$.

This evidence suggests that, provided $m$ is small ($\leqslant 0{\cdot}30$) and the nozzle is smoothly faired into the throat, the practical values of $\alpha_2$ are very close to 1. It therefore seems justifiable to make the assumption, used henceforth, that $\alpha_2 = 1$ and, because of the small variations of $\alpha_2$ in practice, that the associated variations in $C$ are negligible.

Thus, in order to predict $C$ from equation (5), it only remains to know the value of $K_L$.

*Predicting $K_L$ by "Equivalent Pipe" Theory*

In order to predict $K_L$, the mechanism of entry-nozzle losses must be studied in a little more detail. The analysis is restricted to the conical type of meter because there are more experimental data available for the effect of size on $C$ for this simple shape.

From *Fig. 1* it is apparent that between the pressure measuring stations 1 and 2 there are three distinct parts of the meter (ignoring the faired transition curves between cone and cylinder) :—

Part ($a$). Cylindrical pipe, diameter $D$, length $\frac{D}{2}$.

,, ($b$). Conical pipe tapering from $D$ to $d$.

,, ($c$). Cylindrical pipe, diameter $d$, length $\frac{d}{2}$.

It is difficult to calculate accurately the losses in parts ($b$) and ($c$)

because of the pressure gradients caused by the convergence and the resulting doubt about the boundary-layer growth. All that can be said is that the convergence will tend to increase the skin-friction coefficient above the normal pipe-flow values, and that most of the friction loss will occur in the narrow end of the nozzle and in the throat where the velocities are highest. Several attempts have been made to calculate the losses in the conical contraction but until a complete allowance can be made for the effects of convergence, no reliance can be placed upon the results.

For practical use, it is more reliable to make an analysis of experimental data and thus obtain an empirical formula for predicting $K_L$. For this purpose, the Author has found it convenient to refer the nozzle losses to those in an equivalent pipe having the same diameter $d$ as the throat of the meter. Thus, if the total head loss is written as $\Delta h_L$ (between stations 1 and 2):

$$\sum_{1}^{2} \frac{\lambda \,.\, \Delta x \,.\, v^2}{2gd} = \Delta h_L = \lambda \,.\, \frac{x}{d} \,.\, \frac{V_2^2}{2g} \quad . \quad . \quad . \quad . \quad (6)$$

(Venturi meter) (equivalent pipe)

This can be written non-dimensionally as:

$$K_L = \frac{\Delta h_L}{V_2^2/2g} = \lambda \,.\, \frac{x}{d}^{*} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Then all that must be known to predict $K_L$ is the length $x$ of equivalent pipe, diameter $d$, giving the same loss as the Venturi meter between pressure measuring stations 1 and 2. This assumes that the friction and kinetic losses can be included under one friction coefficient $\lambda$. Since the kinetic losses should be relatively small compared with the friction losses in such a convergence, this would seem to be justifiable as a first approximation. It is also borne out by the good results obtained in practice, as will be seen later.

By analogy with pipe flow it would be expected that $K_L$ and therefore $C$ both vary with Reynolds number ($R_d$) and roughness ratio ($\epsilon_v/d$), where $\epsilon_v$ denotes the effective roughness † of the meter. It will now be shown that this is borne out in practice. $C$ certainly varies with $R_d$, and the variation with $\epsilon_v/d$ can happen in two ways :—

(*a*) For a meter of fixed size, $C$ would be expected to vary with surface material $\epsilon_v$. Beitler,[24] Witte,[25] and others have demonstrated the large variations in Venturi-nozzle coefficients caused by roughening the nozzle surface. For Venturi meters,

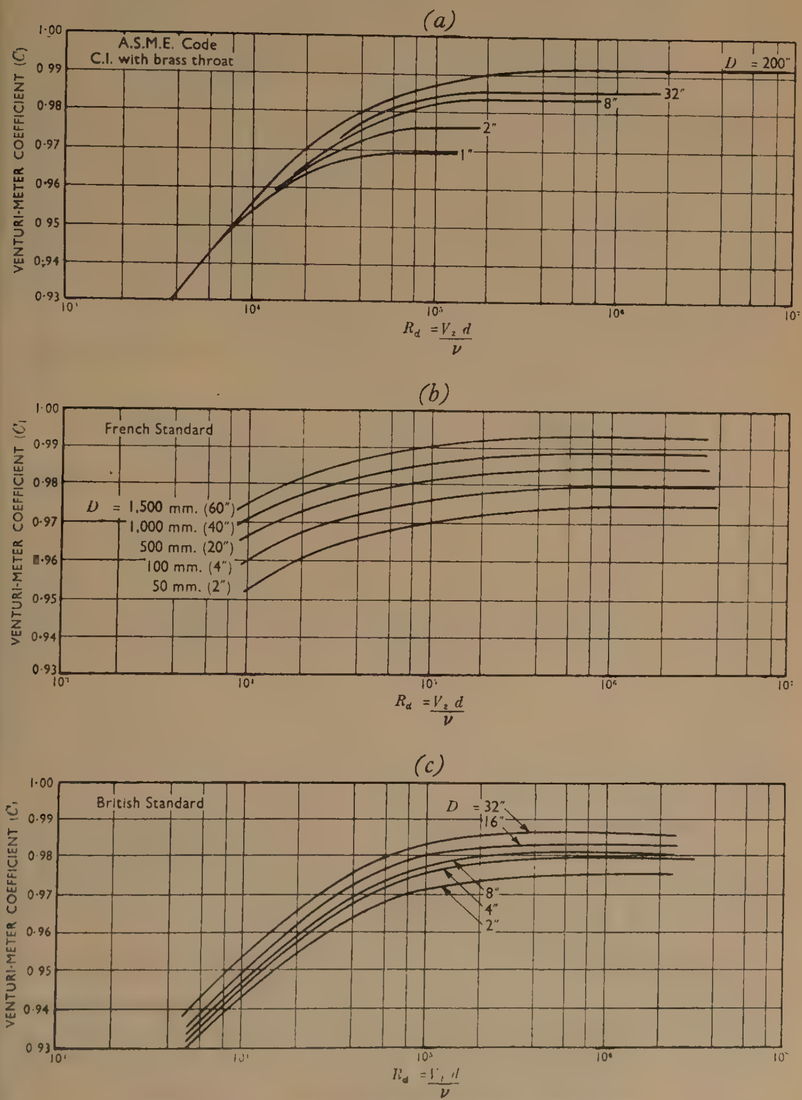
* A similar result can be obtained by dimensional analysis.

† This must not be confused with the previous "external" roughness factor giving variations of $C$ with roughness of the upstream pipe (cf. $\epsilon/D$ and $\epsilon_v/d$). Also the throat diameter is now used for Reynolds number $R_d$, and roughness ratio $\epsilon_v/d$, because the entry-nozzle losses are more influenced by the narrow end of the nozzle.

Schlag [26] has shown that machining the nozzles of bronze and cast-iron meters increases $C$ by $\frac{1}{2}$ to 1 per cent.

(*b*) For a family of geometrically similar meters ($m$ = constant) made of the same material ($\epsilon_v$ constant) the roughness ratio

*Figs 6*




Comparison of Variations of $C$ with Size and Reynolds Number for a Conical Venturi Meter ($m = 0{\cdot}25$)

$\epsilon_v/d$ will vary with diameter $d$. Reliable experimental values for such a case exist for Herschel-type Venturi meters ($m = 0{\cdot}25$) having cast-iron nozzles and bronze throat liners. The figures are published by the A.S.M.E.[8] and are based on Pardoe's original systematic tests.[27] The variations of $C$ with $R_d$ for various sizes of meter given in reference 8 are shown in *Fig. 6* (*a*). It is quite clear that, in addition to varying with $R_d$, $C$ varies with $d$, which supports the above assumptions for $K_L$.

In addition, the British [7] and French [6] Codes include tentative standards for conical meters which also give recommended variations of $C$ with size and Reynolds number. These are reproduced in *Figs 6* (*b*) and (*c*) for comparison with the American data (*Fig. 6* (*a*)). It will be seen that there is an important difference between the American, and French, and British standards. At Reynolds numbers of less than $10^4$, the American values of $C$ are independent of size, whereas the British, and to a much greater extent the French, vary with size in this range.

It would be expected, by analogy with pipe-flow laws, that the variations of $C$ can be divided into three distinct zones :—

(1) $C$ varies with $R_d$ only (independent of size and roughness).
(2) $C$ varies with $R_d$ and roughness (in this case, size).
(3) $C$ varies with roughness only (independent of $R_d$, varies with size).

*Fig. 6* (*a*) certainly shows these zones, but *Figs 6* (*b*) and (*c*) show only zones (2) and (3).

However, at this stage, the primary consideration is zone (3), where $C$ is independent of Reynolds number and in which zone Venturi meters are most used in practice. A comparison is made of the different variations of $C$ with $d$ given by the three codes and Pardoe's experiments. This comparison is made for $R_d = 10^6$, which is just above the constancy limit for all sizes of meter and is as high as the practical range of velocities will permit for the small meters. It is used henceforth as a convenient Reynolds number for comparing all sizes of meter.

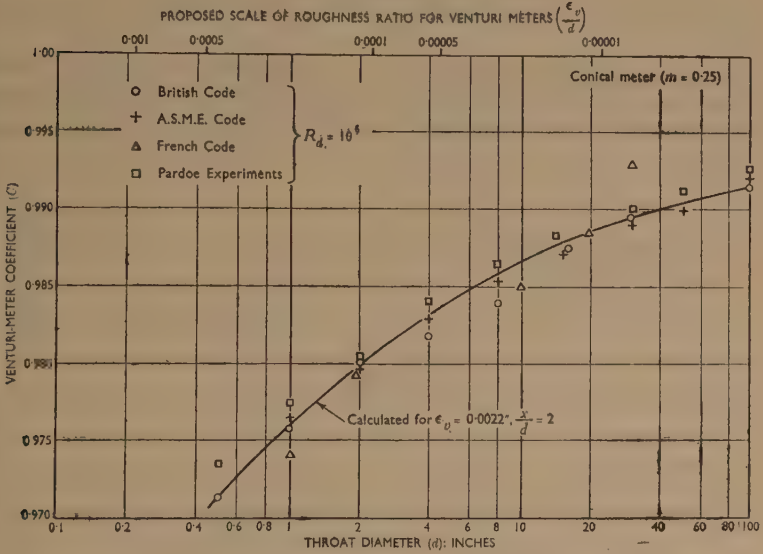
The comparison is shown in *Fig. 7*, where it is seen that, bearing in mind the large scale for $C$, there is little difference between the various data except for the largest diameter (30 inches) point of the French Code. This last point has also been shown to be inconsistent by others (see reference 34) and no great reliance is placed upon it.

A recent survey by Jorissen [34] suggested slightly modified values for the variation of $C$ with $d$. His values (see *Fig. 17* of reference 34) lie above those in *Fig. 7* of this Paper by about $\frac{1}{4}$ per cent of $C$, and were based upon an analysis of experimental results collected by the A.S.M.E. for about 200 conical Venturi meters. In the light of suggestions made in the discussion on his Paper, Jorissen was led to modify his original values for large Venturi meters in order to make them more generally acceptable.

In the same discussion, other proposals were made, notably by Schlag, which seemed to have more general justification than Jorissen's, whose analysis was restricted to American data.

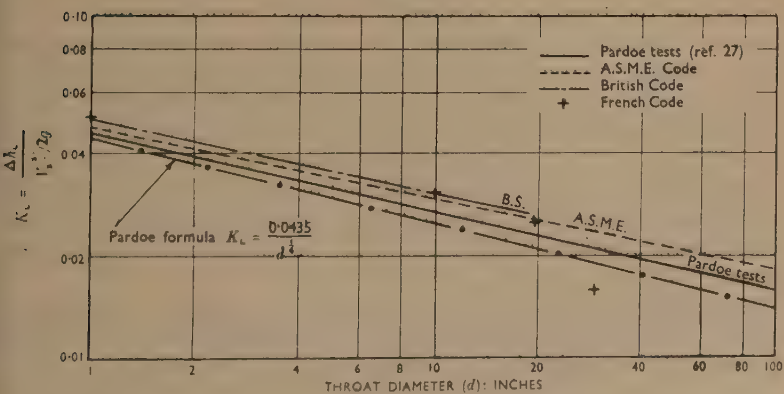
It is obviously too early to make any final pronouncement on the most likely values to use in practice and therefore the present Paper considers

*Fig. 7*




PRACTICAL AND CALCULATED VARIATION OF $C$ WITH THROAT DIAMETER

*Fig. 8*



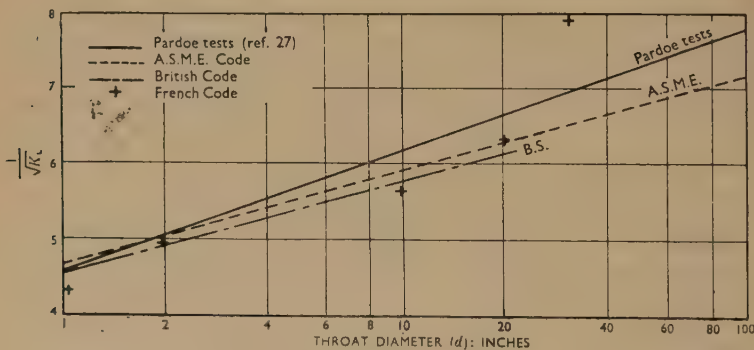

VARIATION OF NOZZLE-LOSS COEFFICIENTS WITH DIAMETER, AS OBTAINED FROM VARIOUS SOURCES

only the code values for $C$, because these are the only generally accepted values now in use. They may be modified later but only by very small amounts which are not likely to affect the argument or the applicability of the theory suggested in this Paper.

These code values of $C$ (at $R_d = 10^6$) are used, as described in Appendix 4, for calculating the corresponding values of $K_L$ from equation (5). An analysis of these values yields an empirical relation for calculating $K_L$ in terms of an equivalent pipe on the basis of existing roughness formulae.

It is shown that the variations of $K_L$ with $d$ are so similar to those expected from " equivalent-pipe theory " and the pipe-roughness formulae of both Fromm [28] (*Fig. 8*) and Von Karman [16] (*Fig. 9*) that it seems most

*Fig. 9*




VARIATION OF $\frac{1}{\sqrt{K_L}}$ WITH log $d$

probable that the apparent variations of $K_L$ and $C$ with $d$ are really variations with hydraulic roughness $\epsilon_v/d$. Not only are the trends consistent with pipe-roughness laws but the empirical constants are of the same order. In fact, for the Venturi meters quoted, it seems that the values of $K_L$ (and thus $C$) can be predicted with good accuracy by equation (7) if the values $\epsilon_v = 0{\cdot}0022$ and $x/d = 2$ are used in conjunction with the Karman roughness formulae for $\lambda$. The $C$ values predicted in this way have been plotted as a full line curve in *Fig. 7*, where it is seen that the agreement between calculated and measured values is excellent, the maximum deviation being less than $\pm$ 0·1 per cent.

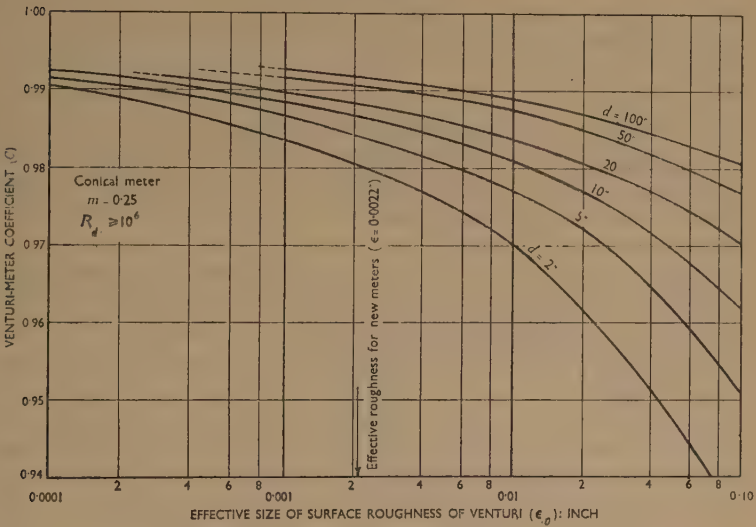
Having shown that $C$ can be thus related to $\epsilon_v$ and $\epsilon_v/d$, an alternative roughness scale has been added to *Fig. 7*, giving the variation of $C$ with $\epsilon_v/d$, based on the previously found mean value of $\epsilon_v = 0{\cdot}0022$.

From this curve of $C$ against $\epsilon_v/d$, a family of curves for various sizes of meter can be plotted, showing the variations of $C$ with $\epsilon_v$, the effective meter roughness. This is shown in *Fig. 10* where, for example, it can be

seen that if the roughness of a 10-inch-by-5-inch meter increases from $\epsilon_v = 0{\cdot}001$ inches to 0·01 inch, then $C$ will decrease from 0·987 to 0·977. In general, it is seen that $C$ decreases with increasing meter roughness and that the decrease is greater for small meters than for large ones.

These constants will probably need modification as more experimental

*Fig. 10*




VARIATION OF METER COEFFICIENT WITH SURFACE ROUGHNESS

data become available on conical and other types of meter, but the theory is useful in relating $C$ to meter roughness.

*Variations of C with Roughness and Time*

It is shown in Appendix III that $C$ can be affected by variations in the roughness of the upstream piping, and in Appendix IV it is demonstrated that $C$ is influenced by internal surface roughness in a manner consistent with existing pipe-flow laws. If the effective surface roughness of the meter is known, then it has been shown possible to predict the value of $C$.

The problem which initially stimulated this analysis was that of Venturi-meter coefficients changing with time, and the argument has now reached a stage when this can be considered in more detail. It must be remembered, however, that although the "internal" and "external" effects of roughening act in opposite directions, the latter is usually negligible compared with the former and so the net result in practice is usually that $C$ decreases with time.

References 24 and 25 demonstrate the effect of controlled roughness

changes on nozzle coefficients, and Schlag [26] showed that machining a cast Venturi-meter nozzle could increase $C$ by 1 per cent. The more natural and uncontrollable problems of roughening caused by rust, corrosion, and nodulation have not yet been studied so carefully, although Schlag [26] and Camichel [31] have demonstrated that rust can decrease $C$ by 1 per cent in a few weeks.

Allen [32] and Pardoe [33] have both quoted cases where the coefficients of conical Venturi meters dropped steadily with time (about 1 per cent decrease in 3 years) and when cleaned, the meter coefficients went back to their initial values.

As regards nodulation, the Author is indebted to Professor H. Gerber for information on a hydro-electric scheme in the Tyrol where the water was particularly hard. Because of the nature of the installation it was not possible to get absolute values but, by using current meters in the penstock to calibrate the Venturi meter, a change in $C$ of 4 per cent in a few years was apparent.

Linford [35] has shown that the growth of weed on the piping can also affect Venturi-meter coefficients, and quantitative data in the same effect in water piping are given by Kruger.[2] All these references are examples of the various ways in which the effective surface roughness can be altered in time by natural causes. Such changes are necessarily difficult to correlate, and the only type of change which has been tackled quantitatively is that of nodulation. Colebrook and White [1] have shown that the increasing surface roughness of water pipes caused by the deposition of lime or other nodules can be represented by an empirical formula of the type :

$$\epsilon_T = \epsilon_0 + \gamma T \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where $\epsilon_0$ denotes the initial pipe roughness,
$\epsilon_T$ ,, the roughness at time $T$ years after installation, and
$\gamma$ ,, a constant depending on the pipe material and the $pH$ value of the water.

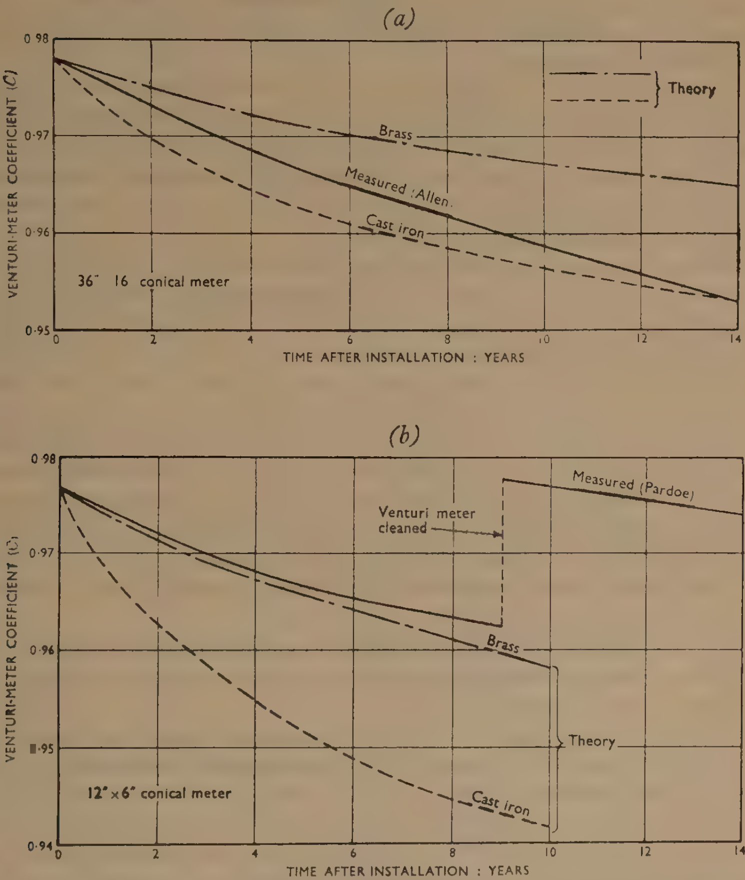
From an analysis of British and American data, Colebrook and White [1] give a value $\gamma = 0{\cdot}025$ inch per annum for the roughening of cast-iron pipes in water of average alkalinity ($pH = 7$).

Therefore, on this basis, it should be possible to estimate the increasing values of $\epsilon_v$ with time for Venturi meters and hence, by the simple theory given in section (2) of Appendix IV, to calculate the corresponding variation of $C$ with time.

There are little quantitative experimental data available on this aspect, largely because it is not often possible to calibrate Venturi meters after installation. However, Allen [32] describes such changes in connexion with a 36-inch-by-16-inch conical Venturi meter installed in the Alden Hydraulic Laboratory in the United States. Calibrations were made over a period of 30 years and it was shown that the values of $C$ were related to the state of surface of the Venturi meter. For instance, it was found that $C$ always

decreased with time, and after the surface was cleaned $C$ immediately increased again. The measured decrease with time after cleaning is shown in *Fig. 11* (*a*). The initial value of $C = 0{\cdot}978$ compared with code values of 0·987 for such a meter indicates that the surface roughness after cleaning ($T = 0$, *Fig. 11* (*a*)) did not correspond to that of a new meter. In fact,

*Figs 11*




COMPARISON BETWEEN MEASURED AND CALCULATED CHANGE OF COEFFICIENT WITH TIME AFTER INSTALLATION

a calculation shows that, to give a coefficient of 0·978, the value of $\epsilon_0$ must have been 0·029 inch—that is, ten times that of new cast iron. This is quite possible because the time zero of *Fig. 11* (*a*) (after cleaning) is actually 27 years after the meter was originally installed.

Using this initial value of $\epsilon_0 = 0{\cdot}029$ inch so as to give the measured

value of $C$, the theoretical changes of $C$ with time have been calculated from equations (5), (8), and (27). Because the meter has a cast-iron nozzle and a brass throat it is difficult to say which is the better representative material, and so two calculations have been made, one using $\gamma$ for cast iron and the other that for brass. For the roughening of brass surfaces, unpublished data from the National Bureau of Standards have been used, which suggested a value of $\gamma$ about one-third that of cast iron. Thus, instead of $\gamma = 0{\cdot}025$ inch per annum for cast iron, a value for brass of $\gamma = 0{\cdot}008$ has been used.

Using the values of $\epsilon_v$ resulting from equation (8) in equation (7) gives the variation of $K_L$ with time which can then be used in equation (5) to give $C$ as a function of time. In equation (5), typical values of $\alpha_1 = 1{\cdot}04$ and $\alpha_2 = 1{\cdot}00$ were used as before. The resulting theoretical variations of $C$ for 36-inch-by-16-inch brass and cast-iron meters are shown for comparison with the measurements in *Fig. 11* (*a*).

In the discussion on Allen's Paper, Pardoe [33] also gives some experimental data for the change in coefficient with time for a 12-inch-by-6-inch conical Venturi meter installed in the University of Pennsylvania Hydraulic Laboratory. Immediately after installation, the meter gave a coefficient of 0·977 as opposed to 0·984 given by the A.S.M.E. Code,[8] once again indicating a rougher surface than for the meters quoted in the Code. The measured variation of $C$ with time is plotted in *Fig. 11* (*a*), and clearly shows how the coefficient returned to its initial value immediately after cleaning. Calculations of changes of $C$ with time for brass and cast-iron 12-inch-by-6-inch meters have been made in a similar way to those for the 36-inch-by-16-inch meter. The initial roughness was found to be $\epsilon_o = 0{\cdot}014$ inch, which is nearer to that of cast iron than the 36-inch meter. The resulting curves are plotted in *Fig. 11* (*b*).

It is seen from *Figs. 11* (*a*) and (*b*) that calculated and measured trends are similar, all curves showing a steady decrease of $C$ with time at a rate between 0·2 and 1 per cent per year.

Because of the lack of detailed information about the shape and roughness of the Venturi meters and of the assumptions about water quality and rates of roughening made in the calculations, it is not justifiable to make any detailed analysis of *Figs 11* (*a*) and (*b*). However, certain trends are apparent which can be expected to be generally true. For instance, it is seen that, because of the higher rate of nodulation, the coefficients of cast-iron meters decrease more rapidly than brass ones for the same quality of water. Also, for Venturi meters made of the same material, changes of $C$ with time are larger for the smaller meters because, with similar changes in $\epsilon_v$, the relative changes $\epsilon_v/d$ with time are much greater for small meters.

All these calculations have been made for meters which were relatively rough to begin with. If the value of $\epsilon_v = 0{\cdot}0022$ inch, as recommended in Appendix IV from an analysis of the codes, had been used, the calculated

changes would have been greater than those of *Figs 11* (*a*) and (*b*), particularly in the first year or two after installation.

## Conclusion

The simple type of theory developed gives correct trends and fair agreement for the few cases quoted. It serves to explain many phenomena encountered in practice and illustrates the order of changes in $C$ which can be caused by variations in surface roughness of the Venturi meter itself, a factor which has not been dealt with until now.

Such changes have been shown to be appreciable and may account for some of the scatter obtained by experimenters in the past, because variations with roughness and time have always been neglected. To obtain consistent results it would be better to use meters having standard surface finishes, preferably chemically inert and highly resistant to changes with time. Even so, chemical deposition from the water or the formation of bacteria films may still prove troublesome.[35]

The changes of $C$ with time are likely to be important in test and research laboratories and, where high accuracy of flow measurement is required, due care must be taken. In certain industrial cases such as boiler feed-water, heating and cooling circuits, and water supply-schemes where the water is hard, it is quite likely that the calibration of Venturi meters, particularly small ones, will have changed by a considerable percentage in a few years. Although it is therefore desirable to recalibrate such meters periodically, this is often inconvenient, and all that can be done is to make some allowance for " ageing " as suggested in this Paper. Even with its present limitations, the semi-empirical method suggested is likely to give more reliable values of $C$ a few years after installation than by using the original $C$ value of the meter when new. For large meters and where the water quality is good, changes in $C$ with time may not be important.

Further systematic experimental data are required before the general applicability of the method can be established, and it is hoped that carefully controlled roughness experiments will be made to establish the best values of $\epsilon_v$ and $x/d$ to use. However, the theory does offer a possibility of calculation where previously none existed, and gives reasonable agreement with the little experimental data available on conical meters. There seems to be no reason why a similar approach cannot be used for I.S.A.-type meters when sufficient experimental evidence is available.

## Acknowledgements

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The Paper is accompanied by eight sheets of diagrams, from which the Figures in the text have been prepared, and by the following Appendices.

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## APPENDIX I

### Notation

| Symbol | Description | Relation | Units |
|---|---|---|---|
| $A_1$ | Cross-sectional area at station 1 | $A_1 = \frac{\pi D^2}{4}$ | square feet |
| $A_2$ | ,, ,, ,, 2 | $A_2 = \frac{\pi d^2}{4}$ | square feet |
| $C$ | Coefficient of Venturi meter . | $Q = \frac{CA_2}{\sqrt{1-m^2}}\sqrt{\frac{2g(P_1-P_2)}{w}}$ | . . |
| $d$ | Throat diameter (station 2) and diameter of equivalent pipe. | | feet or inches |
| $D$ | Main pipe diameter (station 1) | | feet or inches |

## APPENDIX I—(*continued*)

### Notation

| Symbol | Description | Relation | Units |
|---|---|---|---|
| $g$ | Acceleration due to gravity . | $g = 32{\cdot}2$ ft/sec.$^2$ | ft/sec.$^2$ |
| $h$ | Head loss in a length of pipe . | | feet of water |
| $h_1$ | Mean total head at station 1—main. | | feet of water |
| $h_2$ | Mean total head at station 2—throat. | | feet of water |
| $h_L$ | Total head loss between stations 1 and 2. | | feet of water |
| $k'$ | Surface roughness constant (Fromm). | | — |
| $K_L$ | Mean nozzle-loss coefficient (stations 1–2). | $K_L = \dfrac{h_L}{V_2^2/2g}$ | — |
| $l$ | Length of pipe . . . . . | | |
| $m$ | Contraction ratio . . . . | $m = (d/D)^2 = \beta^2$ | — |
| $P_1$ | Mean static pressure at station 1 | | lb per sq. foot |
| $P_2$ | Mean throat static pressure (station 2). | | lb. per sq. foot |
| $Q$ | Volumetric flow. . . . . | $Q = A_2V_2$ | cusecs |
| $r$ | Local radius . . . . . . | | feet |
| $R_1$ | Main pipe radius (station 1) . | $R_1 = D/2$ | feet |
| $R_2$ | Throat radius (station 2) . . | $R_2 = d/2$ | feet |
| $R_D$ | Main pipe Reynolds' number (station 1). | $R_D = V_1D/\nu$ | — |
| $R_d$ | Throat Reynolds' number (station 2). | $R_d = V_2d/\nu$ | — |
| $T$ | Time . . . . . . . . . | $\epsilon = \epsilon_0 + \gamma T$ | years |
| $v$ | Local velocity . . . . . . | | ft/sec. |
| $V_1$ | Mean velocity in pipe . . . | $V_1 = Q/A_1$ | ft/sec. |
| $V_2$ | Mean velocity in throat . . | $V_2 = Q/A_2$ | ft/sec. |
| $w$ | Specific weight of water . . | $w = 62{\cdot}4$ lb./ft$^3$ | lb. per cubic foot |
| $x$ | Length of "equivalent pipe". | | feet |
| $\alpha_1$ | Kinetic energy coefficient at station 1. | $\alpha_1 = 2\int_0^1 \left(\dfrac{v_1}{V_1}\right)^3 . \dfrac{r_1}{R_1} . d\left(\dfrac{r_1}{R_1}\right)$ | — |
| $\alpha_2$ | Kinetic energy coefficient at station 2. | $\alpha_2 = 2\int_0^1 \left(\dfrac{v_2}{V_2}\right)^3 . \dfrac{r_2}{R_2} . d\left(\dfrac{r_2}{R_2}\right)$ | — |
| $\beta$ | Diameter ratio . . . . . | $\beta = d/D$ | — |
| $\gamma$ | Coefficient for roughness increase with time. | $\epsilon = \epsilon_0 + \gamma T$ | inches per annum |
| $\epsilon$ | Effective size of surface roughness of pipe. | | inches |
| $\epsilon_v$ | Effective size of surface roughness of Venturi. | | inches |
| $\phi$ | Total angle of conical diffuser. | | degrees |
| $\nu$ | Kinematic viscosity of water . | | ft$^2$/sec. |
| $\lambda$ | Pipe friction coefficient. . . | $h = \dfrac{\lambda . l . V^2}{2gd}$ | — |

## APPENDIX II

### Relative Importance of Factors Influencing $C$

Considering the variations of $C$ with the factors in equation (5), differentiating together with the approximation that $C = 1$, gives :

$$\frac{\partial C}{\partial \alpha_1} = \frac{C^3}{2} \cdot \frac{m^2}{1 - m^2} \simeq \frac{1}{2} \cdot \frac{m^2}{1 - m^2} > 0 \quad . \quad . \quad (9)$$

$$\frac{\partial C}{\partial m} = - \frac{mC}{1 - m^2} \cdot (1 - \alpha_1 C^2) \simeq - \frac{m}{1 - m^2} \cdot (1 - \alpha_1 C^2) > 0$$

usually $\alpha_1 C^2 > 1$ (10)

$$\frac{\partial C}{\partial K_L} = - \frac{C^3}{2} \cdot \frac{1}{1 - m^2} \simeq - \frac{1}{2} \cdot \frac{1}{1 - m^2} < 0 \quad . \quad (11)$$

$$\frac{\partial C}{\partial \alpha_2} = - \frac{C^3}{2} \cdot \frac{1}{1 - m^2} \simeq - \frac{1}{2} \cdot \frac{1}{1 - m^2} < 0 \quad . \quad (12)$$

Thus it can be said that

as $\alpha_1$ increases $C$ increases (uneven entry velocity distribution increases $C$)
as $m$ ,, $C$ ,, (effect is small, as will be seen later)
as $K_L$ ,, $C$ decreases (increasing nozzle loss decreases $C$)
as $\alpha_2$ ,, $C$ ,, (uneven throat velocity distribution decreases $C$)

All the above trends are borne out in practice, as will be seen from references 5, 6, 7, 8, and 12.

From equations (9), (10), (11), and (12) it is apparent that, to minimize all these effects upon $C$, $m$ should be as small as possible and it is usual to try and keep $m$ below about 0·3.

Comparing the relative importance of the various effects, denoted by $E$ with the appropriate subscript, from equations (9), (10), (11), and (12) the following ratios are obtained :—

$$\frac{E_m}{E_{\alpha_1}} = - \frac{2}{m}(1 - \alpha_1 C^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$\frac{E_{K_L}}{E_{\alpha_1}} = - \frac{1}{m^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$\frac{E_{\alpha_2}}{E_{\alpha_1}} = - \frac{1}{m^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Taking as a typical example a meter having $m = 0{\cdot}25$, which is a commonly used value in practice, $\alpha_1$ can lie between 1·02 and 1·04, and $C$ between 0·975 and 0·990, depending on size and Reynolds number, and so (13), (14), and (15) give :

$$- 0{\cdot}16 < \frac{E_m}{E_{\alpha_1}} < + 0{\cdot}16 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$\frac{E_{K_L}}{E_{\alpha_1}} = \frac{E_{\alpha_2}}{E_{\alpha_1}} = - 16 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Thus it is seen that the effect of $m$ is less than $\alpha_1$, which in its turn is much less than the effects of $K_L$ and $\alpha_2$ (which are sixteen times $\alpha_1$ and in the opposite direction).

These orders of magnitude may be taken as typical of those generally found in practice.

## APPENDIX III

### Theoretical Relations for $\alpha_1$ and $C$

For laminar flow ($R_D < 2{,}000$), the velocity distribution is of the classical Poiseuille type and is parabolic with the maximum velocity twice the mean. Such a distribution

is shown in *Fig. 3* and $\alpha_1$ has a constant value of 2 for all Reynolds numbers within the laminar range.

For turbulent flow, whether in the "smooth" or "rough" ranges, it has been shown [16, 17] that the velocity distribution in a circular pipe can be approximated by the expression :

$$\frac{v}{V} = 1 + 2{\cdot}5\sqrt{\frac{\lambda}{8}}\left[\frac{3}{2} + \log\left(1 - \frac{r}{R}\right)\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where $\lambda$ is the pipe friction coefficient defined by $\lambda = \dfrac{2ghD}{lV^2}$.

Examples of this type of velocity distribution for turbulent flow are given in *Fig. 3* for comparison with the laminar case.

Equation (18) does not hold very near the wall where $r/R \to 1$ but the errors involved are usually neglected. The expression satisfies the flow integral in spite of the difficulties near the wall and, when used in equation (3), gives :

$$\alpha_1 = 1 + 2{\cdot}93\lambda - 1{\cdot}55\lambda^{3/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

which is the same expression obtained by Streeter.[17] The resulting variation of $\alpha_1$ with $\lambda$ is plotted in *Fig. 4*.

Changes of $C$ caused by changes in $\alpha_1$ are related by equation (9), which can be rewritten as :

$$\frac{\partial c}{c} = \frac{1}{2} \cdot \frac{m^2}{1 - m^2} \cdot \partial\alpha_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

A suitable linear approximation to equation (19) and the curve in *Fig. 4*, for small values of $\lambda$, is

$$\partial\alpha_1 = 2{\cdot}5 \,.\, \partial\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

which can now be substituted in equation (20) to give :

$$\frac{\partial c}{c} = \frac{1{\cdot}25m^2}{1 - m^2} \,.\, \partial\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Thus, by equation (22), the percentage increase in $C$ can be calculated approximately for any increase $\partial\lambda$ from that of a smooth pipe. This increase may arise in practice because either (1) the Venturi meter is installed in a rougher circuit than that in which it was calibrated ; or (2) the circuit, although initially smooth, may increase its roughness with time.

As an example, taking the extremes of upstream pipe bore quoted in the codes (2 inches and 8 inches) it is possible, from reference 19, to make a comparison between $\lambda$ for smooth pipes and rough ones corresponding to cast iron (for which Moody [19] gives $\epsilon = 0{\cdot}01$ inch). The values of $\lambda$ are compared in Table 1 for Reynolds numbers above the constancy limit of the Venturi meters but at the same time giving reasonable water velocities as would be used in practice.

TABLE 1

| | | 2-inch-dia. cast-iron pipe | | | 8-inch-dia. cast-iron pipe | | |
|---|---|---|---|---|---|---|---|
| $R_D$ | Smooth | $\epsilon/D$ | $\lambda_{CI}$ | $\partial\lambda = \lambda_{CI} - \lambda_{SM}$ | $\epsilon/D$ | $\lambda_{CI}$ | $\partial\lambda = \lambda_{CI} - \lambda_{SM}$ |
| $10^5$ | 0·018 | 0·005 | 0·031 | 0·013 | 0·00125 | 0·023 | 0·005 |
| $2{\cdot}5 \times 10^5$ | 0·015 | 0·005 | 0·030 | 0·015 | | | |
| $10^6$ | 0·012 | | | | 0·00125 | 0·020 | 0·008 |

Using equation (22) and the $\partial\lambda$ values of Table 1, the corresponding changes in $C$ have been computed and are compared in *Figs 5* (*a*) and (*b*) with the recommended code values for 2-inch and 8-inch pipes.

## APPENDIX IV

### Nozzle Losses in the "Rough" Regime ($R_d = 10^6$)

(1) *$K_L$ Values using the Fromm Roughness Formula*

The practical variations of $C$ with $d$ plotted in *Fig. 7* may now be used to calculate corresponding curves of $K_L$ against $d$. This is done by using equation (5) which, when re-written, gives $K_L$ in terms of $C$.

$$K_L = \frac{1 - m^2}{C^2} + \alpha_2 - \alpha_1 \cdot m^2 \quad \text{(cf. equation (5))}$$

In the above equation it has been assumed that $\alpha_2 = 1{\cdot}00$, and that $\alpha_1 = 1{\cdot}04$ which corresponds to the chosen Reynolds number ($R_d = 10^6$, giving $R_D = 0{\cdot}5 \times 10^6$ and hence $\alpha_1 = 1{\cdot}04$ from *Fig. 4*).

The resulting values of $K_L$ are plotted against $d$ in *Fig. 8*, where it is seen that log $K_L$ against log $d$ gives a straight line in all cases (ignoring the last point of the French Code because of its apparent inconsistency). This agrees with the empirical formula for $K_L$ quoted by Pardoe (in the discussion on reference 14) which gives $K_L = \dfrac{0{\cdot}0435}{d^{\frac{1}{4}}}$ where $d$ is in inches. This curve is plotted for comparison in *Fig. 8*, and is seen to lie below the curves obtained from the codes although its slope is similar.

Thus it seems that one form of expression for $K_L$ is :

$$K_L = \frac{\text{constant}}{d^n} \qquad (23)$$

This is very interesting because with the analysis made in this Paper (equation (7)) and the values from the older and simpler of the two main roughness formulae, namely that of Fromm,[28] the same type of expression for $K_L$ can be obtained.

The formula of Fromm gives the friction coefficient as :

$$\lambda = 10^{-2}\left(\frac{k'}{d}\right)^{0{\cdot}316} \qquad (24)$$

where $k'$ denotes a constant depending on the particular surface roughness.

From equation (7) this gives :

$$K_L = 10^{-2} \cdot \frac{x}{d}\left(\frac{k'}{d}\right)^{0{\cdot}316} \qquad (25)$$

which is also of the form :

$$K_L = \frac{\text{const}}{d^n} \quad \text{(cf. equation (23))}$$

provided that $\dfrac{x}{d}$ is constant for any one meter.

It is therefore seen that equations (23) and (25) give the same form of relation between $K_L$ and $d$, thus demonstrating that $K_L$ based on equivalent pipe theory behaves in a manner consistent with pipe-flow laws in the rough regime. In fact, by comparing the theoretical variation of $K_L$ (equation (25)) with the experimental curves of *Fig. 8* it is possible to obtain the empirical values of $x/d$ for predicting $K_L$ correctly. To do this, a particular value of $k'$ must be assumed in equation (24), and the nearest value to bronze quoted by Fromm has been used. This was for asphalted steel pipes, for which he gave $k' = 1{\cdot}5m$.

The results of this comparison are shown in Table 2.

It is seen that all the Venturi meter experimental values give similar values of $n$, the mean being 0·22 as opposed to the Fromm formula which gives $n = 0{\cdot}316$.*

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* This is probably because the losses in the entry nozzle are not entirely skin-friction losses. There are also secondary losses, caused by the contraction, included in $\lambda$ used for the equivalent-pipe theory, and so there is no reason why $n$ should be the same as for flow in a simple pipe. The Pardoe formula of *Fig. 8* gives $n = 0{\cdot}24$, which is in good agreement with the figures obtained in this Paper.

TABLE 2

| Source | $n$ | Calculated for a roughness $x/d$ equivalent to asphalted steel |
|---|---|---|
| Pardoe (ref. 27) . . | 0·24 | 1·26 |
| A.S.M.E. Code . . | 0·20 | 1·31 |
| British Code . . . | 0·22 | 1·39 |
| French Code . . . | 0·23 | 1·42 |
| Mean . . . . . . | 0·22 | 1·34 |
| Fromm . . . . . | 0·316 | — |

The effective lengths $x/d$ agree quite well and it seems that the mean value is about 1·3 for the particular roughness assumed.

The quantitative use of the Fromm roughness formula is limited for present purposes because the $k'$ values are not easily related to the effective surface roughness $\epsilon$. For this reason the analysis now turns to the more complicated but more exact and generally applicable roughness formula of Karman-Nikuradse [16, 29, 30] involving $\epsilon$.

(2) *$K_L$ Values using the Karman-Nikuradse Roughness Formula*

This formula was developed later than that of Fromm and, having the advantage of Nikuradse's experiments,[29, 30] gives $\lambda$ in terms of the effective "sand" roughness of the surface, $\epsilon$.

For fully rough pipe flow, the Karman-Nikuradse formula gives:

$$\frac{1}{\sqrt{\lambda}} = 1{\cdot}138 + 2\log_{10}\frac{d}{\epsilon} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

as the relation between the friction coefficient and the hydraulic roughness ratio. Combining this with equation (21) for $K_L$ and writing $\epsilon$ as $\epsilon_v$ (the effective roughness of the Venturi meter) gives :

$$\frac{1}{\sqrt{K_L}} = \frac{(1{\cdot}138 - 2\log_{10}\epsilon_v)}{\sqrt{x/d}} + \frac{2}{\sqrt{x/d}}\log_{10} d \quad . \quad . \quad . \quad (27)$$

$$\text{or } \frac{1}{\sqrt{K_L}} = A + B\log_{10} d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

$$\text{where } \frac{x}{d} = \left(\frac{2}{B}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

$$\text{and } \log_{10}\epsilon_v = 0{\cdot}569 - \frac{A}{B} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

The experimental values of $K_L$ obtained from *Fig. 7* by calculation are plotted in the same form as equation (28), that is, $\frac{1}{\sqrt{K_L}}$ against $\log_{10} d$ in *Fig. 9*. Once again it is seen that the same linear relation as suggested by roughness theory is obtained. The Pardoe, A.S.M.E., and British Code values give clearly defined lines whereas the French figures, although showing the same linear tendency, exhibit more scatter. Because of this, the French Code values have only been represented in *Fig. 9* by a few spot points.

A comparison is made in Table 3 of the effective values of $x/d$ and $\epsilon_v$ calculated from equations (29) and (30) respectively for the various curves in *Fig. 9*.

It is seen that this time the effective values of $x/d$ are higher than those given in Table 2 by the Fromm formula. Whereas this gave $x/d = 1{\cdot}4$, the Karman-Nikuradse formula gives $x/d = 2{\cdot}0$. However, the "Nikuradse" value is independent of any

TABLE 3

| Source | $x/d$ | $\epsilon_v$ : inches |
|---|---|---|
| Pardoe . . . . . . . | 1·60 | 0·0044 |
| A.S.M.E. Code . . . . . | 2·20 | 0·0013 |
| British Code . . . . . . | 2·52 | 0·0010 |
| French Code . . . . . | 1·93 | 0·0030 |
| Mean . . . . . . . . | 2·06 | 0·0022 † |

assumed surface roughness and so should be more reliable than the " Fromm " value, which involved assuming a certain value for $k'$ (equation (25)).

Using the Karman-Nikuradse approach it is necessary to know $\epsilon_v$ as well as $x/d$ (see equation (27)) in order to calculate $K_L$, and from Table 2 it seems that a suitable mean value is $\epsilon_v = 0{\cdot}0022$ inch†. This would seem to be of a reasonable order for such meters, as will be seen by comparison with the probable roughness values quoted by Moody [19] and set out in Table 4. The effective roughness must be greater than that of a pipe of the same material because the convergence and the kinetic losses included in $\lambda$ will tend to increase the apparent skin friction. It is seen that the calculated mean value of $\epsilon$ for the Venturi meters happens to be similar to that of wrought iron or steel piping.

TABLE 4.—ROUGHNESS VALUES

| Material | |
|---|---|
| Cast iron . . . . . . . . . | 0·010 |
| Galvanized iron . . . . . . | 0·007 |
| Asphalted cast iron . . . . | 0·005 |
| Steel and wrought iron . . . | 0·0018 |
| $\epsilon_v$ for Venturi meters . . . . | 0·0022 |

† This is not the arithmetic mean of the $\epsilon_v$ values in Table 2 but the effective mean obtained from equation (29) and the arithmetic mean values of $A$ and $B$, thus giving a mean curve through the experimental points.

Paper No. 5943

# "Prestressed Concrete Beams: the Economical Shape of Section"

by

**Professor Reginald George Robertson, M.A., M.I.C.E.**

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## Introduction

When determining the relative economy of various alternative designs for a beam there are three main considerations :—

(1) The total cost of the steel cables in place.
(2) The total cost of the concrete, including shuttering and any mild-steel reinforcing.
(3) The effect of variation in beam depth on the cost of the whole work involved.

The depth governs the cable size directly, since for economy the cables have to be as low as possible, so that the lever arm depends on the depth ; it is seldom economic to have the cables higher than their lowest position (except in cases of reversal of live load) since the material below the cable is not efficiently disposed.

The variation of the area of cross-section as the depth varies requires clarification : this is the purpose of the present Paper.

## The Design Chart

By analysing thin I-section beams of varying proportions but with the same thickness for flanges and web, a chart (*Fig. 1*) was drawn up, connecting non-dimensional coefficients for the moment due to the superimposed load alone, with the area of section, in terms of the span and concrete stress, for various web-thickness-to-depth ratios.

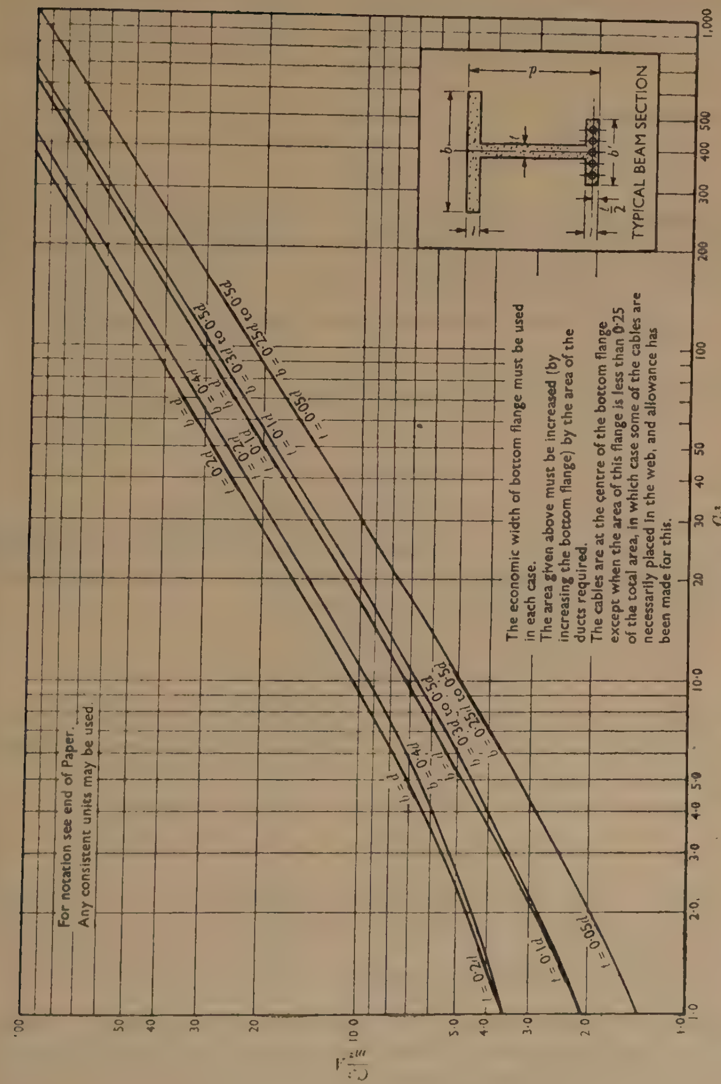
It was found that the area was controlled by the thickness-to-depth ratio of the web, so that if a minimum thickness was specified, a very rapid estimation of the variation of area with depth was possible.

## The Minimum Depth

The minimum depth is often governed by the maximum allowable live load deflexion, which is given as $\frac{\text{span}}{800}$ by A.A.S.H.O., with a footnote

* For notation, see p. 247.

*Fig. 1*




AREAS OF PRESTRESSED CONCRETE BEAMS OF ECONOMIC SECTION FOR VARIOUS $t/d$ RATIOS, FOR $\eta c_1 = c_2$ AND $\eta = 0{\cdot}85$

that, for continuous beams, the equivalent span can be taken as the distance between points of contraflecture under dead load.

For uniform loading on a simply supported span, the value of $\frac{l}{d}$ can be found as $\frac{l}{d} = 9{\cdot}6\,\frac{\delta}{l}\cdot\frac{E}{C}\cdot\frac{y}{d}$.* Hence, if $\frac{\delta}{l} = \frac{1}{800}$ and $C$ denotes the stress due to live load only:

$$\max.\ \frac{l}{d} = \frac{12}{1{,}000}\cdot\frac{y}{d}\cdot\frac{E}{C}$$

In a prestressed concrete beam, the lower fibre stress is zero under full load, and since the stress variation is due to live load alone, the value of $C$ is the maximum stress allowable.

Long spans require T-section beams for which $\frac{y}{d}$ is about $\frac{2}{3}$.

Hence, by the A.A.H.S.O. formula, max. $\frac{l}{d} = \frac{8}{1{,}000}\cdot\frac{E}{C}$.

## Oscillation

Deflexion may also be limited by the oscillation period.

The oscillation period for a simply supported beam is approximately (deflexion in feet)$^{\frac{1}{2}}$, using the total deflexion due to live and dead loads, and ignoring any reduction due to prestressing.

## The Economic Depth

The minimum web thickness can be used with various depths and stresses to find the area of section from *Fig. 1*. The area of section gives the dead load moment and hence the working load moment is obtained.

The moment resisted by the web is found by assuming the lowest position for the cable centroid for which the lever arm of the web is $\left(\frac{2}{3}d - \frac{t}{2}\right)$ and the force in the web is $\frac{1}{4}C_2 d.t$; the remainder of the working load moment is resisted by the top flange, which gives the net area of the top flange; the mean stress in the top flange is $C_2\left(1 - \frac{t}{2d}\right)$. The net area required for the bottom flange is the remainder of the total area already found. If the bottom flange is found to be larger than the top flange the reason is that the cable has been placed too low for the specified stresses. This is rectified by using the mean flange area for each flange: the cable pull is increased (see below) due to the increased top flange, and its position is raised by the distance given by: added cable pull × original top flange lever arm ÷ revised cable pull.

The net area for the flanges is the area on each side of the web. The portion of the web present has to be added before calculating the ultimate moment.

The cable pull required is given by the working stress in the concrete multiplied by $\left(1 - \frac{t}{2d}\right)$ of the area of the top flange added to half the area of the web.

The ultimate moment is given by the ultimate stress in the concrete multiplied by the top-flange gross area multiplied by the lever arm, or by the ultimate stress in the cable multiplied by its area multiplied by the lever arm, whichever is less.

If a compression failure is considered undesirable, the top flange area may have to be increased; this may also be desirable for lateral stability. The effect of such an increase is an increase in the total area, which increases the moment and cable size, which can be readily inserted as a correction; a small reduction in the bottom flange is possible in such cases.

The most suitable design is taken as a standard and the relative economy of alternative designs is found by pricing the changes in concrete and cable area and the change in depth, where relevant.

The effect of varying the working stress can also be valued by making another set of beam depth calculations.

*Example*

Given: $t = 6$ inches; $l = 80$ feet; $m = 10{,}000$ lb./inch; $M = 11{\cdot}5 \times 10^6$ lb.-inches; cable steel working stress $= 145{,}000$ lb. per square inch; cable steel ultimate stress $= 105$ tons per square inch.

(*a*) $C_2$ is taken as 2,000 lb. per square inch.

Then
$$\frac{C_2{}^2 M}{m^3} = 46$$

The value of $\frac{C_2{}^2 A}{m^2}$ is found from *Fig. 1*.

TABLE 1

| Design no. | $d$ | $\frac{t}{d}$ | $\frac{C_2{}^2 A}{m^2}$ | $A$ | Web area | Web lever arm | Web moment $\times 10^{-6}$ | $(M + mA) \times 10^{-6}$ | Flange moment $\times 10^{-6}$ | Flange lever arm | Top flange area (net) | Cable pull $\times 10^{-3}$ | Steel area | Ultimate moment $\times 10^{-6}$ | Bottom flange area (net) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (1) | 40 | 0·15 | 22 | 550 | 240 | 23·7 | 5·7 | 17·0 | 11·3 | 34 | 180 | 580 | 4·0 | 31·5 | 130 |
| (2) | 50 | 0·12 | 19 | 475 | 300 | 30·3 | 9·1 | 16·2 | 7·1 | 44 | 86 | 460 | 3·2 | 32·5 | 89 |

(*b*) $C_2$ is taken as 1,500 lb. per square inch.

Then $$\frac{C_2{}^2M}{m^3} = 25{\cdot}9$$

The value of $\frac{C_2{}^2A}{m^2}$ is found from *Fig. 1*.

TABLE 2

| Design no. | $d$ | $\frac{t}{d}$ | $\frac{C_2{}^2A}{m^2}$ | $A$ | Web area | Web lever arm | Web moment $\times 10^{-6}$ | $(M + mA) \times 10^{-6}$ | Flange moment $\times 10^{-6}$ | Flange lever arm | Top flange area (net) | Cable pull $\times 10^{-3}$ | Steel area | Ultimate moment $\times 10^{-6}$ | Bottom flange area (net) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (3) | 50 | 0·12 | 13 | 578 | 300 | 30·3 | 6·8 | 17·3 | 10·5 | 44 | 170 | 473 | 3·3 | 34 | 108 |
| (4) | 60 | 0·10 | 12 | 533 | 360 | 37·0 | 10·0 | 16·8 | 6·8 | 54 | 89 | 397 | 2·8 | 34 | 84 |

The cost of the cables in design (1) are assumed to be 70 per cent of the cost of the beam.

The other designs could be compared in economy, using design (1) as a basis (see Table 3).

TABLE 3

| Design No. | Change in concrete cost | Change in steel cost | Total change |
|---|---|---|---|
| (2) | $-$ 14% × 30% = $-$ 4% | $-$ 20% × 70% = $-$ 14% | $-$ 18% |
| (3) | + 5% × 30% = + 2% | $-$ 17% × 70% = $-$ 12% | $-$ 10% |
| (4) | $-$ 3% × 30% = $-$ 1% | $-$ 30% × 70% = $-$ 21% | $-$ 22% |

It is evident that the larger depth is more economical in materials, but the effect of an increase in depth would require valuation in most cases.

Provided that depth was not important, the value of high concrete stress is not significant, as can be seen from designs (2) and (4).

The advantage claimed for this method lies in the immediate evaluation of the area and working load moment from the moment due only to superimposed load. The method has been developed further by the compilation of separate charts to give the depth, bottom-flange area, total area, and cable pull for various $b/d$ and $C_1/C_2$ ratios.

The final section can readily be checked by the following equations, which were used in drawing up *Fig. 1*.

*Equations for no tensile stress at any time*

$$P = \frac{y}{d} A C_2$$

$$C_2 = \frac{M + mA}{A} \cdot \frac{d}{y(y + h - g)}$$

$$C_1 = \frac{M + (1 - \eta)mA}{\eta h A}$$

### Continuous Beams

The equations and *Fig. 1* apply to any beam, whether prismatic or not, or simply supported or continuous,[1] provided that they refer to the critical section, and reversal of moment does not occur, or the reverse moment due to superimposed load does not exceed $\frac{1 - \eta}{\eta}$ multiplied by the maximum live plus dead load moments.

When this limit is exceeded a symmetrical section is always economical, if prestressed to give a uniform stress for the mean of the superimposed load moments, since the stress variation will then be equal in top and bottom flanges.

### Notation

$\eta$ denotes $\dfrac{\text{cable pull at working load}}{\text{initial cable pull at anchoring}}$
$C_1$ ,, initial stress in bottom flange
$C_2$ ,, working load stress in top flange
$P$ ,, cable pull at working load
$A$ ,, area of cross-section excluding cable ducts
$d$ ,, depth of section
$y$ ,, bottom fibre distance from centre of gravity of section
$h$ ,, upper core distance $= \dfrac{I}{Ay}$
$l$ ,, span
$m$ ,, moment at time of cable tensioning per unit of cross-sectional area $\left(\therefore m = \text{density} \times \dfrac{l^2}{8}\right)$
$E$ ,, Young's Modulus for concrete at working load
$\delta$ ,, deflexion of beam
$t$ ,, thickness of web
$M$ ,, moment due to loads added after cable tensioning.
$g$ ,, distance from bottom fibre to cable centroid

The Paper is accompanied by one diagram, from which *Fig. 1* has been prepared.

[1] "Prestressed Concrete Beams, a Rational Design Method," *Structural Engineer*, vol. 30, p. 259 (Nov. 1952). *See* Part II, by R. G. Robertson.

Paper No. 5993

# "A Review of Pipe-Friction Data and Formulae, with a Proposed Set of Exponential Formulae based on the Theory of Roughness"

by

**Peter Anton Lamont, M.A., A.M.I.C.E.**

*(Ordered by the Council to be published with written discussion.)* †

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SYNOPSIS

The Paper has been based mainly on a Report [1] prepared by the Author for the International Water Supply Congress at Paris in June 1952, but a number of experimental records, which have subsequently become available, have also been taken into account.

The original report contained a hundred records for new pipes and more than eighty records for old and slimy pipes, which were selected from a total of more than two hundred records examined, some from published and some from hitherto unpublished sources. Each record was tabulated on a uniform basis and plotted on a standard Friction-Factor/Reynolds-Number diagram against the general background of the curves of Prandtl, von Karman, Nikuradse, and Colebrook, which may be described more briefly as "the Theory of Roughness." Such a diagram provides the ideal basis for the representation of experiments carried out with different fluids at different temperatures and for the comparison of experimental results either with the Theory of Roughness or with the Exponential Formulae, which plot as parallel straight lines on the diagram. The report, which comprises nearly three hundred pages with a hundred diagrams is believed to be one of the most complete collections at present available of records for new, old, and slimy pipes and probably the only one in which the experimental records and the formulae with which they were compared, have been plotted throughout on a uniform non-dimensional basis.

The objects of this Paper are briefly :—

(1) To summarize the experimental records.

(2) To compare them with the Theory of Roughness and with certain exponential formulae in common use.

(3) To develop a set of associated exponential formulae based on and expressed in terms of the Theory of Roughness, which will be capable of application, not only to water at normal temperatures, but also to any fluid of known kinematic viscosity.

The records for old and slimy pipes have been dealt with briefly in the Paper, but the detailed examination of age-effects, which was included in the Paris Report, forms the basis of a separate Paper.[2]

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## Method of Summarizing Experimental Records

The procedure adopted in recording the experimental results for each particular class of pipe was, first, to tabulate the results of each experiment

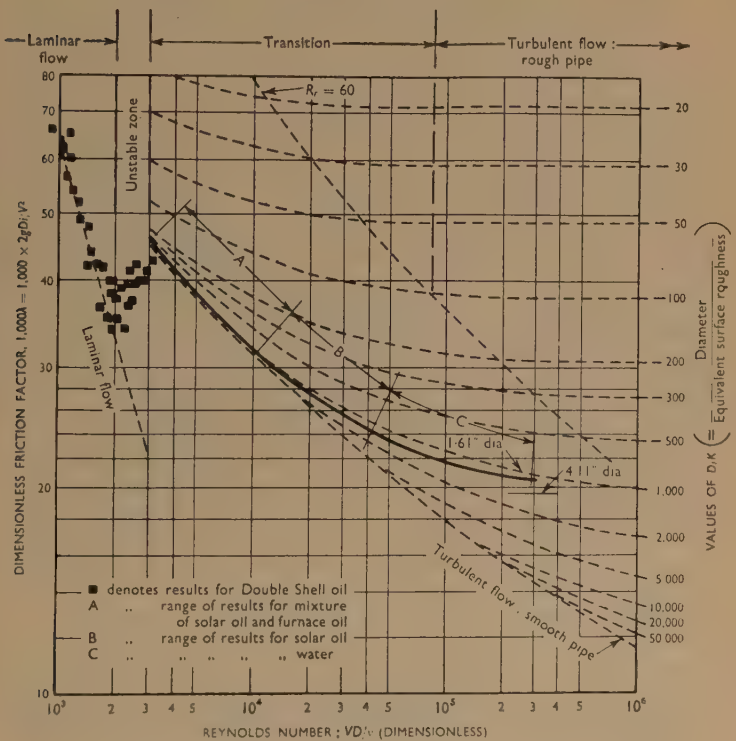
† Correspondence on this Paper should be received at the Institution before the 15th August, 1954, and will be published in Part III of the Proceedings. Contributions should be limited to about 1,200 words.—Sec. I.C.E.

[1] The references are given on p. 273.

on a standard form and then to plot the individual experimental observations on a standard Friction-Factor/Reynolds-Number diagram. The next step was to deduce the value of the equivalent surface roughness by reference to the relative-roughness lines which formed a background to the standard diagram and to record this value in a Table at the foot of the diagram.

To illustrate this procedure, *Fig. 1* has been chosen. This records

*Fig. 1*



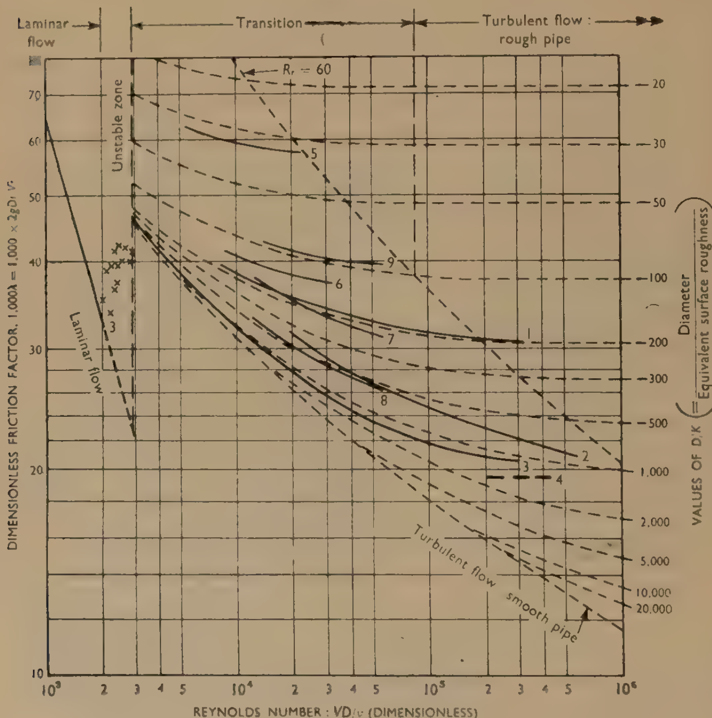

PIPE-FRICTION RECORD FOR SMOOTH NEW GALVANIZED IRON PIPES

(From report by Professor Herbert Addison of laboratory experiments conducted by Dr Ismail at Fouad I University.)

| Dia : inches | Deduced values of *k* : inch | | |
|---|---|---|---|
| | Max. | Min. | Average |
| 1·61 | 0·0016 | 0·0011 | 0·0014 |
| 4·11 | 0·0034 | 0·0023 | 0·0029 |

two experiments on galvanized pipe carried out under the supervision of Professor Herbert Addison and is of particular interest because three different types of oil were employed in addition to water, thus enabling continuous observations to be made from the laminar-flow region through the transition zone virtually to the boundary of the turbulent rough zone.

*Fig. 2*




COLLECTED PIPE-FRICTION RECORDS FOR NEW GALVANIZED IRON PIPES

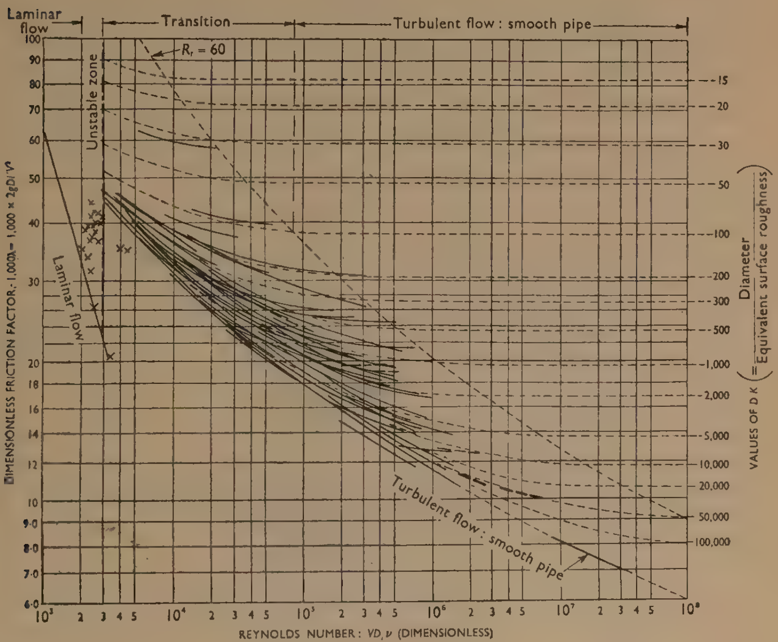
| Reference . . | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Dia : inches . | 2·014 | 4·074 | 1·610 | 4·110 | 0·350 |
| $k$ : inch . . . | 0·013 | 0·0048 | 0·0016 | 0·0034 | 0·0110 |

| Reference . . | 6 | 7 | 8 | 9 |
|---|---|---|---|---|
| Dia : inch . . | 0·485 | 0·626 | 0·850 | 1·042 |
| $k$ : inch . . . | 0·0035 | 0·0031 | 0·0016 | 0·0110 |

The individual records for each class of pipe were then grouped together on a single diagram. *Fig. 2* represents the collected records for new galvanized pipe and is one of eleven similar diagrams prepared for the Paris Report.

Finally, all records for new pipes were reproduced on a single diagram (*Fig.* 3) and summarized in Tables 1 and 2 whilst all records for old and

*Fig. 3*




COLLECTED PIPE-FRICTION RECORDS FOR NEW PIPES: ALL TYPES

(Ninety-seven experiments have been plotted on this diagram. Full lines show experiments with a clearly marked trend; broken lines show experiments with insufficient range to indicate trend; crosses show spot records.)

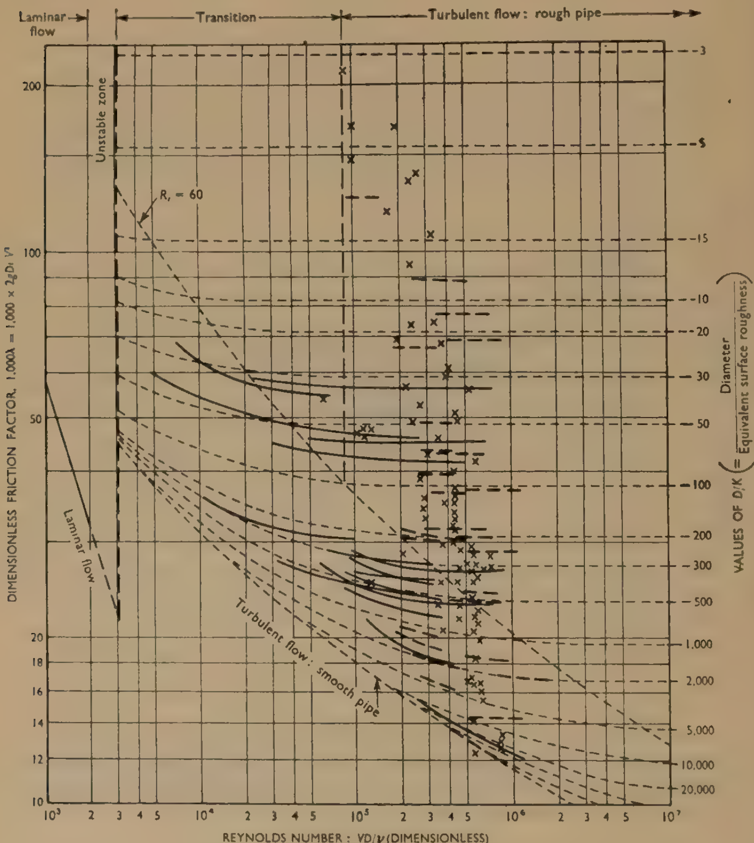
slimy pipes were reproduced on a similar diagram (*Fig. 4*) and summarized in Table 3.

Most of the records summarized in Table 1 were taken from existing published sources, but twenty hitherto unpublished records have also been included. Amongst these were several experiments carried out with galvanized and steel pipes carrying oil, some with pipes of cast iron and aluminium (cold-drawn and extruded), and others with pipes lined with spun concrete, spun bitumen, or with concrete applied in situ by the Tate

process. Records have since become available of experiments carried out on Alkathene pipes,[3] which prove that these pipes are hydraulically smooth.

It will be noted that Table 1 contains no records of spun-iron pipes.

*Fig. 4*




Collected Pipe-Friction Records for Rough, Old, and Slimy Pipes
(Full lines show experiments with clearly marked trend; broken lines show experiments with insufficient range to indicate trend; crosses show spot records.)

Two spot records were examined but were rejected because of uncertainty about the internal diameter. In the absence of such records it is normal to assume that these pipes have the same surface roughness as cast iron.

It will be noted also that the limited number of records for concrete

TABLE 1.—SUMMARY OF EXPERIMENTAL RECORDS FOR NEW PIPES

| Type of pipe | No. of expts | Range of experiments | | | Equivalent surface roughness $k$: inch | | |
|---|---|---|---|---|---|---|---|
| | | Diam: inches | Velocity: ft/sec. | Reynolds Number | Range | Mean | Recommended design value |
| Uncoated cast iron | 3 | 3·24 to 19·68 | 1·175 to 16·168 | $2{\cdot}43\times10^4$ to $7{\cdot}66\times10^5$ | 0·0060 to 0·0130 | 0·0089 | 0·010 |
| Coated cast iron | 14 | 3·83 to 61·0 | 0·3947 to 33·80 | $2{\cdot}88\times10^4$ to $6{\cdot}48\times10^6$ | 0·0007 to 0·0085 | 0·0040 | 0·005 |
| Galvanized iron | 9 | 0·350 to 4·110 | 0·556 to 20·50 | $3{\cdot}40\times10^3$ to $5{\cdot}15\times10^5$ | 0·0016 to 0·0110 | 0·0040 | 0·005 |
| Wrought iron | 18 | 0·3604 to 8·0481 | 0·1844 to 22·50 | $8{\cdot}02\times10^3$ to $9{\cdot}11\times10^5$ | 0·0003 to 0·0040 | 0·0020 | 0·002 |
| Coated steel | 4 | 0·47 to 10·12 | 0·249 to 11·05 | $7{\cdot}57\times10^3$ to $4{\cdot}09\times10^5$ | 0·0006 to 0·0050 | 0·0018 | 0·002 |
| Uncoated steel | 10 | 0·62 to 12·09 | 0·75 to 46·00 | $3{\cdot}75\times10^3$ to $1{\cdot}28\times10^6$ | 0·0003 to 0·0032 | 0·0011 | 0·0015 |
| Asbestos cement | 3 | 2·97 to 5·77 | 0·72 to 5·68 | $1{\cdot}90\times10^4$ to $1{\cdot}80\times10^5$ | Smooth pipe | Smooth pipe | Smooth |
| Spun concrete lined | 6 | 3·61 to 60·0 | 0·53 to 15·65 | $2{\cdot}67\times10^4$ to $1{\cdot}18\times10^6$ | Smooth to 0·0011 | Smooth pipe | Smooth |
| Spun bitumen lined | 7 | 10·000 to 46·25 | 0·648 to 9·42 | $1{\cdot}41\times10^5$ to $1{\cdot}24\times10^6$ | Smooth pipe | Smooth pipe | Smooth |
| Smooth pipe* | 16 | 0·24204 to 3·9995 | 0·145 to 32·30 | $6{\cdot}30\times10^2$ to $8{\cdot}74\times10^5$ | Smooth to indefinite | Smooth pipe | Smooth |
| Concrete | 7 | 13·10 to 21·6 | 2·08 to 20·0 | $2{\cdot}38\times10^5$ to $2{\cdot}77\times10^7$ | Smooth to 0·0195 | | See Table 2 |
| Tate relined pipes | 3 | 13·1 to 23·43 | 2·825 to 5·160 | $2{\cdot}38\times10^5$ to $8{\cdot}30\times10^5$ | 0·0069 to 0·0195 | 0·0130 | 0·015 |

* Smooth pipes include smooth drawn non-ferrous pipes of aluminium, brass, copper, lead, etc. and non-metallic pipes of Alkathene, glass, Saran, etc.

TABLE 2.—SUMMARY OF SCOBEY'S EXPERIMENTS ON CONCRETE PIPES (extracted from Dr Colebrook's thesis 4)

| Type | Range of experiments | | Equivalent surface roughness, $k$: inch | | |
|---|---|---|---|---|---|
| | Diam: inches | Reynolds Number | Range | Mean | Recommended design value |
| *Scobey's Class 1*: Old concrete pipes, mortar not wiped from joints. | 8·0 to 30·0 | $1{\cdot}0\times10^5$ to $6{\cdot}0\times10^5$ | 0·075 to 0·250 | 0·200 | 0·200 |
| *Scobey's Class 2*: Modern dry-mix pipe, monolithic pipe, or tunnel linings made over rough forms. | 12·0 to 120·0 | $1{\cdot}0\times10^5$ to $3{\cdot}0\times10^6$ | 0·031 to 0·069 | 0·050 | 0·050 |
| *Scobey's Class 3*: Small wet-mix pipes in short lengths, dry-mix pipes in long lengths, average monolithic pipes made on steel forms, pressure-made pipes with interior coat of neat cement by mechanical trowel. | 30·5 to 174·0 | $1{\cdot}2\times10^6$ to $7{\cdot}0\times10^6$ | 0·011 to 0·019 | 0·016 | 0·020 |
| *Scobey's Class 4*: Monolithic pipes with joint scars and all irregularities removed, first-class concrete against oiled steel forms. | 30·0 to 216·0 | $6{\cdot}0\times10^5$ to $2{\cdot}8\times10^7$ | 0·0021 to 0·0088 | 0·007 | 0·010 |

TABLE 3.—SUNDRY INFORMATION EXTRACTED FROM RECORDS FOR ROUGH (OLD) PIPES

(1) *Roughness growth rate of coated cast iron pipes.*

| Trend No. | Roughness growth rate: inches per annum | Approx. mean value of Langelier index | Degree of corrosive attack of water | Equivalent surface roughness, $k$: inches | | |
|---|---|---|---|---|---|---|
| | | | | After 30 years | After 60 years | After 100 years |
| 1 | 0·001 | 0 | slight | 0·035 | 0·065 | 0·105 |
| 2 | 0·003 | −1·3 | moderate | 0·095 | 0·185 | 0·305 |
| 3 | 0·010 | −2·6 | appreciable | 0·305 | 0·605 | 1·005 |
| 4 | 0·030 | −3·9 | severe | 0·905 | 1·805 | 3·005 |

pipes summarized in Table 1 did not enable any definite conclusions to be drawn, but the records of the Tate relining process may be of interest and value to waterworks engineers. To the records in Table 1 have been added, in Table 2, the summarized records of thirty-two experiments on concrete pipes carried out by Scobey and analysed by Dr Colebrook in his Thesis.[4]

| | | |
|---|---|---|
| (2) *Roughness of mains after brushing.* | | $k$ : inch |
| Liverpool waterworks, Vyrnwy Aqueduct, second pipeline. Average roughness after each brushing with whalebone brush | | 0·040 |
| (3) *Roughness of mains after scraping* | | $k$ : inch |
| Summary of eighteen records of Sheffield and Newcastle waterworks and three records of Darcy | av. | 0·0185 |
| | max. | 0·030 |
| | min. | 0·014 |
| (4) *Average roughness corresponding to description of tuberculation contained in eighty records of old pipes* | | $k$ : inch |
| Pipes described as untuberculated | | 0·030 |
| ,, ,, ,, slightly tuberculated | | 0·120 |
| ,, ,, ,, appreciably tuberculated | | 0·750 |
| ,, ,, ,, severely tuberculated | | 1·650 |
| (5) *Maximum roughness recorded* | | $k$ : inches |
| Maximum value of $k$ recorded | | 2·70 |
| (6) *Maximum relative roughness recorded* | | $D/K$ |
| Minimum value of $D/K$ recorded | | 4·0 |

## Comparison of Experimental Records with Theory of Roughness

The extremely close agreement between the experimental records and the Theory of Roughness is clearly shown in *Figs 3* and *4*, where the whole of the records for new, old, and slimy pipes have been plotted against the general background of the Theory of Roughness which appears on the diagrams as a series of curved broken lines. The combined evidence of nearly two hundred sets of experiments illustrated on these two diagrams, places the essential soundness of the Theory of Roughness beyond all reasonable doubt.

A detailed examination of the individual records which have been grouped together in *Figs 3* and *4* enables the following conclusions to be drawn, each of which confirms one of the five main features of the Theory of Roughness.

(1) Pipes carrying water or oil adhere closely to the theoretical law for laminar flow at values of the Reynolds Number, $R$, below 2,000 and occasionally follow this law up to $R = 3{,}500$.

(2) Pipes carrying water or oil exhibit signs of instability at values of $R$ between 2,000 and 3,000 and occasionally at values of $R$ up to 4,500.

(3) Certain types of pipe carrying water (see list in Table 1) follow the smooth-pipe law closely. Clean steel pipes carrying oil also follow the smooth-pipe law, owing, no doubt, to the formation of a thin film of wax on the inner wall of the pipe.

(4) Other types of pipe (see list in Table 1) exhibit evidence of roughness and break away from the smooth-pipe curve in a series of transition curves, which appear in most cases to follow the Colebrook relative-roughness curves closely. It is of considerable interest to note that the ninety-seven experiments for new pipes plotted in *Fig. 3* lie almost entirely within the transition zone. This illustrates the importance of this zone for new pipes carrying water and the value of Colebrook's brilliant contribution.[5]

(5) Although many of the experimental records for rough pipes submitted by water authorities and illustrated in *Fig. 4* are within the turbulent rough zone, comparatively few have a range of velocity sufficiently wide to enable a definite trend to be established. The experiments of Darcy on old cast-iron pipes and the experiments of Freeman[6] on old wrought-iron pipes, which are also plotted on *Fig. 4*, are, however, convincing on this point and confirm that the flow is of the turbulent—rough-pipe type, for values of the Roughness Reynolds Number $R_r$ higher than 60. The flattening out of the curves for new pipes (*Fig. 3*) as they approach the boundary of the turbulent rough zone is also significant.

The hydraulically smooth pipes follow the smooth-pipe curve very closely, the variation from the mean being of the order of $\pm 2\frac{1}{2}$ per cent on velocity, which is within the accuracy of most meters. Since such pipes are not usually subject to age effects caused by corrosion, the flow in them may be assessed and predicted with a high degree of accuracy. The only factor which is liable to affect the discharge of such pipes adversely is the formation of slime or silt when carrying certain raw waters.

On the other hand, among the various classes of new iron and steel pipes which exhibit roughness, the variation of roughness within each particular class is appreciable. A study of Table 1 reveals that the rates of the highest to lowest roughness for each particular class of the order of 9:1. If, therefore, a mean value of the roughness is applied to all pipes within the class, the variation from the mean may be as much as $\pm 6$ per

cent on velocity or $\pm 12$ per cent on hydraulic gradient. Part of this variation may be due to experimental discrepancies in the measurement of diameter, velocity, etc. It is possible also that in some cases the pipes may have been a few months old and subject to slight age effects. The probable actual variation from the mean should therefore be appreciably less than indicated above.

Such variations in initial carrying capacity are, in any case, comparatively unimportant in iron pipes carrying water, since the increased roughness arising from tuberculation after even a few years service is usually many times greater than the initial roughness. In such circumstances it is more valuable to be able to assess the roughness after a given period of service than it is to know the precise initial roughness.

## Comparison of Experimental Records with Existing Exponential Formulae

Exponential formulae are used almost universally in Britain for the solution of pipe-friction problems. Amongst those in general use are the formulae of Hazen-Williams, Manning, Barnes,[7] and Scimemi. More recently, Blair[8] has published four formulae, covering a wide range of new pipes. In general terms, such formulae have the important practical advantage of simplicity, but their inherent limitations are clearly revealed when they are plotted on the Friction-Factor/Reynolds-Number diagram against the general background of the Theory of Roughness.

On such a diagram there is ample experimental evidence (see *Figs 3* and *4*) that the curves of equal diameter are concave upwards, but the exponential formulae seek to replace these curves by straight lines. No great error arises over a moderate range of velocity within the experimental limits, but as the prolongation of the straight line is inevitably below the curve, the error increases rapidly when any appreciable degree of extrapolation is attempted and leads in every case to an underestimation of the friction.

The other inherent defect of exponential formulae is that they seek to impose a logarithmic distribution of the diameter lines whereas, in fact, the distribution is more nearly log-logarithmic. Again, no serious error arises if each formula is applied strictly within the range of diameters of the experiments on which it has been based, but since the log-log scale converges more rapidly than the log scale, any attempt to extrapolate appreciably in terms of diameter, either upwards or downwards, leads to an underestimation of the friction.

It is unfortunate that the experimental limits of particular exponential formulae are seldom stated in text-books, since they can be very misleading if applied without regard to these limits. Finally, the exponential formulae which, in general, have been based on experiments with water

at normal temperature, do not give any guidance towards the solution of friction problems in pipes carrying other fluids.

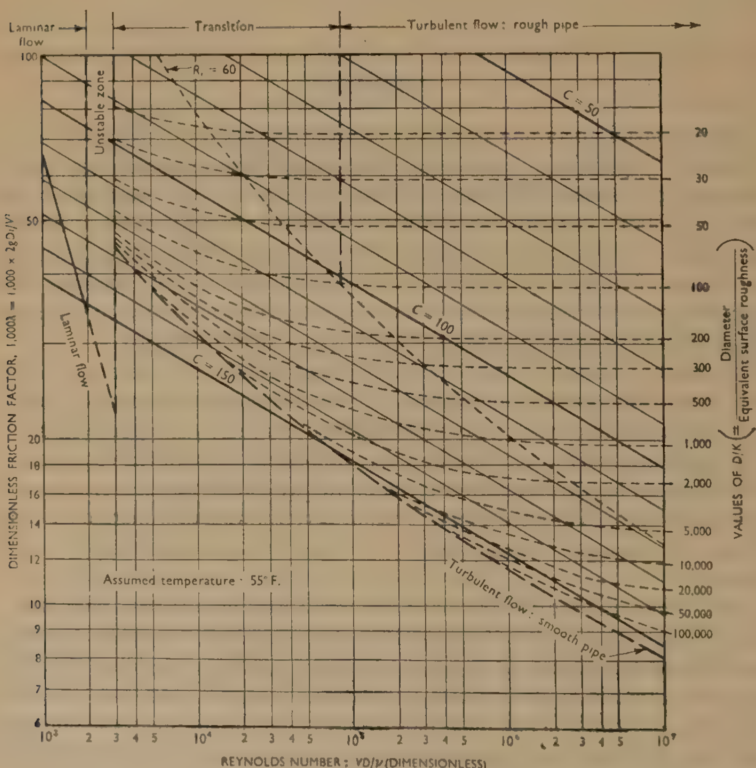
In proceeding to examine certain exponential formulae in some detail, it should be remembered that the above limitations are common to them all.

### *Hazen-Williams Formula*

The Hazen-Williams formula, which has been reproduced on the Friction-Factor/Reynolds-Number diagram in *Fig. 5* may be expressed in ft-lb.-second units as:

$$V = 0{\cdot}55C\ D^{0{\cdot}63}\ i^{0{\cdot}54} \qquad (1)$$

*Fig. 5*




HAZEN-WILLIAMS FORMULA

(The formula has been plotted at $V = 4$ feet per second. There is little difference in the positions of the lines at other velocities.)

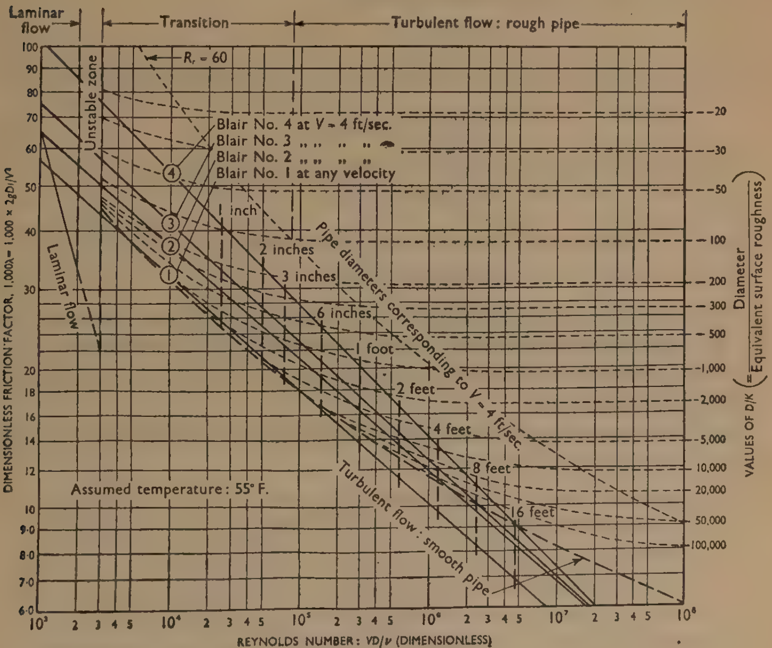
The Hazen-Williams formula is $V = 0{\cdot}55C \,.\, D^{0{\cdot}63} \,.\, i^{0{\cdot}54}$, where $V$ denotes mean velocity in feet per second, $D$ the internal diameter in feet, $i$ the hydraulic gradient; $C$ is a coefficient dependent upon type, condition, and diameter of pipe and, to some extent, upon the velocity.

where $C$ is a coefficient which varies according to the type, condition, and diameter of the pipe and to some extent upon the velocity. It is essentially a formula of the smooth-pipe type, since for any particular value of $C$ and for all diameters, it plots as a single straight line on the diagram.

An inspection of *Fig. 5* reveals that, with a coefficient $C$ of 150–155, the formula is eminently suitable for smooth pipes of large diameter, whilst, with an appropriate coefficient of from 100 to 150, it is reasonably suitable for new pipes in the transition zone.

Although the formula is frequently applied to old pipes in the turbulent rough zone with coefficients of 30 to 100, the steep slope of the lines makes

*Fig. 6*




### Blair Formulae for New Pipes

| Curve No. | Formula | Type of pipe |
|---|---|---|
| 1. | $V = 102{\cdot}6D^{0{\cdot}71}\,.\,i^{0{\cdot}57}$ | Smooth non-ferrous (glass, plastic, copper, brass, lead, etc.) |
| 2. | $V = 85{\cdot}3D^{0{\cdot}69}\,.\,i^{0{\cdot}555}$ | Uncoated steel or wrought iron, asbestos cement, bitumen lined. |
| 3. | $V = 75{\cdot}9D^{0{\cdot}68}\,.\,i^{0{\cdot}54}$ | Bitumen-coated or painted steel; concrete |
| 4. | $V = 62{\cdot}6D^{0{\cdot}67}\,.\,i^{0{\cdot}52}$ | Galvanized, cast iron, coated cast iron. |

it quite unsuitable for application in this zone and the use of the formula with coefficients appreciably below 100 is not recommended.

Although far from being an ideal formula, either for smooth pipes or in the transition zone, the Hazen-Williams formula has, however, been of great service to water engineers, principally because they have become accustomed to its use and have acquired a working knowledge of the approximate value of the coefficient $C$ to be expected when the formula is applied to old cast-iron pipes in their own particular areas of supply.

*Blair Formulae*

Blair's formulae for new pipes, which have been reproduced graphically in *Fig. 6*, may be expressed in ft-lb.-second units as :

No. 1, for smooth non-ferrous pipes

$$V=102{\cdot}6\ D^{0{\cdot}71}\ i^{0{\cdot}57} \qquad (2)$$

No. 2, for bare steel, wrought-iron, asbestos-cement, or bitumen-lined pipes

$$V=85{\cdot}3\ D^{0{\cdot}69}\ i^{0{\cdot}555} \qquad (3)$$

No. 3, for bitumen-coated steel pipes or concrete pipes

$$V=75{\cdot}9\ D^{0{\cdot}68}\ i^{0{\cdot}54} \qquad (4)$$

No. 4, for galvanized, cast-iron, or other coated cast-iron pipes

$$V=62{\cdot}6\ D^{0{\cdot}67}\ i^{0{\cdot}52} \qquad (5)$$

These formulae have been based on a wide range of experimental data and, provided that they are applied to new pipes strictly within the transition zone, they represent an improvement upon most other exponential formulae in general use.

The following ovservations upon the individual formulae are made as a result of a study of *Fig. 6* :—

(1) Formula No. 1 agrees very closely with the smooth-pipe curve for Reynolds Numbers up to $10^5$ but it underestimates the friction above this figure. The discrepancy is still not serious at $R=10^6$, but at $10^7$ the underestimate is approximately 28 per cent rising to 46 per cent at $R=10^8$.

(2) There is comparatively little difference between the results given by formulae Nos 2 and 3, especially at diameters of 12 inches and greater.

(3) Although formula No. 3 is stated to be applicable generally to spun concrete, cast concrete, and cement pipes, it is considered that, so far as cast concrete is concerned, it is strictly applicable only to the smoothest type of cast concrete, corresponding roughly to Scobey's class 4. (See descriptions in Table 2.)

It would underestimate the friction slightly, if applied to Scobey's concrete class 3 and appreciably, if applied to Scobey's classes 2 and 1.

Unlike the Hazen-Williams and Manning formulae, the Blair formulae do not incorporate adjustable coefficients for age-effects.

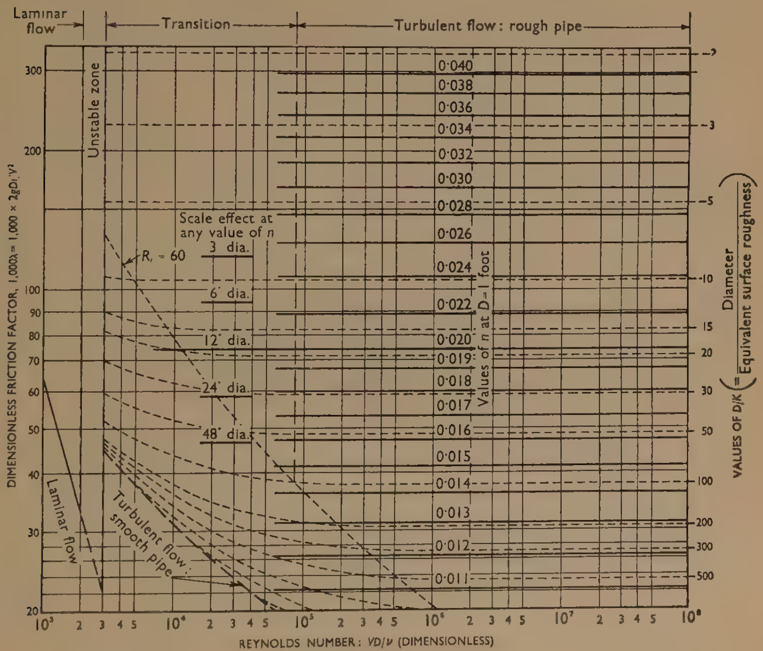
*Manning Formula*

The Manning formula, which has been reproduced graphically in *Fig. 7*, may be expressed in ft-lb.-second units as:

$$V = \frac{1{\cdot}486}{n}\left(\frac{D}{4}\right)^{0{\cdot}667} i^{0{\cdot}5} \qquad \dots\dots\dots \quad (6)$$

where $n$ is a coefficient which varies according to the type and condition of the pipes and to some extent according to the diameter.

*Fig. 7*




Manning Formula $V = \frac{1{\cdot}486}{n}\left(\frac{D}{4}\right)^{0{\cdot}667} . i^{0{\cdot}5}$

(In the above formula, $n$ is a coefficient dependent upon the type, condition, and, to some extent, the diameter of the pipe.)

An inspection of *Fig. 7* reveals that the Manning formula is especially suitable for the turbulent rough zone but is not suitable for the transition zone or for smooth pipes.

If used in the turbulent rough zone, the Manning formula, which has a

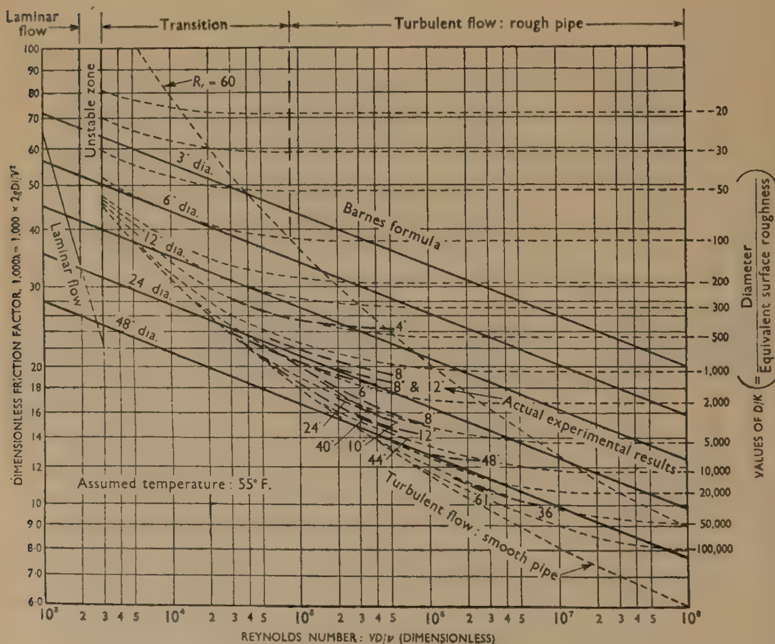
constant which is independent of the velocity, provides a more reliable basis for estimating the friction in old cast-iron pipes than the Hazen-Williams formula.

*Barnes Formula for Asphalted Cast-Iron Pipes*

The Barnes formula for asphalted cast-iron pipes, which has been reproduced graphically in *Fig. 8*, may be expressed in ft-lb.-second units as :

$$V=59{\cdot}85\ D^{0{\cdot}769}\ i^{0{\cdot}529} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

*Fig. 8*




A. A. Barnes's Formula for New Asphalted Cast-Iron Pipes
$V = 59{\cdot}85D^{0{\cdot}769}.\,i^{0{\cdot}529}$

An inspection of *Fig. 8*, where the formula may be compared with the Theory of Roughness and with actual experimental results, reveals that the slope of the diameter lines is suitable for flow in the transition zone but that the spacing of the diameter lines has not been well chosen. The formula agrees reasonably well with experimental results for diameters of 24 inches to 48 inches, but overestimates the friction appreciably if applied to smaller pipes.

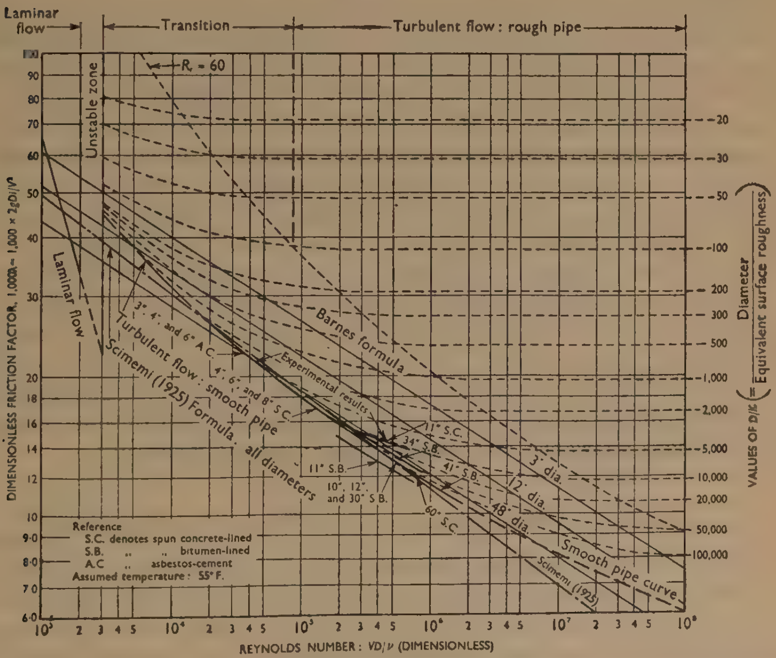
*Barnes Formula for Spun Concrete or Bitumen-Lined Pipes*

The Barnes formula for spun concrete or bitumen-lined pipes, which has been reproduced graphically in *Fig. 9* may be expressed in ft-lb.-second units as :

$$V = 78{\cdot}4\ D^{0{\cdot}72}\ i^{0{\cdot}55} \qquad (8)$$

A comparison of the formula with the smooth-pipe curve and with the experimental results on *Fig. 9* reveals that it agrees reasonably well for large pipes of 30 inches to 60 inches diameter, but that it overestimates

*Fig. 9*




BARNES FORMULA FOR SPUN CONCRETE OR SPUN BITUMEN-LINED PIPES, AND SCIMEMI (1925) FORMULA FOR ASBESTOS-CEMENT PIPES

the friction appreciably if applied to pipes of smaller diameter. The formula is not strictly of the smooth-pipe type since, unlike the Scimemi formula, which is also shown on *Fig. 9*, it does not reproduce as a single straight line on the diagram.

*Scimemi Formula for Asbestos-Cement Pipes*

The Scimemi formula for asbestos-cement pipes (described by Professor Scimemi as his 1925 Formula) may be expressed in ft-lb.-second units as :

$$V = 93{\cdot}7\ D^{0{\cdot}68}\ i^{0{\cdot}56} \qquad (9)$$

A comparison of the formula, which is of the smooth-pipe type, with the smooth-pipe curve and with the experimental results on *Fig. 9*, reveals that it agrees closely over a wide range of Reynolds Numbers. It is a very suitable formula for application in the range $R=10^4$ to $R=10^6$, not only to asbestos-cement pipes but also to other smooth pipes and spun lined pipes.

More recently Professor Scimemi has carried out a series of experiments on asbestos-cement pipes which had been in service for periods of from 16 to 27 years and has developed a new formula (described as his 1950 formula) which gives a discharge 4·2 per cent lower than his 1925 formula, but which is still of the smooth-pipe type with the same indices for $D$ and $i$ as in the 1925 formula. The difference is small and Professor Scimemi attributes it, not to age effects, but to imperfections to the metering method used in his original experiments. It is, however, difficult to reconcile this conclusion with the results of the carefully conducted experiments on new asbestos-cement pipes carried out at the National Physical Laboratory in 1937 which agreed very closely with Scimemi's 1925 formula. As Professor Scimemi mentions "discoloration," "coatings of yellowish film," "slight softening of the pipe surface" and "covered with a thin veil of a dark substance" in describing the internal condition of some of the old pipes tested, it would seem reasonable to conclude that age-effects were at least partly responsible for the slight difference between the two sets of experiments. For practical design purposes, whether the Scimemi 1925 formula (equation (9) ) is used with the customary 5 per cent allowance for age-effects, or whether the Scimemi 1950 formula is used without such allowance, the results will be virtually identical.

## Proposed New Formulae based on Theory of Roughness

Since the Friction-Factor/Reynolds-Number diagram and the Theory of Roughness provide such a convenient testing ground for exponential formulae, they would also appear to provide a logical basis for devising exponential formulae for particular classes of pipe. By this means it should be possible to adjust the slope and spacing of the lines of diameter so as to minimize the overall errors.

This approach has already been adopted for particular types of pipe by Blasius, Colebrook,[5] Prosser,[9] and others, and was also used by the Author in devising a formula for hydraulically smooth pipes for the Manual of British Water Supply Practice.[10]

Although graphical solutions of the Theory of Roughness (see Fig. 11, Plate 1,) provide a rapid, accurate, and convenient method for the practical solution of pipe-friction problems for water and other fluids, a useful purpose would also be served if it were possible to devise a set of mutually associated exponential formulae, based on and expressed in terms of the Theory of Roughness and having a comparable accuracy. With the aid

of such formulae it would be possible to solve pipe-friction problems on a slide-rule having a log-log scale or by means of nomograms or simple logarithmic diagrams of the type usually associated with exponential formulae. Such formulae would be essentially of three types—smooth-pipe, transition, and turbulent-rough—and, ideally, three formulae only would be required, one for each type.

The Author has devised a set of such formulae, but in order to deal with the very wide range of roughness and velocity encountered in practice with a maximum error of approximately ±5 per cent it has been found necessary to increase the number of formulae to seven. Two of these are for smooth pipes, two for the transition zone and three for the turbulent rough zone. With the addition of the Poiseuille Formula for laminar flow, it is therefore possible to express the whole practical range of pipe friction by means of eight basic formulae.

The proposed exponential formulae, which have been shown graphically in Figs 10, Plate 1, and summarized in Tables 4 to 6, incorporate the following features believed to be new :—

(1) They have been deduced not from individual test results, but directly from the Theory of Roughness as reproduced on the Friction-Factor/Reynolds-Number Diagram.
(2) They have been expressed in terms of surface roughness and kinematic viscosity, so as to be applicable to any type of pipe carrying any fluid of known kinematic viscosity.
(3) They are linked together (see Figs 10, Plate 1) to cover with equal accuracy the widest possible range of roughness and velocity, each formula having clearly defined boundaries.

Alternative methods of expressing the formulae to give solutions for quantity, hydraulic gradient, and diameter have been summarized in the

TABLE 4.—PROPOSED TURBULENT-ROUGH-ZONE FORMULAE

| Ref. No. | Formula | Limits $D/K$ | Limits $R$ | Accuracy $V$ | Accuracy $i$ |
|---|---|---|---|---|---|
| R.1 | $\lambda = 0{\cdot}529/(D/K)^{0{\cdot}73}$ <br> $V = 11{\cdot}03\, D^{0{\cdot}865}\, i^{0{\cdot}5}/K^{0{\cdot}365}$ | 2 to 10 | $>3\times10^3$ | ±2·3% | ±4·6% |
| R.2 | $\lambda = 0{\cdot}2578/(D/K)^{0{\cdot}414}$ <br> $V = 15{\cdot}8\, D^{0{\cdot}707}\, i^{0{\cdot}5}/K^{0{\cdot}207}$ | 10 to 200 | $>3\times10^3$ | ±2·7% | ±5·4% |
| R.3 | $\lambda = 0{\cdot}0933/(D/K)^{0{\cdot}22}$ <br> $V = 26{\cdot}28\, D^{0{\cdot}61}\, i^{0{\cdot}5}/K^{0{\cdot}11}$ | 200 to 20,000 | $>8\times10^4$ | ±2·3% | ±4·6% |

TABLE 5.—PROPOSED TRANSITION-ZONE FORMULAE

| Ref. No. | Formula | Limits | | Accuracy | |
|---|---|---|---|---|---|
| | | $D/K$ | $R$ | $V$ | $i$ |
| T.2 | $\lambda = 0{\cdot}459/R^{0\cdot115}(D/K)^{0\cdot2745}$<br>$V = \dfrac{13{\cdot}8\ D^{0\cdot737}\ i^{0\cdot53}}{\nu^{0\cdot061}\ K^{0\cdot1456}}$<br>For water at 55°F.<br>$V = 27{\cdot}45\ D^{0\cdot737}\ i^{0\cdot53}/K^{0\cdot1456}$ | 10 to 200 | $>3\times10^3$<br>$<8\times10^4$ | $\pm2{\cdot}5\%$ | $\pm5{\cdot}0\%$ |
| T.3 | $\lambda = 0{\cdot}2149/R^{0\cdot115}(D/K)^{0\cdot129}$<br>$V = \dfrac{20{\cdot}6\ D^{0\cdot66}\ i^{0\cdot53}}{\nu^{0\cdot061}K^{0\cdot0684}}$<br>For water at 55°F.<br>$V = 40{\cdot}95\ D^{0\cdot66}\ i^{0\cdot53}/K^{0\cdot0684}$ | 200 to 20,000 | $>3\times10^3$<br>$<5\times10^6$ | $\pm3{\cdot}5\%$ | $\pm7{\cdot}0\%$ |

TABLE 6.—PROPOSED SMOOTH-PIPE FORMULAE

| Ref. No. | Formula | Limits | Accuracy | |
|---|---|---|---|---|
| | | $R$ | $V$ | $i$ |
| S.3 | $\lambda = 0{\cdot}262/R^{0\cdot2292}$<br>$V = 22{\cdot}4\ D^{0\cdot6935}\ i^{0\cdot5645}/\nu^{0\cdot1295}$<br>For water at 55°F.<br>$V = 95{\cdot}5\ D^{0\cdot6935}\ i^{0\cdot5645}$ | $>3\times10^3$<br>$<3\times10^5$ | $\pm2{\cdot}3\%$ over $R=3\times$ | $\pm4{\cdot}6\%$ range $10^3$ to $10^6$ |
| S.4 | $\lambda = 0{\cdot}1059/R^{0\cdot158}$<br>$V = 32{\cdot}5\ D^{0\cdot629}\ i^{0\cdot543}/\nu^{0\cdot0858}$<br>For water at 55°F.<br>$V = 85{\cdot}4\ D^{0\cdot629}\ i^{0\cdot543}$ | $>3\times10^5$<br>$<10^8$ | $\pm1{\cdot}9\%$ over $R=10^5$ | $\pm3{\cdot}8\%$ range to $10^8$ |

TABLE 7.—POISEUILLE THEORETICAL FORMULA FOR LAMINAR FLOW

| Formula | Limits | Accuracy | |
|---|---|---|---|
| | $R$ | $V$ | $i$ |
| $\lambda = 64/R$<br>$V = 1{\cdot}006\ D^2 i/\nu$<br>For water at 55°F.<br>$V = 77{,}400\ D^2 i$ | $<2\times10^3$ | at least as great as that of smooth-pipe formulae | |

*Units in Tables 4-7 inclusive*

$V$ denotes velocity in ft/sec.
$K$ ,, equivalent surface roughness in ft
$i$ ,, hydraulic gradient (dimensionless)
$D$ ,, internal diameter in ft
$\nu$ ,, kinematic viscosity in ft$^2$/sec.
$R$ ,, $VD/\nu$=Reynolds Number (dimensionless)
$\lambda$ or $(2gDi/V^2)$ denotes friction factor (dimensionless)

Appendix. For the sake of completeness, the Poiseuille formula has been summarized in Table 7.

The general scheme for the application of the formulae will be seen from their arrangement in Figs 10, Plate 1, but it has also been summarized in Table 8.

TABLE 8.—APPLICATION OF PROPOSED EXPONENTIAL FORMULAE

| Group | Range of $D/K$ | Formula applicable | | |
|---|---|---|---|---|
| | | Smooth pipe | Transition zone | Turbulent rough zone |
| 1 | $D/K$ 2–10 | — | — | R.1 |
| 2 | $D/K$ 10–200 | — | T.2 | R.2 |
| 3 | $D/K$ 200–20,000 | S.3 | T.3 | R.3 |
| 4 | Smooth | S.4 | — | — |

Note :—The above Table applies for $R>3{,}000$. For $R<2{,}000$ the Poiseuille formula applies throughout.

For pipes exhibiting roughness the method of selecting the appropriate formula is as follows :—

(1) The appropriate group is determined by calculating the relative roughness $D/K$.

(2) The appropriate formula in the group is *the one which gives the highest solution for the hydraulic gradient or diameter or the lowest solution for the velocity or quantity.* In other words, the most conservative solution applies throughout. This may be proved by inspection of Figs 10, Plate 1, and illustrates the principle that exponential formulae underestimate the friction when used outside their proper range.

(3) With water at normal temperatures, the appropriate formula in each group can usually be determined by inspection. The formula most likely to apply at normal velocities in each

group has been underlined in Table 8. At extremely low velocities the most likely formula should be checked against the formula on its left and at extremely high velocities it should be checked against the formula on its right.

For smooth pipes, Formula S.3 or S.4 should be used according to the Reynolds Number. For water at normal temperatures, Formula S.3 is generally suitable for pipes up to 12 inches diameter approximately and formula S.4 for pipes more than 12 inches diameter. Without reference to the Reynolds Number or diameter of pipe, the appropriate formula is again the one which gives the highest solution for the hydraulic gradient or diameter or the lowest solution for velocity or quantity.

Although a few new pipes of small diameter come within the range of Group 2, the majority are in Group 3. By substituting the appropriate values of roughness and kinematic viscosity it is possible to devise simple exponential formulae suitable for particular classes of pipe carrying particular fluids. The following formulae for water at 55°F. illustrate this principle :

Galvanized iron, coated cast iron, of 1 inch diameter upwards :

| | | |
|---|---|---|
| S.3 | Smooth | $V = 17{\cdot}0\ d^{0{\cdot}6935}\ i^{0{\cdot}5645}$ |
| T.3 | Transition | $\underline{V = 13{\cdot}52\ d^{0{\cdot}66}\ i^{0{\cdot}53}}$ |
| R.3 | Rough | $V = 13{\cdot}61\ d^{0{\cdot}61}\ i^{0{\cdot}5}$ |

Scobey's concrete Class 3 and Tate relined pipes of 4 inch diameter upwards :

| | | |
|---|---|---|
| S.3 | Smooth | $V = 17{\cdot}0\ d^{0{\cdot}6935}\ i^{0{\cdot}5645}$ |
| T.3 | Transition | $\underline{V = 12{\cdot}30\ d^{0{\cdot}66}\ i^{0{\cdot}53}}$ |
| R.3 | Rough | $V = 11{\cdot}69\ d^{0{\cdot}61}\ i^{0{\cdot}5}$ |

In the above formulae $V$ is in feet per second and $d$ is in inches. The most probable formula has been underlined and need only be checked against the alternative formulae at extreme velocities (smooth for low velocities ; rough for high velocities).

The formulae may also be used for liquids other than water by substituting the appropriate values for viscosity and roughness. For example, the formulae for new steel pipes ($k$=0·002 inch) from 0·4 inch to 40 inches diameter, carrying diesel oil with a kinematic viscosity of 6 centistokes ($6{\cdot}44 \times 10^{-5}$ ft²/sec. units), may be expressed as :

For $R < 2 \times 10^3$

Poiseuille, laminar : $V = 108{\cdot}5\ d^2 i$

For $R > 3 \times 10^3$

S.3 Smooth : $V = 13{\cdot}84\ d^{0{\cdot}6935}\ i^{0{\cdot}5645}$

T.3 Transition : $V = 13{\cdot}08\ d^{0{\cdot}66}\ i^{0{\cdot}53}$

where $V$ is in feet per second and $d$ is in inches. For values of $R$ below

2,000 the Poiseuille formula applies. For values of $R$ above 3,000, the formula (S.3 or T.3) giving the lowest solution for $V$ is the appropriate one. The rough formula R.3 has not been included because it would not apply at velocities likely to occur in practice.

*Example 1*

As an example of the use of the formulae with new pipes, an investigation has been made into the velocity in a 1-inch-diameter coated steel or wrought-iron pipe ($k$=0·002 inch; $d/k$=500; Group 3) carrying water at 55°F. over a wide range of hydraulic gradients, and the solutions have been compared with Blair formula No. 2 and the Hazen-Williams formula ($C$=135).

| | | | | | |
|---|---|---|---|---|---|
| Hydraulic gradient: | 0·002 | 0·01 | 0·10 | 1 | 10 |
| Approximate Reynolds Number: | $3{\cdot}3\times10^3$ | $8{\cdot}2{\cdot}10^3$ | $2{\cdot}7\times10^4$ | $9{\cdot}43\times10^4$ | $3{\cdot}04\times10^5$ |
| Velocity by Roughness Theory: | 0·500 | 1·23 | 4·32 | 14·50 | 47·4 |
| Calculated velocity, ft/sec: | | | | | |
| 1. Proposed smooth formula, S.3: | 0·51 | 1·265 | 4·63 | 17·00 | 61·4 |
| 2. Proposed transition formula, T.3: | 0·53 | 1·250 | 4·26 | 14·40 | 51·4 |
| 3. Proposed rough formula, R.3: | 0·67 | 1·505 | 4·76 | 15·05 | 47·5 |
| *Compare* | | | | | |
| 4. Blair formula No. 2: | 0·49 | 1·193 | 4·29 | 15·40 | 55·1 |
| 5. Hazen-Williams formula ($C$=135): | 0·54 | 1·295 | 4·48 | 15·52 | 53·8 |

(Solutions: lowest values of $V$ have been underlined.)

The results illustrate the method of choosing the lowest solution for velocity from formulae S.3, T.3, and R.3. The solution obtained from these formulae is within 3 per cent of the roughness theory over the extreme range of hydraulic gradients selected. The Blair formula and the Hazen-Williams formula are reasonably accurate within most of the transition zone but overestimate the velocity appreciably at the upper end of the velocity range.

*Example 2*

As an example of the use of the formulae for smooth pipes, an investigation has been made into the velocity in a 12-inch-diameter smooth pipe (say spun concrete or bitumen lined) carrying water at 55°F. over a wide range of hydraulic gradients and the results compared with the Blair, Hazen-Williams, and Scimemi formulae. This example has been deliberately chosen to include both formulae S.3 and S.4.

| | | | | | | |
|---|---|---|---|---|---|---|
| Hydraulic gradient : | 0·00001 | 0·0001 | 0·001 | 0·01 | 0·1 | 1·0 |
| Approx. Reynolds Number : | $1{\cdot}11\times10^4$ | $4{\cdot}03\times10^4$ | $1{\cdot}49\times10^5$ | $5{\cdot}38\times10^5$ | $1{\cdot}88\times10^6$ | $6{\cdot}56\times10^6$ |
| Velocity by Roughness Theory : | 0·1455 | 0·541 | 1·963 | 7·035 | 24·9 | 86·0 |
| Calculated velocity ft/sec : | | | | | | |
| Smooth formula, S.3 : | 0·1438 | 0·524 | 1·934 | 7·08 | 26·0 | 95·5 |
| Smooth formula, S.4 : | 0·1645 | 0·573 | 2·01 | 7·00 | 24·45 | 85·4 |
| *Compare* | | | | | | |
| Blair Formula No. 1 : | 0·1445 | 0·537 | 2·005 | 7·43 | 27·62 | 102·6 |
| Hazen-Williams Formula ($C$=150) : | 0·1645 | 0·569 | 1·985 | 6·86 | 23·82 | 82·5 |
| Scimemi Formula : | 0·1482 | 0·541 | 1·956 | 7·09 | 25·80 | 93·7 |

(Solutions : lowest values of $V$ have been underlined.)

The results again illustrate the method of choosing the lower solution for velocity from formulae S.3 and S.4. The solution obtained from the formulae is within 2 per cent of the smooth-pipe law over the extreme range of velocity covered. The Blair, Hazen-Williams, and Scimemi formulae are all within 2 per cent of the smooth-pipe curve over part of the velocity range only.

*Example 3*

As an example of the use of the formulae for fluids other than water, an investigation has been made into the velocity in a 4-inch-diameter coated steel pipe ($k$=0·002 inch : $d/k$=2,000 : Group 3) carrying diesel oil with a kinematic viscosity of 6 centistokes ($6{\cdot}44\times10^{-5}$ ft²/sec.) over a wide range of hydraulic gradients.

| | | | | | |
|---|---|---|---|---|---|
| Hydraulic gradient : | 0·0001 | 0·001 | 0·01 | 0·1 | 1·0 |
| Approx. Reynolds Number : | 895 | $3\times10^3$ | $1{\cdot}2\times10^4$ | $5\times10^4$ | $1{\cdot}65\times10^5$ |
| Velocity by Roughness Theory : | 0·173 | 0·71 | 2·74 | 9·60 | 33·2 |
| Calculated velocity, ft/sec : | | | | | |
| Poiseuille Formula, laminar : | 0·173 | — | — | — | — |
| Smooth formula, S.3 : | — | 0·73 | 2·81 | 9·84 | 36·15 |
| Transition formula, T.3 : | — | 0·84 | 2·85 | 9·65 | 32·75 |

(Solutions : lower values of $V$ have been underlined.)

The results illustrate the close agreement between the appropriate formula and the theory of roughness ; the maximum difference being approximately 4 per cent.

## Practical Design Construction

*Minor Pipeline Losses*

The recommended values of surface roughness given in Tables 1 to 3 include the effect of joints, but in the practical design of pipelines allowance must also be made for the effect of bends, valves, tapers, etc. and for inlet and exit losses. Wherever possible these should be calculated separately by means of Tables similar to that published in the Manual of British Water Supply Practice.[10] In waterworks trunk mains the effect of these losses normally varies between 2 and 20 per cent of the friction head, the relative effect being greater in short mains and in smooth mains.

In the absence of a more precise determination, an allowance of 5 per cent of the friction head will usually be sufficient in the case of cast-iron mains designed for age effect. For spun lined and other hydraulically smooth mains where little or no age effect is to be expected, the allowance should be increased to 10 per cent.

*Age Effects*

The extent to which the carrying capacity of a waterworks pipeline deteriorates in service depends principally upon the effect of the water on the walls of the pipeline, which may take the form of corrosion or other chemical action, erosion, the deposit of sediment, the formation of slime, or a combination of these factors.

In hydraulically smooth non-ferrous pipes or spun lined pipes carrying clean water little or no age effect is to be expected and an allowance of 5 per cent (on quantity) should be sufficient. The same allowance should also be ample in concrete pipes carrying clean water, provided that the water is not chemically agressive to cement.

In spun lined and concrete pipes carrying raw upland water which has a tendency to deposit a coating of slime on the pipe walls, the reduction of carrying capacity after 10 or 20 years' service may be 25 per cent or, in extreme cases, 50 per cent of the original capacity.

In coated cast-iron mains carrying treated or raw water the reduction of capacity after 30 years' service may be from 15 per cent to 70 per cent of the original capacity according to the degree of corrosive attack and the diameter of the main.

The conception of the uniform rate of growth of roughness[11] appears to be the most rational method of assessing age-effects in cast-iron pipes and in pipes subject to slime formation. The roughness figures for cast-iron pipes given in Table 3(1), which are based on British records, may be of service in design.

The subject of age-effects has been dealt with more fully by the Author in another Paper.[2]

*Design Diagrams*

As an aid to practical design, Figs 10 to 14, Plates 1 and 2, have been prepared. Figs 10 to 13 give graphical solutions of the proposed exponential formulae for pipes carrying water over a wide range of diameter and velocity. They could also be used for other fluids if the solutions were corrected for viscosity according to the details in Appendix 1. Figs 14, which is a graphical solution of the Theory of Roughness, is applicable not only to water but to any fluid of known viscosity.

## Summary and Conclusions

*Theory of Roughness*

The Theory of Roughness, which is supported by ample experimental evidence provides the most rational basis for the solution of pipe-friction problems. Although admirably concise mathematically, the dimensionless equations comprising the theory are generally considered to be too complicated for normal design purposes, but graphical solutions such as that shown in Fig. 14,* Plate 2 provide a rapid and convenient method of applying the theory to practical design problems with water and other fluids.

*Existing Exponential Formulae*

The Hazen-Williams formula and Blair's formulae are reasonably accurate when applied to new pipes carrying water within the transition zone. The Scimemi formula for asbestos-cement pipes is also a good formula, not only for asbestos cement pipes but for smooth pipes and spun lined pipes generally. The Barnes formulae for asphalted cast-iron and for spun lined pipes have severe limitations and their use is not recommended.

The Manning formula is of the correct type for the turbulent rough zone and is more suitable for old pipes than the Hazen-Williams formula.

*Proposed Exponential Formulae*

The proposed formulae, which are expressed in the same terms as the Theory of Roughness, provide a method for the solution of pipe-friction problems with water and other liquids over a very wide range of roughness and velocity by means of a log-log slide rule, logarithmic tables or the design charts given in Figs 10 to 13, Plates 1 and 2. It is not suggested that they represent an improvement upon the graphical solution of the Theory of Roughness (Fig. 14, Plate 2) to which they are a straight-line approximation, but they give substantially the same results over the entire range of roughness and velocity and enable direct solutions to be obtained in every case, whereas solutions by successive approximation have to be employed in certain cases (velocity, quantity, or diameter unknown) with Fig. 14.

* Large-size unfolded copies of Fig. 14 may be obtained through the Secretary of the Institution.

They may also be used to deduce simple exponential formulae for pipes carrying particular fluids (for example steel pipes carrying crude oil) and are of a form from which simple logarithmic design charts can be made.

In comparing the proposed formulae with existing exponential formulae of the smooth-pipe and transition-zone types for pipes carrying water, it is not claimed that they are more accurate within the proper working range of existing formulae. Their advantage lies in the fact that their range of application is clearly defined, each formula being replaced by another at the limit of its range. On the other hand, the range of application of existing formulae is seldom defined and there is a danger that they may be misapplied outside their proper range. The proposed formulae have the important additional advantage over existing exponential formulae of being applicable to fluids other than water.

In the turbulent rough zone, it is considered that the three proposed rough-pipe formulae (R.1, R.2, and R.3), which cover an extremely wide range of relative roughness (2 to 20,000), are an improvement upon the Manning formula, the coefficient of which is not truly independent of the diameter and which, unlike the surface roughness, cannot be predicted by any rational method.

## Acknowledgements

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The Paper is accompanied by sixteen sheets of diagrams and charts, from some of which folding Plates 1 and 2 and the Figures in the text have been prepared, and by the following Appendix.

## APPENDIX

### LAMONT'S PROPOSED EXPONENTIAL FORMULAE

### ALTERNATIVE METHODS ON EXPRESSING FORMULAE ARRANGED IN GROUPS ; EACH GROUP IN ASCENDING ORDER OF VELOCITY

*Units*

$V$ denotes mean velocity in feet per second.
$G$ ,, rate of flow in Imperial gallons per minute.
$d$ ,, internal diameter of pipe in inches.
$i$ ,, hydraulic gradient (dimensionless).
$k$ ,, equivalent surface roughness in inches.
$\nu$ ,, kinematic viscosity of fluid in sq. ft per sec. (taken as $1{\cdot}3 \times 10^{-5}$ for water at 55°F.).

*GROUP 1* (*D/K* from 2 to 10)

Rough Formula R.1, applicable to any fluid, including water at 55°F.

$$V = 3{\cdot}185\, d^{0.865}\, i^{0.5}/k^{0.365}$$
$$G = 6{\cdot}48\, d^{2.865}\, i^{0.5}/k^{0.365}$$

$$i = k^{0.73}\, V^2/10{\cdot}13\, d^{0.173} = k^{0.73}\, G^2/42\, d^{5.73}$$
$$d = k^{0.422}\, V^{1.157}/3{\cdot}815\, i^{0.578} = k^{0.1275}\, G^{0.35}/1{\cdot}92\, i^{0.175}$$

*GROUP 2* (*D/K* from 10 to 200)

Transition Formula T.2

For any fluid

$$V = 3{\cdot}175\, d^{0.737}\, i^{0.53}/\nu^{0.061} k^{0.1456}$$
$$G = 6{\cdot}46\, d^{2.737} i^{0.53}/\nu^{0.061} k^{0.1456}$$

$$i = \nu^{0.115} k^{0.275} V^{1.885}/8{\cdot}85\, d^{1.39} = \nu^{0.115} k^{0.275} G^{1.885}/33{\cdot}85\, d^{5.17}$$
$$d = \nu^{0.0828} k^{0.1975} V^{1.357}/4{\cdot}8\, i^{0.72} = \nu^{0.0223} k^{0.0582} G^{0.365}/1{\cdot}979\, i^{0.194}$$

For water at 55°F.

$$V = 6{\cdot}315\, d^{0.737}\, i^{0.53}/k^{0.1456}$$
$$G = 12{\cdot}85\, d^{2.737}\, i^{0.53}/k^{0.1456}$$

$$i = k^{0.275} V^{1.885}/32{\cdot}3\, d^{1.39} = k^{0.275} G^{1.885}/123{\cdot}5\, d^{5.17}$$
$$d = k^{0.1975} V^{1.357}/12{\cdot}19\, i^{0.72} = k^{0.0582} G^{0.365}/2{\cdot}546\, i^{0.194}$$

## PLATE I
## PIPE-FRICTION FORMULAE

| T.2 | T.3 |
|---|---|
| $K^{-0.1456}\,D^{0.737}\,i^{0.53}$ | $V=20{\cdot}6\nu^{-0.061}\,K^{-0.0684}\,D^{0.66}\,i^{0.53}$ |
| $-0.1456\,D^{0.737}\,i^{0.53}$ | $V=40{\cdot}95\,K^{-0.0684}\,D^{0.66}\,i^{0.53}$ |
| 2<br>0–200<br>$)^3$ to $8\times10^4$ | 3<br>200–20,000<br>$3\times10^3$ to $5\times10^6$ |
| ±2·5%<br>±5·0% | ±3·5%<br>±7·0% |

Foot-lb.-second units

ction with smooth- and rough-pipe diagrams. Select ic gradient or diameter, or lowest value of velocity or he diagrams.

n iron main (I.D.= 3·18″) carries 1,800 g.p.h. Find (A) when smooth and new ($k$ = 0·005″), and after 30 slightly corrosive water ($k_{30}$ = 0·035″); and (C) with ter ($k_{30}$ = 0·095″).

| S.4 |
|---|
| $-0.0858\,D^{0.6}$ |
| $\cdot4\,D^{0.629}\,i$ |
| $=10^5$ to 1( |
| ±1·9%<br>±3·8% |

)5 = 636 (Group 3)
$i$ = 3·5/1,000
: $i$ = 3·2/1,000
035 = 91 (Group 2)
$i$ = 5·0/1,000
$i$ = 4·9/1,000
095 = 32·2 (Group 2)
$i$ = 6·5/1,000
$i$ = 7·4/1,000

her figure in each case) are underlined.

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gram for pi

P. A. LAMONT

LONDON

Rough Formula R.2, applicable to any fluid, including water at 55°F.

$$V = 4{\cdot}56\ d^{0{\cdot}707}\ i^{0.5}/k^{0{\cdot}207}$$
$$G = 9{\cdot}27\ d^{2{\cdot}707}\ i^{0.5}/k^{0{\cdot}207}$$

$$i = k^{0{\cdot}414} V^2/20{\cdot}8\ d^{1{\cdot}414} = k^{0{\cdot}414}\ G^2/86\ d^{5{\cdot}414}$$
$$d = k^{0{\cdot}293} V^{1{\cdot}414}/8{\cdot}55\ i^{0{\cdot}707} = k^{0{\cdot}0765}\ G^{0{\cdot}37}/2{\cdot}275\ i^{0{\cdot}185}$$

*GROUP 3* ($D/K$ from 200 to 20,000)

Smooth Formula S.3

For any fluid

$$V = 3{\cdot}965\ d^{0{\cdot}6935}\ i^{0{\cdot}5645}/\nu^{0{\cdot}1295}$$
$$G = 8{\cdot}063\ d^{2{\cdot}6935}\ i^{0.5645}/\nu^{0{\cdot}1295}$$

$$i = \nu^{0{\cdot}229} V^{1{\cdot}77}/11{\cdot}45\ d^{1{\cdot}23} = \nu^{0.229} G^{1{\cdot}77}/40{\cdot}4\ d^{4{\cdot}77}$$
$$d = \nu^{0{\cdot}187} V^{1{\cdot}44}/7{\cdot}29\ i^{0{\cdot}813} = \nu^{0{\cdot}0481} G^{0.372}/2{\cdot}17\ i^{0{\cdot}210}$$

For water at 55°F.

$$V = 17\ d^{0{\cdot}6935}\ i^{0{\cdot}5645}$$
$$G = 34{\cdot}6\ d^{2{\cdot}6935}\ i^{0{\cdot}5645}$$
$$i = V^{1{\cdot}77}/151\ d^{1{\cdot}23} = G^{1{\cdot}77}/532\ d^{4{\cdot}77}$$
$$d = V^{1{\cdot}44}/59{\cdot}6\ i^{0{\cdot}813} = G^{0{\cdot}372}/3{\cdot}73\ i^{0{\cdot}210}$$

Transition Formula T.3

For any fluid

$$V = 4{\cdot}74\ d^{0{\cdot}66}\ i^{0{\cdot}53}/\nu^{0.061} k^{0{\cdot}0684}$$
$$G = 9{\cdot}65\ d^{2{\cdot}66}\ i^{0.53}/\nu^{0{\cdot}061} k^{0{\cdot}0684}$$

$$i = \nu^{0{\cdot}115} k^{0.129} V^{1{\cdot}885}/18{\cdot}9\ d^{1{\cdot}244} = \nu^{0{\cdot}115} k^{0.129} G^{1{\cdot}885}/72\ d^{5{\cdot}02}$$
$$d = \nu^{0{\cdot}0924} k^{0{\cdot}1037} V^{1.515}/10{\cdot}6\ i^{0{\cdot}803} = \nu^{0{\cdot}0229} k^{0{\cdot}0257} G^{0.376}/2{\cdot}345\ i^{0{\cdot}199}$$

For water at 55°F.

$$V = 9{\cdot}42\ d^{0{\cdot}66}\ i^{0.53}/k^{0{\cdot}0684}$$
$$G = 19{\cdot}15\ d^{2{\cdot}66}\ i^{0.53}/k^{0{\cdot}0684}$$

$$i = k^{0{\cdot}129} V^{1{\cdot}885}/69{\cdot}0\ d^{1{\cdot}244} = k^{0.129} G^{1{\cdot}885}/262\ d^{5{\cdot}02}$$
$$d = k^{0{\cdot}1037} V^{1{\cdot}515}/30\ i^{0{\cdot}803} = k^{0{\cdot}0257} G^{0{\cdot}376}/3{\cdot}035\ i^{0{\cdot}199}$$

Rough Formula R.3, applicable to any fluid, including water at 55°F.

$$V = 7{\cdot}59\ d^{0{\cdot}61}\ i^{0.5}/k^{0{\cdot}11}$$
$$G = 15{\cdot}42\ d^{2{\cdot}61}\ i^{0.5}/k^{0{\cdot}11}$$

$$i = k^{0{\cdot}22} V^2/57{\cdot}7\ d^{1{\cdot}22} = k^{0{\cdot}22} G^2/238\ d^{5{\cdot}22}$$
$$d = k^{0{\cdot}18} V^{1{\cdot}64}/27{\cdot}7\ i^{0{\cdot}82} = k^{0{\cdot}0422} G^{0{\cdot}384}/2{\cdot}85\ i^{0{\cdot}192}$$

*GROUP 4* (Large Smooth Pipes $R = 3 \times 10^5$–$10^8$)

Smooth Formula S.4

For any fluid

$$V = 6{\cdot}81\ d^{0{\cdot}629}\ i^{0{\cdot}543}/\nu^{0{\cdot}0858}$$
$$G = 13{\cdot}85\ d^{2{\cdot}629}\ i^{0{\cdot}543}/\nu^{0{\cdot}0858}$$

$$i = \nu^{0{\cdot}158} V^{1.84}/34{\cdot}25\ d^{1{\cdot}159} = \nu^{0{\cdot}158} G^{1{\cdot}84}/126{\cdot}5\ d^{4.84}$$
$$d = \nu^{0{\cdot}1365} V^{1{\cdot}59}/21{\cdot}15\ i^{0{\cdot}863} = \nu^{0{\cdot}0327} G^{0{\cdot}38}/2{\cdot}72\ i^{0{\cdot}207}$$

For water at 55°F.

$$V = 17{\cdot}89\ d^{0{\cdot}629}\ i^{0{\cdot}543}$$
$$G = 36{\cdot}35\ d^{2{\cdot}629}\ i^{0{\cdot}543}$$

$$i = V^{1{\cdot}84}/203\ d^{1{\cdot}159} = G^{1{\cdot}84}/749\ d^{4{\cdot}84}$$
$$d = V^{1.59}/98{\cdot}2\ i^{0{\cdot}863} = G^{0{\cdot}38}/3{\cdot}93\ i^{0{\cdot}207}$$

Note :—For small and medium smooth pipes in the range $R = 3 \times 10^3$ to $3 \times 10^5$ use Smooth Formula S.3.

Paper No. 5964

# "The Elimination of Moments in Shell Roofs by Prestressing"

by

**Professor William Thomas Marshall, B.Sc., Ph.D., M.I.C.E.**

(*Ordered by the Council to be published with written discussion.*) †

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## SYNOPSIS

The main difficulty in the design of shell roofs is the calculation of the transverse bending moment. This moment is caused by the deflexion of the shell under loading. Deflexions of opposite magnitude to those from the applied load can be induced by prestressing, and it should therefore be possible in shell-roof design to eliminate the deflexion due to the applied load. If such deflexion is eliminated, then the secondary stresses in the shell will be eliminated and there will be no transverse bending moments. The design will therefore be very much simplified.

The Paper shows that, whilst it is not generally possible to eliminate the secondary stresses by prestressing, it is nevertheless possible to eliminate the transverse bending moments, since the final stresses at all points in the shell can be made equal to the membrane stress plus a constant value. This constant is what would be obtained if a load were applied along the axis of the shell. This latter load can produce no transverse bending. A worked example is given, showing how the cables can be arranged so that transverse bending is eliminated at all points in the shell.

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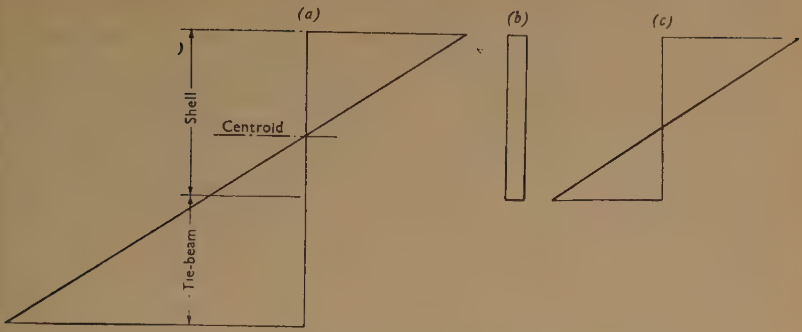
Long shells of thicknesses commonly used in roof construction can be designed as beams giving a distribution of longitudinal stress (generally referred to as $T_1$) at mid-span, as shown in *Fig. 1* (*a*). If the shell is designed as a thin membrane, then the stress distribution in the shell itself, that is, without the edge beams, is as shown in *Fig. 1* (*b*). The membrane condition has no bending moment and the difference (as shown in *Fig. 1* (*c*)) between the beam stress as shown in *Fig. 1* (*a*) and the membrane stress shown in *Fig. 1* (*b*) represents the "secondary" stress produced by the deformations of the shell under loading. This is seen to be almost linear in distribution. In addition to altering very considerably the distribution of the longitudinal stress, the deformations give rise to bending moments. The transverse bending moment, $M_2$, is generally a critical factor in the design of shells. The calculation of $M_2$ is the main difficulty in shell design, and the object of this short Paper is to show how the value of $M_2$ can be made negligible by prestressing.

† Correspondence on this Paper should be received at the Institution by the 15th August, 1954, and will be published in Part III of the Proceedings. Contributions should be limited to about 1,200 words.—Sec. I.C.E.

*Figs 1*




If the suffix $b$ denotes stress under load but without prestress ;
$m$ ,, membrane stress ;
$s$ ,, secondary stress ;
$p$ ,, prestress ;
and $f$ ,, the final stress after the prestress has been applied,

then $b = m + s$ or $s = b - m$

and $f = b + p = m + s + p$.

Theoretically, any appropriate prestress, $p$, can be applied and if $p = -s$ then the final stress $f = m$, the membrane stress. If this state of affairs exists, then the final deflected form of the shell is the same as its initial unloaded form and no secondary bending occurs. This means that the $M_2$ term vanishes and the design is considerably simplified.

There are, however, difficulties in the application of a prestress $p$ which is equal and opposite to $s$. Taking the shell shown in *Fig. 2* as a beam, the stresses induced by a prestressing force $P$ are given by

$$p = \frac{P}{A}\left(1 \pm \frac{ey}{k^2}\right)$$

where $A$ denotes cross-sectional area of beam (that is, shell plus edge beams) ;
$Ak^2$ ,, second moment of area of beam section about an axis through its centroid ;
$e$ ,, eccentricity of prestressing force measured from this axis ;
and $y$ ,, distance of fibre under investigation from the centroid.

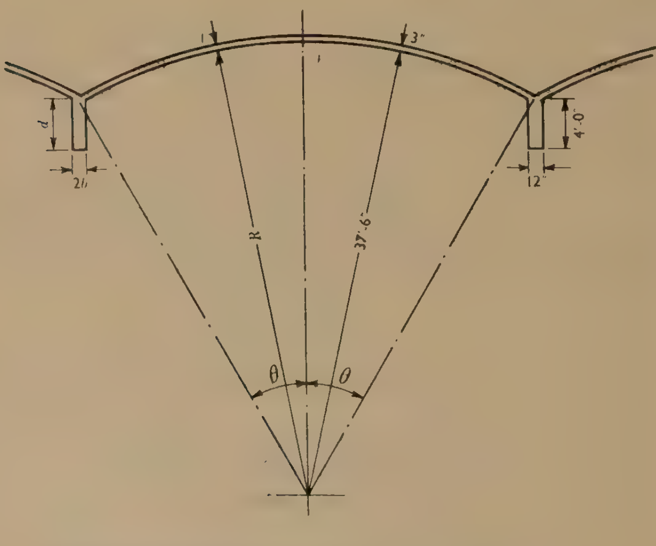
The fibre at which the prestress $p$ is zero is at a distance $\frac{k^2}{e}$ above the centroid, and it is seen from *Fig. 1* (*c*) that, if the prestress is at all points to be equal to $-s$, then the value of $\frac{k^2}{e}$ must be small. The value of $k^2$

depends on the geometry of the shell whilst $e$ is governed by the necessity of keeping the cables within the edge beams.

Calculations on a few shells of normal dimensions showed that in general it is not possible to have a value of $e$ which is large enough to enable the designer to make $p = -s$. It was found, however, that it is generally possible to make $p = -s + c$ where $c$ is a stress which is constant for all values of $y$. Such a stress is equivalent to that produced by a load acting horizontally along the centroid of the beam. Neglecting the effects of buckling, such a load can produce no deformations at right angles to its line of application and does not, therefore, alter the deflected form of the shell. In other words, the final deflected form of the shell corresponds to the membrane condition and the secondary bending is eliminated.

The amount of prestress and eccentricity is calculated in the first instance for the mid-span section. As the supports are approached the value of the prestress falls and its distribution is different from that at mid-span. The total prestressing force cannot vary, of course, but by adjusting the position of the cables the eccentricity can be made so that at all sections of the span $p = -s + c$.

*Figs 2*




## Example showing Application of Method

In a previous Paper,[1] the Author used as an example the shell shown in *Fig. 2*, which has a thickness of 3 inches, a radius of curvature of 37 feet

[1] W. T. Marshall, "A Method of Determining the Secondary Stresses in Cylindrical Shell Roofs." *J. Instn Civ. Engrs*, vol. 33, p. 126 (December 1949).

6 inches, a span of 118 feet, and a central angle of 60 degrees (0·5236 radian). The valley beams are 12 inches wide and 4 feet deep and the shell is taken as one of a series. With the notation used in the *Fig. 2*, $A = 2(Rt\theta + bd) = 13{\cdot}8$ square feet; $I = 112{\cdot}23$ feet$^4$ units, hence $k^2 = 8{\cdot}15$ feet.$^2$ The distance $x$ of the centroid from the springing is 1·87 foot. The rise of the shell is 5·03 feet, giving the distances of the most stressed compressive and tensile fibres from the centroid as 3·16 and 5·87 feet respectively.

The dead weight of the shell and valley beams is 1,986 lb. per foot run, the central bending moment is $\left(\frac{1{,}986 \times 118^2}{8}\right)$ foot-lb., and the stresses at the crown and springing of the shell produce loads of 24,300 and 14,360 lb. per foot width at these sections, the former being compressive and the latter tensile. The secondary stresses are the difference between the beam stresses and the membrane stresses. In Table 1, the values of

TABLE 1.—VALUES OF STRESSES

| Distance from centre of span | Position | Stresses: lb. per foot width | | |
|---|---|---|---|---|
| | | Beam | Membrane | Secondary |
| 0 | Crown<br>Centroid<br>Springing | −24,300<br>0<br>+14,360 | −3,330<br>−3,040<br>−2,880 | −20,970<br>+ 3,040<br>+17,240 |
| $\frac{L}{8}$ | Crown<br>Centroid<br>Springing | −22,750<br>0<br>+13,440 | −3,120<br>−2,850<br>−2,700 | −19,630<br>+ 2,850<br>+16,140 |
| $\frac{L}{4}$ | Crown<br>Centroid<br>Springing | −18,220<br>0<br>−10,760 | −2,500<br>−2,280<br>−2,160 | −15,720<br>+ 2,280<br>+12,930 |
| $\frac{3L}{8}$ | Crown<br>Centroid<br>Springing | −10,620<br>0<br>+ 6,280 | −1,460<br>−1,332<br>−1,260 | − 9,160<br>+ 1,332<br>+ 7,540 |

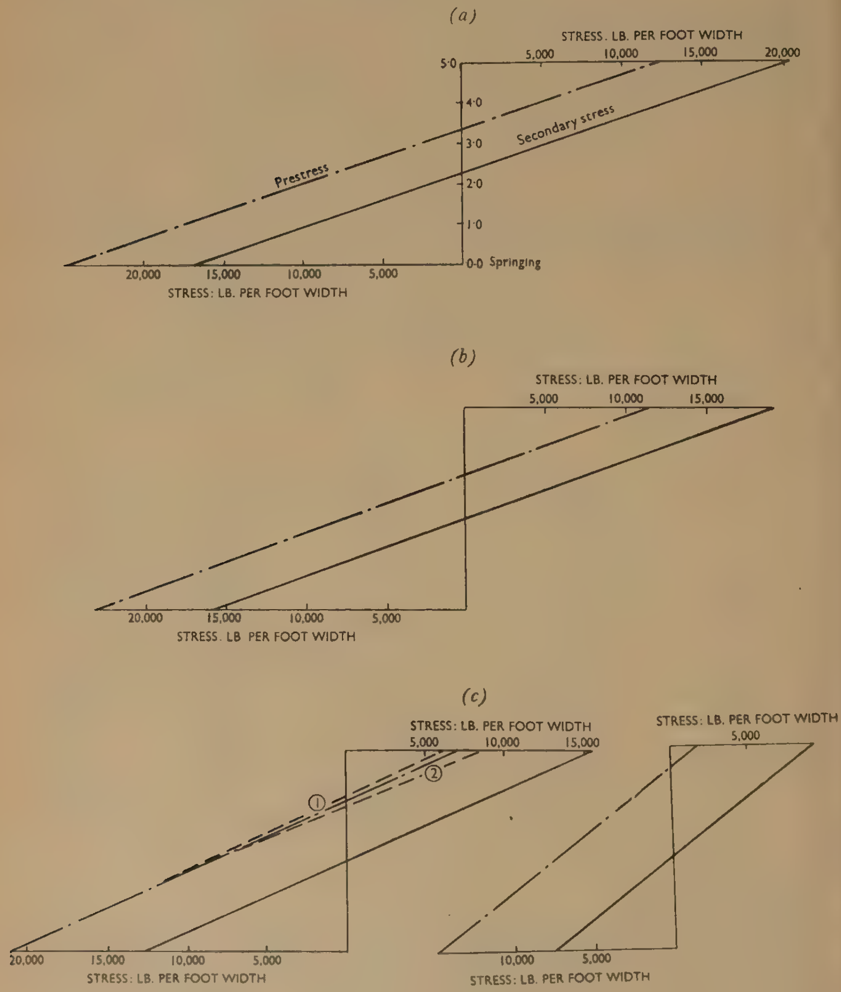
the stresses are given for the crown, centroid, and springing at different positions on the span.

The distribution of secondary stress at each section is shown diagrammatically in *Figs 3* (*a*) to *3* (*d*) respectively.

The centre of the prestressing cable is taken as being 4 inches above the beam soffit, thus $e = 4{\cdot}00 + 1{\cdot}87 - 0{\cdot}33 = 5{\cdot}54$ feet. With this eccentricity the position of zero prestress is at $\frac{8{\cdot}15}{5{\cdot}54} = 1{\cdot}47$ foot above the centroid. A line is now drawn through this point parallel to the secondary

stress distribution. This line gives the distribution of prestress necessary to ensure a constant difference between the secondary stress and prestress, thus making the final stress at all points equal to the membrane stress plus a constant.

*Figs 3*




At mid-span it is necessary for the prestress to be equal to 13,000 lb. per foot width, or a stress of 52,000 lb. per square foot. Thus the total prestressing force required is given by the equation :

$$52{,}000 = \frac{P}{13{\cdot}8}\left(-1 + \frac{5{\cdot}54 \times 3{\cdot}16}{8{\cdot}15}\right)$$

giving $P = 626{,}000$ lb.

This prestressing force is constant along the beam. The distribution of prestress required, namely, $= -s +$ constant, varies from section to section, depending on the distribution of $s$. This variation in distribution of prestress can be produced by bending up the cable. The calculation of the required eccentricity at any given section is best carried out by a simple trial-and-error process. A Table similar to that shown in Table 2 is prepared, giving the relationship between $e$, the eccentricity of the cable measured from the centroid, the intensity of fibre prestress produced by the prestressing force calculated for mid-span, and $y_n$, the distance of the point of zero prestress from the centroid.

TABLE 2.—PRESTRESS DISTRIBUTION WITH VARYING $e$

| $e$ : feet | 5·2 | 5·0 | 4·8 | 4·5 | 4·0 | 3·5 | 3·0 |
|---|---|---|---|---|---|---|---|
| Fibre stress : lb. per foot width | 11,600 | 10,680 | 9,780 | 8,460 | 6,270 | 4,080 | 1,860 |
| $y_n$ : feet . . . | 1·57 | 1·63 | 1·70 | 1·81 | 2·04 | 2·33 | 2·72 |

As an example, consider the section distant $\frac{L}{4}$ from the centre, the distribution of stress being as shown in *Fig. 3* (*c*). In order to find the required $e$, the line marked 1 is first drawn, corresponding with $e = 4{\cdot}0$. It is seen that this line is not satisfactory since it is not parallel to the secondary stress line. Line 2 is then drawn to correspond with $e = 4{\cdot}5$. It is seen that this line is not satisfactory, but that the line of prestress distribution parallel to the secondary stress distribution lies between lines 1 and 2. This line has a value of $y_n = 1{\cdot}92$, corresponding with $e = 4{\cdot}25$.

The other values of $e$ are as follows. At $\frac{L}{8}$ from centre, $e = 5{\cdot}3$ feet; at $\frac{3L}{8}$ from centre, $e = 3{\cdot}0$ feet. With these values of $e$ and a prestressing force of 626,000 lb. the final forces in the shell are the membrane force plus a compression of the order of 8,000 lb. per foot width. This compression must be added to the membrane stress at any point to get the final stress. It tends to reduce the tensile stresses and is therefore an advantage from the reinforced-concrete viewpoint.

In the example, the only load taken is the dead load. In practice a snow load has generally to be allowed for. This additional load produces

secondary stresses which cannot be counterbalanced by the original prestress, and therefore produces a transverse moment. Such moments for normal shells are, however, very much smaller than those which would occur under the total load had no prestress been applied, and require little reinforcement to resist them. Alternatively the amount of prestress can be made equal to the mean between the values required to eliminate transverse moments under normal loading alone and with snow loading. This produces transverse moments of opposite sign under dead load to those which occur under the snow load, but in each case the magnitude is less than that occurring if the prestress eliminated only the dead load moment.

The Paper is accompanied by one sheet of diagrams, from which the Figures in the text have been prepared.

Paper No. 5982

# "The Use of Ultrasonic Vibrations in Public Health Engineering"

by

**H. R. Oakley, M.Sc.(Eng.), A.M.I.C.E.,**
**J. A. Philpott, Ph.D., B.Sc.(Eng.), Grad.I.C.E., and**
**Z. B. Abdalla, M.Sc.(Eng.), Stud.I.C.E.**

(*Ordered by the Council to be published in abridged form.*) †

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SYNOPSIS

Established methods for the treatment of water and sewerage are of proved efficiency and reliability. Nevertheless, there is a continuous search for new and better methods.

The propagation in water of waves of ultrasonic frequency is known to give rise to a number of interesting phenomena, and in recent years such vibrations have been applied to problems as diverse as food processing and laundering. The Paper describes briefly experimental research into the application of ultrasonic vibrations to a range of public health engineering problems. It deals with the effect of the vibrations on the aeration of water, on bacteria and algae, and on the sedimentation of suspended solids.

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## Introduction

Present day purification of water for domestic supply, and the treatment of sewerage for safe disposal into rivers and streams, is largely carried out by established methods, the economy and efficiency of which is amply demonstrated by the results achieved in transforming polluted waters into cheap and safe potable supplies. Nevertheless, the search for new and better methods is continuous, and in public health engineering, as in other branches of applied science, there is an ever widening field of investigation and research directed towards the effective application of new knowledge and experience.

Public health engineering has been taught at University College, London, since the institution and endowment in 1895 of the Chadwick professorship and laboratory. The laboratory, which was badly damaged during the 1939–1945 war, has recently been re-equipped, and research in this branch of engineering has been recommenced as one of the seven postgraduate groups of the Department of Civil and Municipal Engineering. One of the

† The full MS. and illustrations may be seen in the Institution Library. Sec. I.C.E.

first investigations, undertaken at the request of the Institution of Water Engineers, was to examine the possible uses of ultrasonic vibrations in water purification.

## Ultrasonic Vibrations

The human ear is sensitive to sound waves of frequencies from about 100 to 18,000 cycles per second. Vibrations of a similar type, but beyond the top limit of the audible range, are termed ultrasonic, and can be produced up to frequencies of $5 \times 10^8$ cycles per second.

The physical behaviour of ultrasonic vibrations in air, water, and other mediums was extensively studied in the two decades immediately before the 1939–1945 war, and a number of interesting phenomena have been observed.[1] Unfortunately, very few quantitative results are available which enable the value of ultrasonic vibrations in the various applications to be assessed.

The investigations described in this Paper fall into three main groups : (1) the use of vibrations to increase the rate of solution of oxygen from air bubbles, (2) the effect on bacteria and algae in water, and (3) methods of increasing the rate of sedimentation of suspended particles.

Vibrations of four frequencies were used : 19·5 kilocycles per second (just above the sonic range), generated by a magneto-striction assembly, having a maximum output of 150 watts on a flat face 12·5 centimetres square ; and 300, 650, and 1,000 kilocycles per second, generated by quartz crystal transducers, with maximum outputs of 25 watts on a face of 4·75 centimetres diameter. Details of the apparatus used to generate and measure the vibrations are given elsewhere,[2, 3]

## The Aeration of Water

Many of the effects known to be produced by ultrasonic vibrations in water are associated with the presence of dissolved gas[4] and, under certain conditions, the vibrations may effect a slow degassing of the water.[5] Little is known, however, of the influence of the vibrations on the rate of solution of a gas from free rising bubbles. It was thought that the rate of aeration might be effected in several ways, of which the most probable were the disturbance of the bubble interface, and the modification of the bubble size. Both effects were, in fact, observed. Increases in the rate of solution of oxygen across a unit area of surface of up to 50 per cent were determined for irradiation with vibrations of 19·5 kilocycles frequency. A more important fact, however, was found to be the modification of bubble size. Using a single stream of bubbles from a capillary jet, it was observed that the bubble size was significantly reduced by vibrations of 19·5 kilocycles frequency, and a substantial increase in the total oxygen dissolved from a

[1] The references are given on p. 287.

unit volume of air resulted. The effect appeared to arise from the pressure conditions in the ultrasonic field causing smaller bubbles than normal to leave the capillary jet.

When bubble streams from diffuser plates were used, an opposite effect occurred; considerable aggregation of the smaller bubbles (below 0·5 millimetre diameter) took place in the ultrasonic field at all four frequencies. This resulted in large decreases (up to 60 per cent), in the quantities of oxygen dissolved. The effect arising from any increase in the rate of mass transfer across a unit area of bubble surface was outweighed by the reduction in total bubble area which resulted.

## Biological Applications

The lethal effect of ultrasonic vibrations on living organisms in water was reported as early as 1927.[6] In recent years, a number of workers have investigated various aspects of this phenomenon, using in the main high ultrasonic concentrations of the order of 5 watts per square centimetre or greater. The production of these intensities presents considerable difficulties on a plant scale, and one of the objects of the present work was to ascertain the effects of vibrations of a lower energy level.

In the first series of tests, a sample of water from the River Thames at Westminster Bridge was used as representative of a heavily polluted lowland river water, and 37° C. agar plate-counts were made after various times of irradiation (at constant temperature) for all four available frequencies. In most samples tested an initial increase in the plate-count was observed; this was thought to be caused by the vibrations dispersing the bacteria existing in clumps. A progressive decrease in the plate-count then followed, while irradiation continued for all four frequencies; periods of up to 2 hours, however, did not give complete sterilization.

A second series of tests was made, in which the sample was first subjected to the vibrations, and then stored at room temperature. A logarithmic decline in the total number of bacteria surviving the direct irradiation took place for a period up to 96 hours; this was followed by a vigorous aftergrowth, and the final bacterial count was little different from that of the control sample. These experiments appear to throw some light on the lethal mechanism, since it is known that ultrasonic vibrations in water can synthesize small quantities of hydrogen peroxide and ozone, and it was thought that this residual germicide was responsible for the reduction in the bacterial count observed.

A number of pure bacterial cultures of various types in Ringers solution were also irradiated. At the 300 kilocycle frequency, 98 per cent destruction of *Bacterium aerogenes* and *Bacterium coli* type 1 was achieved after 2 hours treatment, whilst *Nocardia Cuniculi* was unaffected. Other types exhibited various degrees of resistance, a result generally in accordance with other published information.[7, 8]

Little is known of the effect of ultrasonic vibrations on algae, and a number of pure cultures of varieties of differing physical characteristics were subjected to ultrasonic treatment. It was found that the only satisfactory method of determining the effect was by reculture of the specimens; microscopic examination and staining techniques proved unreliable. Significant destruction was only obtained with *Euglena*, a class of organism mid-way between an algae and a protozoa. The small unicellular organisms *Chlorella* and *Navicula* were unaffected by the vibrations, but the filamentous variety *Anabeana* was fragmented and the cellular clumps of *Chlamydomonas* and *Scenedesmus* were dispersed; both effects occurred, however, without causing an appreciable reduction in the rate of growth.

In all these experiments, both with algae and bacteria, the 300 kilocycles frequency was the most effective, but the time of irradiation for appreciable effects was considerable. The results suggested that the destruction of micro-organisms arose both from the direct physical effects associated with cavitation and the variations of pressure within the liquid, and from secondary effects attributed to the formation of hydrogen peroxide and ozone in the water.

## Effect on Sedimentation

It is known that under different conditions, ultrasonic vibrations can have either a dispersive or a coagulative action on systems of aqueous suspensions, but few qualitative results are available. Early experiments by the Institution of Water Engineers illustrate the dispersive effect.[9]

It is generally agreed, however, that the coagulation of suspended matter results from its concentration at the nodes or antinodes of a field of stationary waves. The movement to the nodal points results from the radiation pressure of the waves on the suspended particles, the pressure being zero at the nodal points and of maximum value at the midpoints. The relative densities of the particles and liquid are important, light particles (for example, gas bubbles) moving to the nodes whilst relatively dense particles move to the antinodes.[10] This concentration of particles at the nodal points increases the number of orthokinetic collisions and, if these collisions are effective in producing union, an increased rate of sedimentation will result.

The variables which affect the results include the frequency, intensity, and field pattern of the ultrasonic vibrations, and the particle size, specific gravity, and flocculation characteristics of the suspended material. In the first series of tests, suspensions of anthracite, calcium carbonate, and quartz were allowed to settle under quiescent conditions in the ultrasonic field. Appreciably increased settling rates were obtained for the two former materials but for the quartz a longer sedimentation time was required. There was some indication that the smaller the initial particle size, the higher the optimum frequency, and the highest available frequency was

probably too low to affect the smaller particles which are, in practice, the most difficult to settle. Within the limits 250–1,000 p.p.m., the initial concentration of the suspended solid had no bearing on the results obtained.

A series of continuous flow experiments with calcium carbonate and resuspended sewage sludge in water were also carried out. A certain degree of improved sedimentation was obtained, and the most effective frequency was 1,000 kilocycles for both materials. In general flow velocities of 1 to 2 centimetres per second proved most suitable, since, at higher velocities, turbulence overcame the force of radiation pressure. The radiation pressure present in the ultrasonic field increases with vibrational energy, but there is an optimum value above which disturbances from streaming of the suspension away from the crystal face and cavitation effects prevent sedimentation; these disturbances were present at all frequencies and intensities with powdered quartz as the suspended material.

## General Conclusions

Although in all the fields investigated, some positive results were obtained, the efficiency of the processes was at no time sufficient to suggest that full-scale application could be economic. A particular difficulty arises in that the most effective frequency appears to be in the order of 300 kilocycles per second; vibrations of this frequency are rapidly attenuated in turbid waters, and can be produced only at limited intensity and over a small area by means of fragile piezo-electric crystals. It was concluded that the practical applications of ultrasonic vibrations in public health engineering are not likely to be profitable until better methods of producing vibrations of the required frequency are developed.

## Acknowledgements

The work desciibed in this Paper was carried out in the Department of Civil and Municipal Engineering, University College, London, under the direction of Professor H. J. Collins, M.C., M.Sc., M.I.C.E.

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The Paper is accompanied by six photographs, two sheets of diagrams and four sheets of Tables.

# CORRESPONDENCE

## on a Paper published in Proceedings, Part III, August 1953

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Paper No. 5894

## "Further Research in Reinforced Concrete, and its Application to Ultimate Load Design" †

by

**Professor Arthur Lemprière Lancey Baker, B.Sc.Tech., M.I.C.E.**

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### Correspondence

**Dr G. G. Meyerhof** of Montreal, Canada, observed that the Paper presented encouraging evidence that, at ultimate load, the behaviour of the beams and shells tested by the Author compared well with estimates made by the Author on the basis of the plastic theory and the suggested safe limiting design values. It would seem, however, that further research was essential before that theory could be applied to the analysis of the more redundant multi-storey multi-bay framed reinforced-concrete structures by using statically determinate component frames with plastic hinges. That applied particularly to the bending-moment/rotation relationships of "tensile" plastic hinges, since those would govern the amount of moment redistribution of beams in practice. Under a uniformly distributed load that condition would be critical at the supports, where both the bending moments and shearing forces were maximum, so that neither the maximum plastic moment nor the maximum plastic shearing force, calculated separately, could be resisted at the same time, and the rotation capacity of the section might thus be severely limited. Even less was known, at present, about the "plastic" behaviour of columns subjected to bending moments, direct forces, and shearing forces.

Dr Meyerhof agreed with the Author that differential settlement of the foundations, as such, was unlikely to affect the collapse load of building frames and mainly aggravated the required capacity of plastic hinges to rotation at the fully plastic moment. It should be remembered, however, that horizontal movement and tilting of foundations would generally occur, in addition to settlement. Dr Meyerhof had recently carried out, [17]

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† Proc. Instn Civ. Engrs, Part III, vol. 2, p. 269 (August 1953).

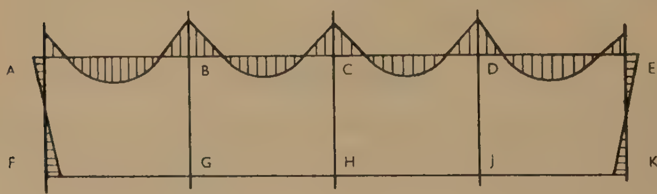
[17] References 17 to 25 are given on p. 320.

at the Building Research Station, some tests with model portal frames on clay and sand. He had found that, under both (*a*) vertical load and (*b*) combined vertical and horizontal loads, the failing load and the mode of structural collapse of frames might be influenced to a considerable extent by lateral and rotational foundation movements. That was to be expected theoretically, when the structure, its foundations, and underlying soil, were considered as an integral unit—as they should be in studying the effect of foundation movements on the behaviour of structures.

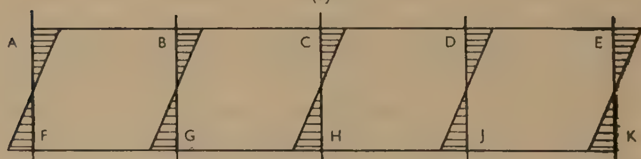
**Professor W. T. Marshall** asked for further information on plastic hinges in multi-storey frames. The hinges shown in *Fig. 2* were stated "... to occur at sections where wind moments and vertical load moments have maximum values and combine together." For any storey of a

*Fig. 24*

(*a*)




(*b*)




four-bay frame the moments due to vertical load were as shown in *Fig. 24* (*a*) and those due to wind loading blowing from left to right were as shown in *Fig. 24* (*b*). Thus the maximum moments at the joints in the stanchions from the combined loading were at E and K, whilst those at A and F were the smallest. Consequently if the plastic hinges were to form at points of maximum moment, those in the outer stanchions would form at E and K, and not at F and K as shown in *Fig. 2* of the Paper.

Secondly, referring to the construction of a shell from segments which were joined by prestressing, he pointed out that no mention was made, in the Paper, of the reinforcement used in each of the precast-segments. Had they been unreinforced ? If so, and since the only connexion between the segments (apart from the prestressing wires) had been dowels inserted to resist shear, there could obviously have been little or no transverse bending in the shell. That provided experimental evidence in support of

Professor Marshall's own opinions [18] on the elimination of transverse moments by prestressing. Because of the construction of the shell it was difficult to understand the remark, on p. 306, that " . . . the shell developed transverse stresses in accordance with elastic theory simplified by neglecting torsion and longitudinal bending." Was not the elastic theory, thus simplified, similar to Schorer's theory mentioned earlier in the same paragraph ? If, as stated, the stresses agreed with those calculated by Schorer's theory then there must have been tranverse bending present which could be resisted only by the friction between the segments since the steel-plate dowles would be unable to resist an applied bending moment. The paragraph was confusing since it was mentioned earlier that the transverse forces were " . . . the membrane forces with small eccentricities." If the same set of transverse forces was referred to on each occasion, did that mean that the stresses calculated by Schorer's method were equal to the membrane stresses acting with a small eccentricity ?

Finally, there was no mention of the method of loading used in testing the prestressed shell, but for the shell tested by Gouda a sand-bag loading was shown in *Figs 20* and *21*. What type of loading had been taken, in calculating the theoretical stresses and displacements shown in *Figs 19* and *22* ? Professor Marshall did not think that it could be stated, with any degree of assurance, what had been the intensity of pressure distribution on the shell from sand-bag loading. Owing to the internal friction of the sand, some arching action would, in all probability, take place, giving a pressure distribution which would certainly not be uniform. The use of timber walings and the insertion of a number of ropes tied to anchors above the supports seemed to indicate that loads had acted in directions other than vertically downward.

**Dr R. Gartner**, of Cape Town, thought the Paper showed that the plastic-hinge method was practical not only for the calculation of highly indeterminate structures, but also to even out the bending-moment diagram and thus would give a pleasing construction. It put the often used " approximation " on a definite basis.

Did Professor Baker consider ultimate-load design throughout, or only in the adjustment of the moment line through plasticity as Dr Gartner had done ? [19] If ultimate load design was used throughout, how did Professor Baker propose to design the ordinary concrete sections since that was still very controversial ? If, however, ultimate load design was used only for the calculation of the indeterminate forces, could it be assumed that, on account of the similarity of the plastic and elastic bending-moment diagrams (*Figs 8*), working loads were used ? If that assumption was correct, then by what factor of safety should the rotation-angle of the plastic hinges be multiplied ?

In the example of the twenty-five-storey 4-bay building frame, Dr Gartner drew attention to the fact that the lower floor columns would probably have large direct loads, so that for the whole length of those

columns the sections might be entirely in compression. That being so, a plastic hinge point had very little meaning. In such a case the elastic, rather than plastic, conditions should be applied for the calculation of the indeterminate forces. Since the lower part of the frame would be stripped before the upper part was cast, only the live loads on the lower part would influence the upper storeys. Moreover, since the concrete of upper and lower storeys would be of different ages, especially during construction, $E$ could not be constant throughout.

In the example of *Fig. 6* (*i*), the moment 1-2-$c$ had a minus sign. That was rather misleading, since it was contrary to the convention.

It would have been interesting to have had, in *Fig. 11*, curves for normal and wide-spaced links for purpose of comparison.

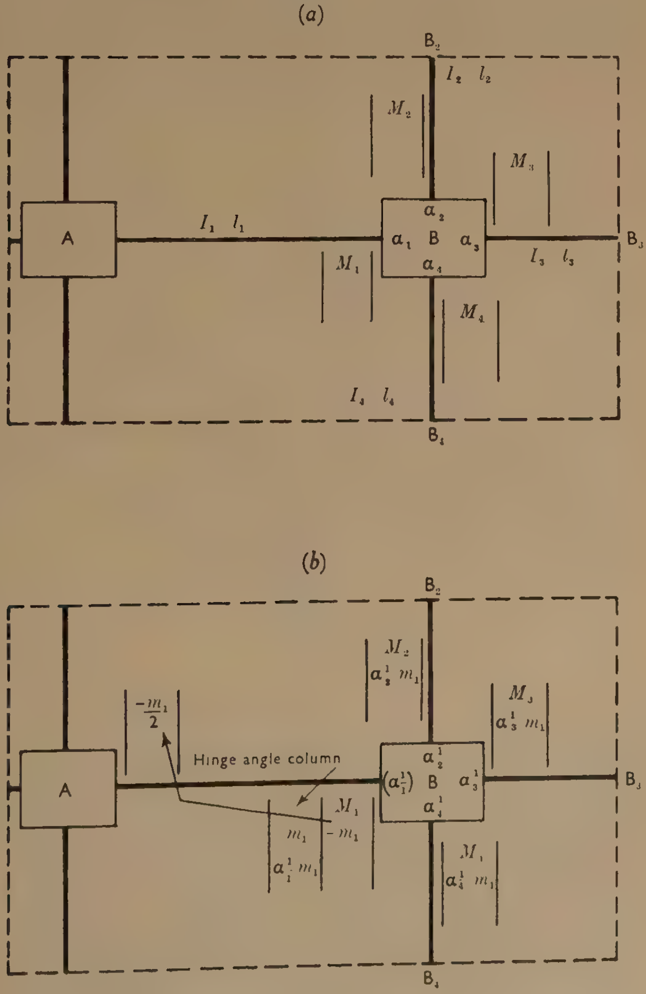
**Dr M. R. Horne** stated that the plastic theory of mild-steel continuous structures afforded a simple basis for design because, except when calculating deflexions, it was unnecessary to consider the slope/deflexion equations. Unfortunately, that advantage was lost when it became necessary to limit angular discontinuities at "plastic hinges," as was the case when dealing with reinforced concrete. The Author, by assuming plastic hinges and corresponding full plastic moments at a sufficient number of positions to render the structure statically determinate, had avoided the most fearsome aspects of a formal elastic analysis, but his method was still far removed from the simplicity of plastic-design methods for structures of ductile materials. His approach involved a three-fold trial-and-error process. It was necessary to assume both the positions of the hinges and the values of the full plastic moments. Those assumptions had to be modified by trial to satisfy three conditions: relative rotations at hinges should be everywhere of the correct sign; they should be nowhere excessive; and there should be an economic distribution of bending moments. It seemed that considerable skill might be required to perform those processes satisfactorily.

A degree of complexity in the design procedure might be inevitable, but engineers would have their own preferences in choosing between possible approaches to the problem. Those who were familiar with moment-distribution analysis for elastic structures might prefer to employ an adaptation of that technique rather than the one given by the Author. The moment-distribution process might be readily adapted to take account of "plastic hinges," the angles of discontinuity at plastic-hinge positions being systematically recorded as the analysis proceeded. Moreover, the positions of the hinges themselves did not have to be chosen merely by good judgement. The amended moment-distribution method so obtained might be considered to be intermediate in character between ordinary elastic moment distribution and the plastic moment-distribution method described in a Paper by Dr Horne.[20]

A and B in *Fig. 25* (*a*) represented two adjacent joints in a structure. The moments of inertia and the lengths of the members meeting at B

were $I_1$ and $l_1$, etc. as shown. The stiffnesses were then $K_1$, $K_2$, $K_3$, and $K_4$, where $K_1 = \dfrac{I_1}{l_1}$ . . . etc. The moment-distribution factors $\alpha_1$, $\alpha_2$, $\alpha_3$, and $\alpha_4$ at the joint were then given by $\alpha_1 = \dfrac{K_1}{K_1 + K_2 + K_3 + K_4}$ etc. Ordinary elastic moment distribution was then assumed to give

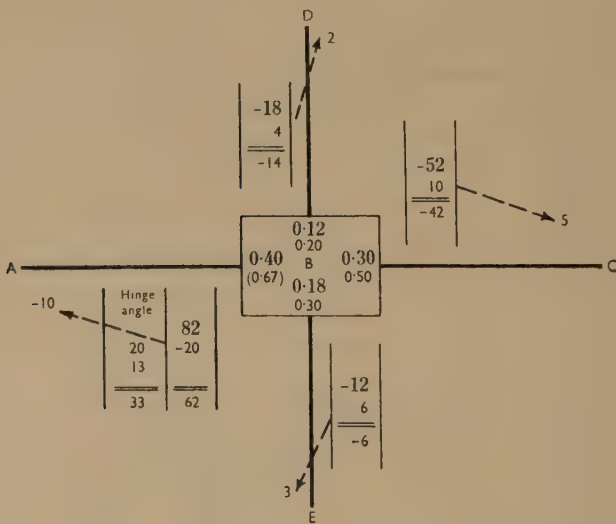
*Figs 25*




$$\alpha_1^1 = \frac{\alpha_1}{1-\alpha_1}, \quad \alpha_2^1 = \frac{\alpha_2}{1-\alpha_1}, \quad \alpha_3^1 = \frac{\alpha_3}{1-\alpha_1}, \quad \alpha_4^1 = \frac{\alpha_4}{1-\alpha_1}$$

end moments $M_1$, $M_2$, $M_3$, and $M_4$, which were positive when they acted clockwise on the member. If the joint was balanced $M_1 + M_2 + M_3 + M_4 = O$. In the case where the bending moment $M_1$ in beam AB was considered excessive and was to be reduced by an amount $m_1$ (by introducing a plastic hinge at the end B of that beam), the distribution factors were first replaced by revised factors $\alpha_1'$, $\alpha_2'$, $\alpha_3'$, and $\alpha_4'$. Those were obtained from $\alpha_1$, $\alpha_2$, $\alpha_3$, and $\alpha_4$ by dividing each by $(1 - \alpha_1)$ (see *Fig. 25* (*b*)). The bending monent $m_1$ was subtracted from the moment $M_1$ and distributed to the other three members, using the factors $\alpha_2'$ $\alpha_3'$, and $\alpha_4'$. There would then be an angle of discontinuity at the plastic hinge, and that was recorded in the column headed " hinge angle." The change

*Fig. 26*




of moment $m_1$ at the end of beam AB was recorded in that column with a change of sign, together with the quantity $\alpha_1'$ $m_1$, obtained by multiplying the out-of-balance moment $m_1$ by the distribution factor $\alpha_1'$. The sum of the two moments $(m_1 + \alpha_1' m_1)$, when multiplied by $4EK_1$, would then give the total angle of discontinuity at the hinge. That might be proved quite readily, the quantity $(4EK_1)m_1$ representing the additional counter-clockwise rotation of the end B of the beam AB, whilst $(4EK_1)\alpha_1' m_1$, was the additional clockwise rotation of the joint B. The changes in bending moments of $- m_1$ in AB, $\alpha_2' m_1$ in $BB_2$, $\alpha_3' m_1$ in $BB_3$ and $\alpha_4' m_1$ in $BB_4$ were multiplied by 0·5 and " carried over " to adjacent joints in the usual way.

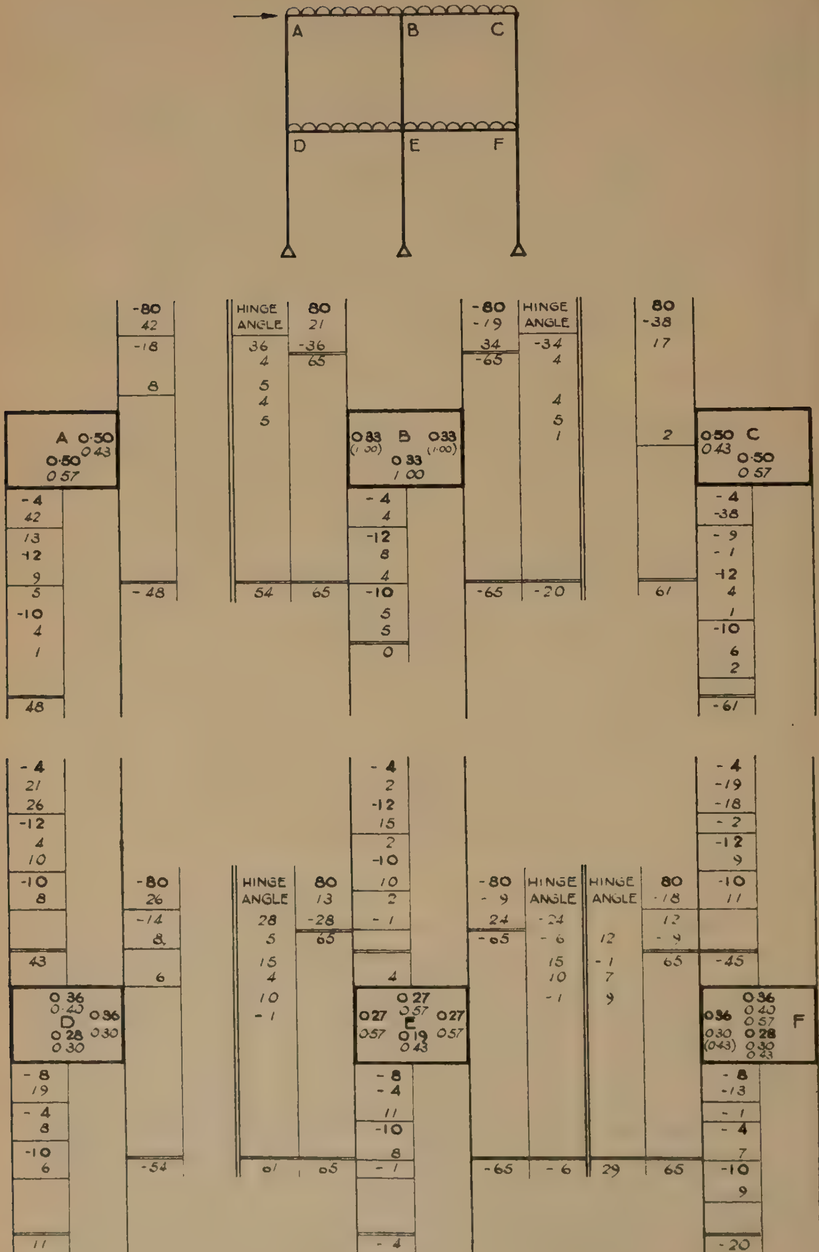
A numerical example of the above process was given in *Fig. 26*, which showed the balancing of a four-member joint in a building frame. The numbers in larger type were the balanced elastic moments and distribution factors. If it was desired to reduce the bending moment in the beam AB from 82 to 62 by introducing a plastic hinge at that section, the revised distribution factors shown in smaller type were first calculated, where $0{\cdot}20 = \frac{0{\cdot}12}{1 - 0{\cdot}40}$ etc. The adjusting moment of 20, subtracted from the moment of 82 in beam AB, was then distributed to the other three members, and the hinge angles at end B of beam AB were also recorded. Finally, half the adjustments in the bending moments were carried over to the remote ends of the members as indicated. In order to obtain the actual angle of discontinuity at the hinge, the total of 33 in the hinge-angle column had to be multiplied by the value of $\left(\frac{4EI}{l}\right)$ for the beam.

Dr Horne said that it would make his contribution too lengthy to give further details of the method, but they might be easily worked out by anyone familiar with the principles of elastic-moment distribution. It was unnecessary to achieve a complete balance of bending moments before introducing plastic hinges, since it quickly became apparent which sections would sustain the highest elastic bending moments. The process might be stopped at any stage at which a full balance of joints and balance against sway had been obtained, and the introduction of additional hinges might be stopped as soon as it appeared that excessive hinge discontinuities were beginning to occur. That may well happen before reaching the stage at which there were sufficient hinges to render the structure statically determinate. Under such circumstances, the effect of introducing further hinges would be to reduce the apparent (but not necessarily the real) carrying capacity, and would thus produce a less efficient structure. The design process was thus more under control than in the method suggested by the Author.

The whole working necessary to obtain a design for the two-bay two-storey frame considered by the Author (*Figs 6*) was shown in *Figs 27*. It was found advantageous to introduce hinges at the inner ends of all four beams, and also at the outer end of beam E. The resultant bending moments were shown in *Fig. 28*, which also showed (enclosed in circles) the angles of discontinuity at the five hinge positions. As compared with the plastic moments derived by the Author (see *Fig. 8* (*a*)), the maximum beam moment was decreased (being nowhere greater than 65), while the stanchion moments in the upper storey were increased; there were only five hinge positions compared with nine; the maximum angle of discontinuity was 92, compared with the maximum value of 174. The bending-moment distribution in *Fig. 28* might be improved slightly by assuming further hinge positions, but that hardly appeared worth while.

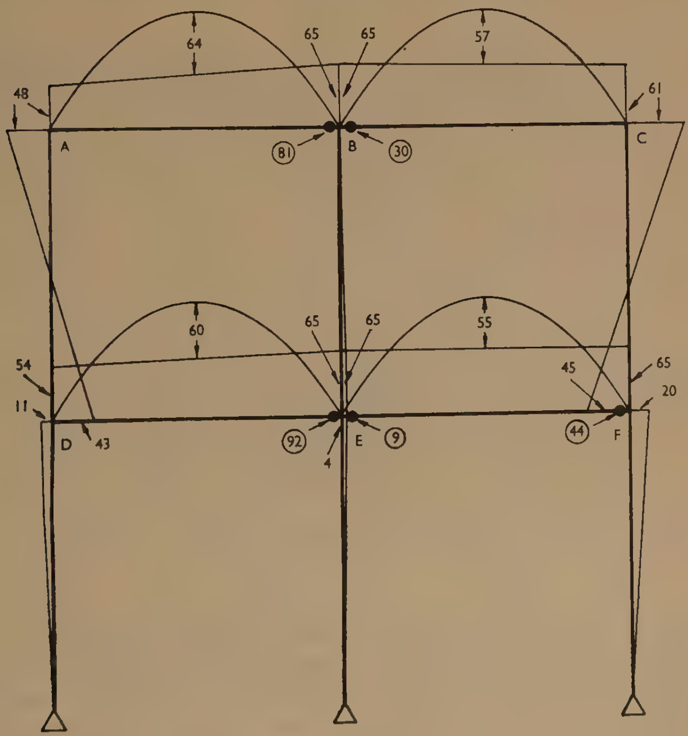
The moment-distribution process had a further advantage over that

*Figs 27*



given by the Author, provided that the most excessive bending moments were reduced first. The hinges were then introduced step by step more or less in the order in which they would form under gradually increasing loads. In the plastic theory of genuinely ductile structures, the order of formation of plastic hinges had no effect on the collapse load, but that was not theoretically true when dealing with structures in which the moment-curvature characteristics rose to a maximum and then fell, as was the case for reinforced-concrete members. While the practical effect might be

*Fig. 28*



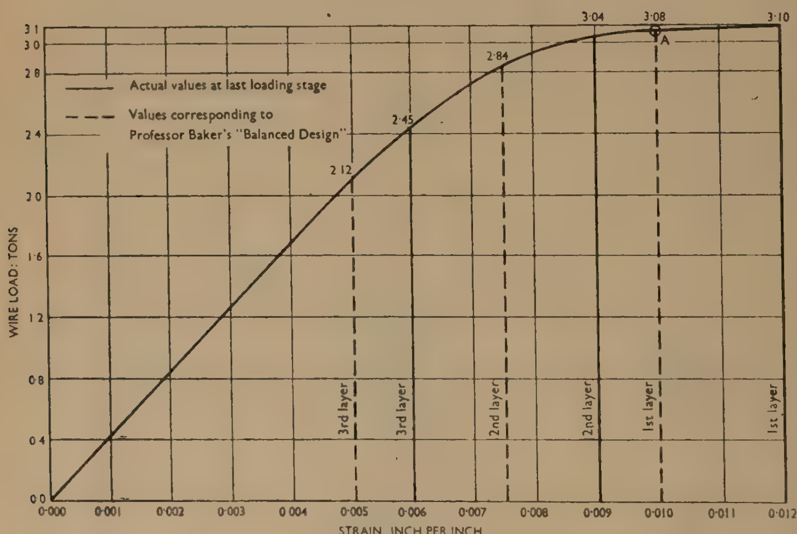

negligible, it seemed advisable, if possible, to take account of order of formation of hinges unless some proof could be given that it was unimportant.

**Dr Hajnal-Kónyi** said that in connexion with the beam tested by Ziauddin at Imperial College, the Author stated, " It is clear, therefore, contrary to Hajnal-Kónyi's expectation, that the position of the neutral axis, prior to failure, is governed mainly by concrete and steel strains, and that when non-prestressed high-tensile steel is used, the ultimate

strength of beams in regard to failure caused by the concrete is very much smaller than when mild steel of similar total strength is used, since before yield, the mild-steel strain is only about a quarter of the value of the high-tensile-steel strain and the neutral axis is at about half the depth." (p. 303). The data contained in the Paper were insufficient for an independent investigation of the test results, but the Author had very kindly supplied him with further data which had enabled him to prepare the following analysis :—

The six wires had been arranged in three layers the distances of which from the bottom edge of the beam had been 1·25, 3·00, and 4·75 inches

*Fig. 29*




respectively. The strains at those levels at the last loading state = (7·075 tons) had been 1·2, 0·9, and 0·6 per cent. (*Fig. 16.*) The corresponding forces in the wires, obtained from the stress-strain diagram (see *Fig. 29*) were 3·10, 3·04, and 2·45 tons. The breaking force per wire was 3·3 tons so that the two wires in the bottom layer developed 94 per cent of their strength. The total force in the wires was 17·18 tons as against a possible maximum of 19·8 tons, which was 87 per cent of the possible maximum. The centre of gravity of the tensile force at failure was at a distance of 2·87 inches from the bottom. The depth of the neutral axis (scaled from an enlarged print of *Fig. 16*) was 1·90 inch. With an assumed rectangular stress block in the compression zone, the lever arm was $10{\cdot}2 - 2{\cdot}87 - \frac{1{\cdot}90}{2} = 6{\cdot}38$ inches. The maximum bending moment, in-

cluding the effect of the dead weight, was $\frac{5\cdot1 \times 10\cdot2 \times 1\cdot04 \times 7^2 \times 12}{8} + 7\cdot075 \times 2{,}240 \times 15 = 4{,}000 + 238{,}000 = 242{,}000$ lb. inches. The calculated tensile force at failure was $\frac{242{,}000}{6\cdot38} = 37{,}900$ lb. as against a force of $2{,}240 \times 17\cdot18 = 38{,}500$ lb. obtained from strain measurements. The difference was 600 lb., or only about 1·5 per cent. That was a very satisfactory agreement between measured and calculated values. The compressive stress in the assumed rectangular stress block was $\frac{37{,}900}{5\cdot1 \times 1\cdot9} = 3{,}910$ lb. per square inch, which was 60 per cent of the cube strength.

As might be seen from *Fig. 16*, there was a definite downwards movement of the neutral axis between the last two loading stages shown on the diagram. From the large scale diagram received from the Author it was clear that the neutral axis moved downwards by 0·2 inch whilst the load was increased from 6·3 tons to 7·075 tons.

The Author stated that "failure occurred by sudden bond slip." In Dr Hajnal-Kónyi's publication, referred to by the Author, as in all his other publications on that subject, he had emphasized the importance of good bond. Bonding of the wires by casting (instead of by grouting) did not in itself guarantee sufficient bond strength. Bond depended on the surface conditions of the wires and on the consolidation of the surrounding concrete. The only criterion was the mode of failure which had proved that the bond had not been sufficient to allow the development of the ultimate strength of the wires (a force of 19·8 tons). In order to balance such a force an increase of the depth of the neutral axis to not more than $\frac{19\cdot8 \times 2{,}240}{3{,}910 \times 5\cdot1} = 2\cdot22$ inches would have been necessary, that was a further downwards movement before failure by maximum 0·32 inch.

At the load at which failure occurred no greater depth than actually observed could have been expected, irrespective of the quantity of the tensile reinforcement, since the tensile force in the steel was balanced by the compressive force in the concrete at an average stress of 60 per cent of the cube strength. The theoretically maximum possible bending moment with good bond would have been $19\cdot8 \times 2{,}240 \times \left(7\cdot2 - \frac{2\cdot22}{2}\right) = 270{,}000$ lb. inches. However, it should be borne in mind that the arrangement of the wires in three layers in such manner that the average strain in the top layer was only about one-half of the average strain in the bottom layer was extremely unfavourable, and rendered the development of the ultimate strength of the wires in all three layers rather unlikely. Under such circumstances it was not correct to base the calculation of the ultimate moment on the assumption that the full strength of all wires was acting

at the centre of gravity of the three layers. For that reason the calculated maximum of 270,000 lb. inches might not have been reached even with good bond, whilst it would have been quite feasible to reach it with a more suitable distribution of the wires.

The ratio $\frac{n}{d}$ at failure was $\frac{1{\cdot}90}{7{\cdot}2} = 0{\cdot}264$. Its theoretical maximum for the given quantity of steel (but not for its actual distribution in the cross-section of the beam) was $\frac{2{\cdot}22}{7{\cdot}2} = 0{\cdot}31$. In order to exceed that value more steel would have to be provided.

It was of interest to work out the maximum bending moment at which the beam should have failed in compression according to the Author's theory. In a discussion on Dr Hajnal-Kónyi's Paper[7] the Author had advocated the use of a point A on the stress/strain diagram (*Fig. 2*, p. 129) corresponding to 1 per cent elongation "as the yield point of wire for design calculation." With the position of the neutral axis as measured at the last loading state ($n = 1{\cdot}9$ inch); the strains and corresponding forces in the wires were given in Table 6.

TABLE 6

| Layer | Strain | Force: tons |
|---|---|---|
| 1 | 0·010 | 3·08 |
| 2 | 0·0075 | 2·84 |
| 3 | 0·005 | 2·12 |

The total force in six wires was $16{\cdot}08 \times 2{,}240 = 36{,}000$ lb. acting at 2·79 inches above the bottom edge of the beam. With the Author's usual assumption $\gamma = 0{\cdot}4$, $a = 10{\cdot}2 - 2{\cdot}79 - 0{\cdot}4 \times 1{\cdot}9 = 6{\cdot}65$ inches and $M = 36{,}000 \times 6{\cdot}65 = 239{,}500$ lb. inches; that was only 1 per cent less than the actual maximum but the agreement was only apparent because of the different mode of failure; bond slip instead of compression.

The results might be summarized as in Table 7.

Case 3 (Table 7) was representative of a beam of an effective depth of 7·2 inches with mild-steel reinforcement of a total yield strength equal to the total tensile strength of the wires (44,400 lb.). Case 1 corresponded to a similar beam in which the total yield strength of mild-steel reinforcement equalled the total strength of wires as assumed for design purposes according to the Author's suggestion (36,000 lb.). The Author's conclusion that "when non-tensioned high-tensile steel is used, the ultimate strength of beams in regard to failure caused by the concrete crushing is

TABLE 7

| Case | Max. $M$: lb. inches | Max. $\frac{M}{bd^2c_u}$ | Ratio | $\frac{n}{d}$ |
|---|---|---|---|---|
| 1. The Author's "balanced" design . . . . . . . . | 239,500 | 0·1395 | $\frac{(1)}{(3)} = 0{\cdot}887$ | 0·264 |
| 2. Actual maximum at bond slip . | 242,000 | 0·141 | $\frac{(2)}{(3)} = 0{\cdot}897$ | 0·264 |
| 3. Theoretical maximum (utilizing full tensile strength) . . . | 270,000 | 0·157 | | $\leqq 0{\cdot}310$ |

very much smaller than when mild steel of similar total strength * is used," was not justified in respect of the beam in question since the values for cases 1 and 2 were not very much smaller than for case 3. In fact, if the Author's suggestion was adopted and his point A was accepted on the stress-strain curve as the limiting value of the steel stress the diametrically opposite conclusion was reached, since a similar beam with mild-steel reinforcement of a total yield strength of 36,000 lb. would have carried about the same load as Mr Ziauddin's beam, in spite of the latter's premature failure by bond slip.

Dr Hajnal-Kónji would like to add that, contrary to the Author's expectation, Mr Ziauddin's beam was not balanced but under-reinforced because with adequate bond and a suitable arrangement of the wires it was possible to reach higher values of $\frac{n}{d}$ and $\frac{M}{bd^2c_u}$ at steel stresses reasonably near the ultimate strength.

**Mr W. W. L. Chan**, referring to experiments at Imperial College, London, on short reinforced-concrete prisms and cylinders with closely spaced binders subjected to eccentric loading, said that some of the results of those experiments had already been referred to by the Author in *Figs 11* and *12*. They had provided favourable support for the use of close binding in extending the straining capacity of compressive plastic hinges.

Test observations indicated that the lateral support of binders became effective as the concrete cover began to spall, that was, when unbound concrete was deformed to its ultimate strain capacity. From that stage onwards, large longitudinal plastic deformations were accompanied by corresponding high lateral strains, thereby causing the binders to be stretched and so induced lateral support for the contained concrete. That enabled very high compressive strains to be accommodated by the concrete without appreciable weakening. When failure finally occurred,

* Dr Hajnal-Kónyi stated that the term "mild steel of similar strength" should read, of course, "mild steel of similar yield strength."

no abrupt collapse took place, but only a gradual drop in load, accompanied by very high creep was observed. Thus it seemed that bound concrete had been changed from a comparatively brittle material into a ductile one.

It was apparent that the extent of ductility of bound concrete depended on the efficiency and quantity of binding used. Tests had indicated that spiral helical binding generally provided more efficient lateral support than is possible with rectangular tied links.

Whilst the investigation had not yet been completed, examination of test results so far had shown that for spiral binding, the ultimate concrete strain varied from a value of about 0·0035 for plain unbound concrete, to 0·0245 for concrete with 4-per cent spiral binding, and having corresponding $\frac{\alpha c'}{C_u}$ values of 0·7 and 1·8 respectively.

For concrete bound with rectangular tied links, ultimate strains ranged from 0·0035 up to 0·0165 for 3-per cent binding, whilst corresponding $\frac{\alpha c'}{C_u}$ values were 0·7 and 1·25 respectively.

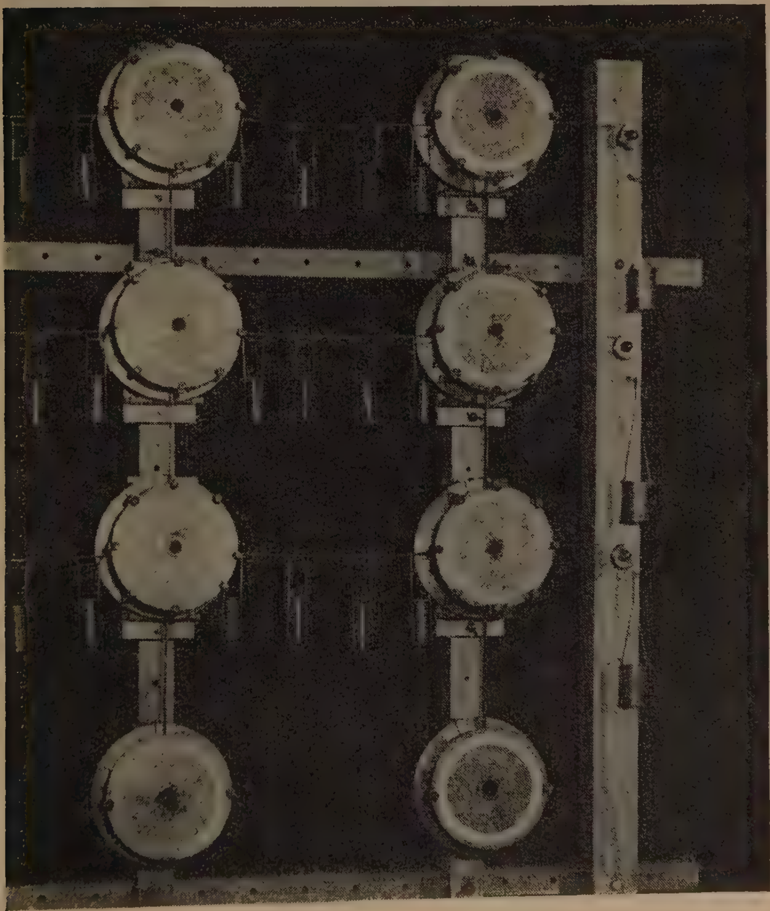
When applying such values of $\alpha c'$ to assess the plastic moment values of bound compressive hinges, it was clear that appropriate reductions in effective concrete area caused by spalling had to be made. When the concrete cover was deep and the concrete strength high, the loss in strength of the hinge owing to spalling might be quite considerable. Hence for a given depth of cover, the choice of high percentages of longitudinal reinforcement, low concrete strength, and large sectional dimensions, would result in minimum loss of hinge strength.

**Mr C. W. Yu** observed that when the number of bays or storeys of a building was great, the procedure of obtaining $A_E$ adjustment as suggested in the " Adjustment of Hinge Moment Values " paragraph of the Paper would not be found to supply the desired reduced $\theta$-values quickly enough. A special model *Figs 30* and *31* had therefore been designed at the Imperial College to seek a better solution.

In the model, the beam members of adjacent bays, and column members of adjacent storeys were connected together by means of what might conveniently be termed " hinge units." Each hinge unit consisted of a central core soldered to a vertical square iron bar. The top of the vertical bar was soldered to a T-block with a longitudinal V-groove cut in the bottom face of the flange, whilst the bottom of the vertical bar was soldered to another cross-bar with a V-groove in its back face. That arrangement enabled the complete hinge unit to rest on a block of wood by means of ball bearings. The block of wood was screwed to a wooden vertical, thus relieving the column member of all its direct loading and also enabling the complete hinge unit to translate horizontally with minimum friction. Four concentric tubes A, B, C, and D, each cut to a special shape were placed inside one another in their appropriate positions so that they could rotate

about the central core (*Fig. 32*). The beam and column members were connected to the concentric tubes by means of slugs soldered to the tubes in such a way that the ends of the members could rotate as in a pin-jointed structure. The angle through which the end of a member rotated was indicated by a pointer attached to the slug and moving along a dial soldered

*Fig. 30*



to the innermost concentric tube. The position of the member attached to that tube was used as a datum. At the other ends of the concentric tubes two strips of spring steel were soldered in appropriate positions. The displacement of those spring-steel strips caused the tube, and consequently

the end of the member attached to it, to rotate through a certain angle. The steel strips were displaced by moving clips, *Figs 33*, along the circumference of the disk soldered conveniently to the central core. Arrange-

*Fig. 31*

ments were made for varying the distance between the wooden verticals (and hence the lengths of the beam members) and the distances between the wooden blocks attached to the verticals (hence the column lengths). The hinge units of the ground floor, being fixed, were of slightly modified

*Fig. 32*

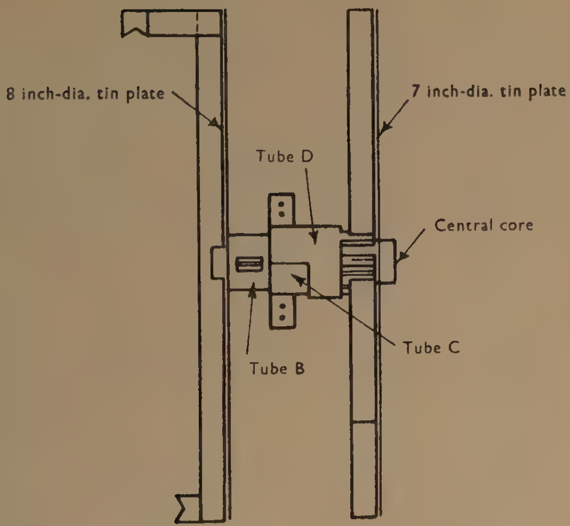
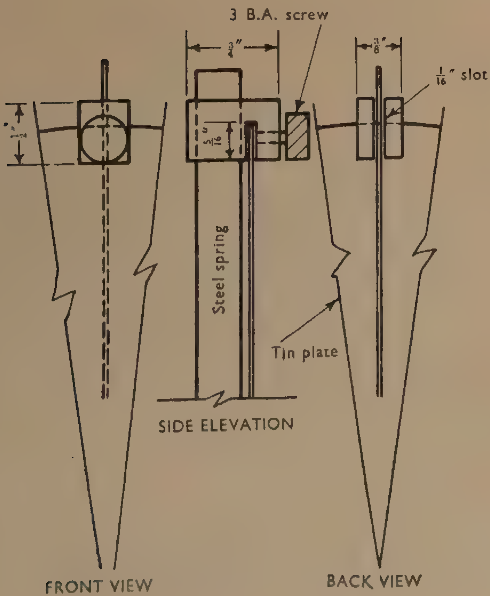

8 inch-dia. tin plate
7 inch-dia. tin plate
Tube D
Central core
Tube C
Tube B

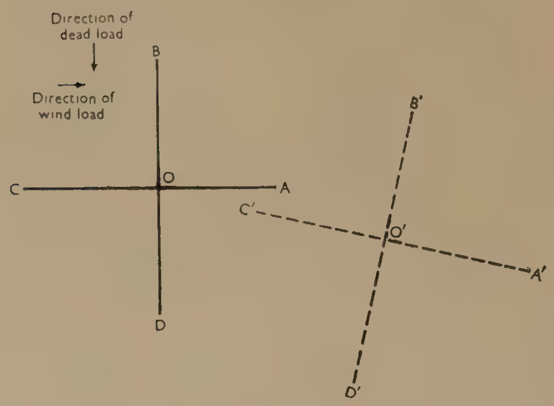
*Fig. 33*


3 B.A. screw
3/4"
3/8"
1/16" slot
1/2"
5/16"
Steel spring
Tin plate
SIDE ELEVATION
FRONT VIEW
BACK VIEW

construction but were basically of the same principle. The model framework was assembled in the following manner. Having decided on the lengths of the beam and column members, the wooden verticals and the wooden blocks were placed in their appropriate positions. The hinge units were then rested on the wooden blocks and connected to one another by the column and beam members.

*Fig. 34* showed diagrammatically four members representing the beams and columns of a structure joined together at O. If the joint O rotated and translated, under the influence of external vertical and lateral loads, then O, A, B, C, and D would move to O′, A′, B′, C′, and D′, but the four

*Fig. 34*




angles would still be 90 degrees if none of the members had been stressed beyond the elastic range. However, if OB were stressed beyond the elastic range then angle B′O′A′ would be > or < 90 degrees ; if OC were stressed beyond the elastic range then angle C′O′A′ would be > or < 180 degrees and if OD were stressed beyond the elastic range angle A′O′D′ would be > or < 90 degrees. Member OA would be least stressed in a joint loaded as shown and hence it was assumed to be elastic in the analysis (as mentioned in the Paper) and might conveniently be used as datum. In the hinge unit the rotation-indicating disk was soldered to the concentric tube to which OA was attached. If the positions of all the pointers were marked zero on the dial when the members were not loaded and all the spring steels not displaced, then as soon as the members were being loaded the pointers would deviate from their zero marks. If the spring steels were displaced, making the rotations of the concentric tubes opposite to the direction of the corresponding members when under external loads, it would be possible to move the pointers back to their zero positions. If

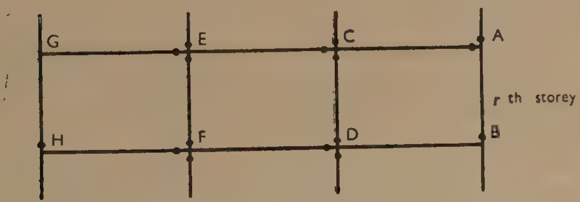
that was attained the moments required to displace the various spring steels would be equal to the moments at the ends of the corresponding members in the elastic stage.

After the hinge unit had been assembled the moment displacement relationship of the various spring steels was calibrated on the disk attached to the central core. The zero positions of the various pointers were marked as previously described. The various spring steels were then displaced to give the desired moments of resistance for the various members as required by the ultimate-load design analysis. All the members were then loaded. The hinge unit was designed and constructed so that rotation and translation of the members were made possible and no sway correction was required. The pointers would not be pointing to zero. Hinge-moment adjustments were then performed until all the pointers were indicating zero. Different sequence of adjusting the hinge moments could then be attempted without involving tedious arithmetic.

While carrying out the preceding operations, at every stage it should be borne in mind that :

(1) the sum of all the moments at a joint must be zero,
(2) the sum of all the column moments of a storey must balance the external moment caused by the external load.

*Fig. 35*




Accordingly, in (1) if the hinge moment of OC was adjusted, OA should be so adjusted to make the joint balanced (see *Figs 34*). If the moment of the lower column hinge of the *r*th floor at joint D as in (2), was altered, then a corresponding equal and opposite alteration for the column moment of *r*th floor at joint G was necessary (see *Fig. 35*).

The model indicated very clearly how the adjustment of a hinge moment could influence the angle of discontinuity of another hinge situated at a distance.

With the use of the model and by further analysis the following results had been arrived at :—

(*a*) The hinge moments of the hinges type *b* of all storeys should be adjusted as a group.
(*b*) The hinge moments of hinges type *a* and *c* other than those of $c_1$ and $c_{n+1}$ of the same storey should be adjusted as a group.

(c) The hinge moments of $c_1$ and $c_{n+1}$ of the same storey should be adjusted as a group.

The sequence of adjustment should be (a) then (b) and (c) of the top storey; then (b) and (c) of the storey below, continuing until the ground floor was reached. Having completed one cycle, if the angles of discontinuity were still considerable, another cycle might be performed, which would doubtless give more desirable results.

In reducing the angle of discontinuity, it was important for every one of them to be reduced to approximately the same value, and most important, of the same sign. If those conditions were not fulfilled, the moments obtained for the various members at the elastic stage might be quite far from their true values.

**Mr J. A. Derrington** observed that it was satisfying to the ordinary designer of reinforced concrete, whose methods were somewhat more elementary, that the Author reached conclusions which were in general accord with the assumptions often used in practice to simplify the solution of the complicated frameworks.

For example, in the case of a beam carrying uniformly distributed load, a designer knew that a total bending moment of $\frac{WL}{8}$ occurred, and had to consider the most economic way of producing the resistance moments of the section at the support and at mid span so that the total figure was as near that value as possible.

Mr Derrington said that far too many buildings of reinforced concrete were still designed on the same basis as that used for structural steel, and many designers considered the balancing of support and span moments as the basis of economy.

In a normal reinforced-concrete frame, it was not always remembered that the floor acted as an integral part of the structure, and a proper realization should be made of the relative effect in resisting bending of the rectangular section at the support and the T-shaped section at mid span.

In the more common type of multi-storey building used for office or dwelling purposes, where live loads were less than the total dead load and concrete members generally under-reinforced, the greatest economy could often be achieved by treating each beam as though simply supported. There was, generally, sufficient restraint provided at the supports by either the distribution steel in the slab or by the column splice-bars which project above floor level and were adequately tied together to prevent serious cracking at all floor levels except the roof.

Using the Author's method, with a calculation which involved many factors at present unfamiliar to the average designer, a proper assessment might be made of the safe limits of the distribution of the free span moment between support and span. It would, however, be of great help to the designer to see those limits assessed in simple terms as a coefficient of span

and load, and to see those values related to strain-gauge readings carried out on actual structures in which the degree of restraint at the supports had been varied. The relatively small limits between which the restraint might be varied from the theoretical figure, allowed by C.P. 114 ($\pm$ 15 per cent) might then be extended and the methods of calculation allowed by many authorities, of designing all units of a frame as completely free or almost completely fixed, made more rational.

With regard to the sway moments produced in a multi-storey building, true appreciation was not always made of the stiffening effects of wall panels which in some circumstances rendered the more complicated methods of analysis unnecessary. Care should be taken that the assumption of a point of contraflexure at half height of a column was used only with a knowledge of how that assumption was justified.

Finally Mr Derrington pointed out that the assumption that the modulus of elasticity of concrete might be represented by a simple figure might produce disturbing results. Until recently the figure taken for concrete had always been about 2,000,000 lb. per square inch, but with the more common use of high-grade concretes it was now appreciated that that value might be increased. However, it was not always realized that the modulus of elasticity of concrete varied considerably and had to include other factors such as creep and shrinkage. It was, therefore, dependent to a large extent on the age of the concrete and the type of loading. In different members of a typical structure the effective modulus might range from 2,000,000 to 8,000,000 lb. per square inch and render highly unsatisfactory any figures prepared on the assumption that it had a constant value.

**Dr K. A. Everard,** of Jamaica, observed that the behaviour of a loaded structure as a whole was governed by the geometric proportions of the component parts and the load deformation properties of the material or materials from which those components were made. Of those parameters, the specification of the load deformation relationship had caused the greatest concern.

Mild structural steel was possibly one of the simplest materials to predict and the simplifying assumption that the stress/strain curve was linear up to a yield point and thence of a form giving continued strain under constant stress had given reasonably accurate and consistent results.

Outside the "elastic range" the behaviour of reinforced-concrete structures had not been nearly so predictable. Indeed, the determination of the stress/strain curve of concrete itself had been of the greatest difficulty. Not only had different results been obtained from the same batch of concrete, but variations in form had been obtained for different mixes tested at different rates of loading.

It was submitted for consideration, however, that that difficulty had been partly because of a wish to obtain a general stress/strain relationship from a land deformation test at a particular rate of loading. It was

thought that the following indirect method might provide a new and useful approach to the problem.

Stress/strain curves of visco-elastic materials, when tested in direct compression, were of a form such that the curvature varies with the rate of straining. Thus it might be seen that a strain was not a direct indication of the intensity of stress present, unless some refeience was made to the rate of straining.

In addition, it might be seen that non-linear behaviour derived from stress/strain tests in direct compression was of no real significance in bending since such behaviour might be caused by time effects alone. Thus non-linear stress distribution in bending might be assumed only if the relationship between stress and rate of straining was non-linear.

Consider the case of a reinforced concrete member which might be assumed to behave as a elasto-plastic member under the action of an increasing bending moment. It would be assumed that the increase of moment was such that the rate of increase of curvature was uniform.

The following conditions must be satisfied :—

(1) Linear strain distribution across section

$$e = e_m \cdot \left(\frac{y}{hd}\right) \quad \ldots \quad (1)$$

(2) Linear rate of straining across secti on

$$= \left(\frac{de}{dt}\right)_m \left(\frac{y}{hd}\right) \quad \ldots \quad (2)$$

Those conditions must be satisfied simultaneously in order that the bending-stress distribution be determined.

It could be seen that when time-dependent deformations are large, a typical stress/strain curve could not be used since a different curve applied to each fibre. However, the problem might be treated as follows.

Prismoidal specimens of concrete were tested at various rates of straining, readings of stress and strain being taken at all stages up to failure.

The results were plotted on the grid system as in *Fig. 36*.

That direct plot might be utilized to evaluate the stress distribution in a reinforced-concrete beam as follows :

(*a*) uniform rate of increase of curvature ;
(*b*) maximum or extreme fibre strain rate, $\partial_m$ ;
(*c*) at any moment, $M_1$ extreme fibre strain, $e_m$.

Then the curve satisfying simultaneously the conditions (1) and (2), would give the required result.

Thus the distribution was determined for any particular concrete, which allowed for computation from strains with creep. The increase in stress owing to the resistance of the reinforcement and the drop of the

neutral axis was not included, however, but that effect was small, in over-reinforced beams, in which variations in the stress/strain relationship for concrete was of the greatest importance.

*Fig. 36*

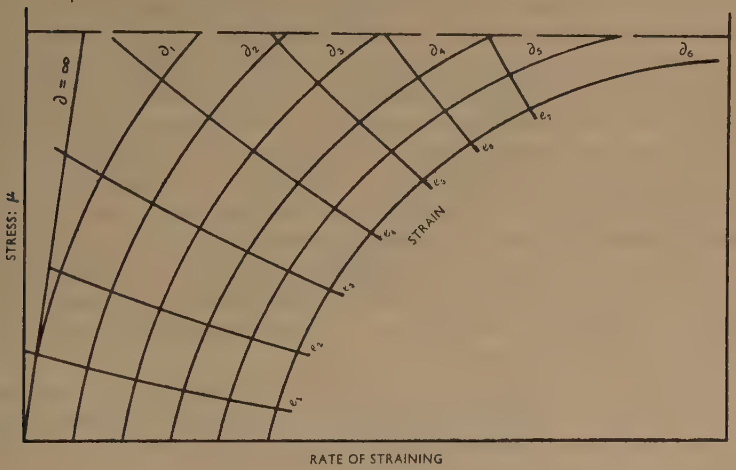
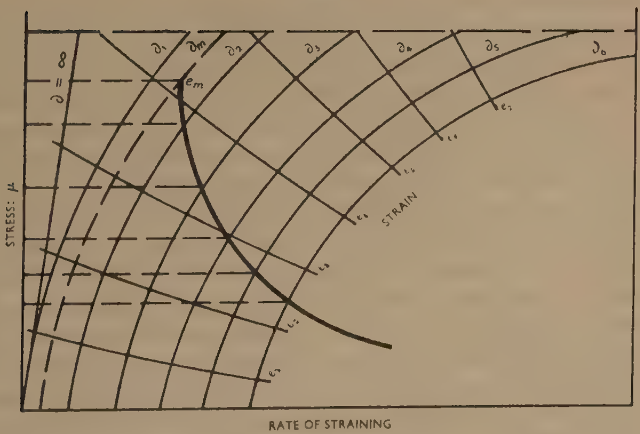



*Fig. 37*




In under-reinforced sections the rise in the neutral axis could be great. It could be shown, however, that the stress/strain curve for steel was the main factor and variations in the relationship for concrete were relatively unimportant.

**Mr O. T. Hun** of Trengganu, Malaya, pointed out that the photograph of the abacus (*Fig. 23*, p. 309) was incorrect. The figures should read as 128 and not 173.

**Professor R. H. Evans** believed that the theory produced by the Author was a very important development of the collapse method of design for mild-steel structures. It departed from the established nomenclature and method of approach. It seemed that the plastic theory would apply to reinforced-concrete frames provided: (*a*) that special reinforced-concrete hinge units were incorporated in the construction; and (*b*) that between those units the members were stressed only elastically. In that respect there was a real difference between the behaviour of steel and reinforced-concrete structures. It was possible in the former to have plastic hinges with full elastic behaviour between the hinges. But in reinforced-concrete frame construction, tensile cracks in the concrete would exist between the assumed hinge units and it was not clear in what way those tensile cracks would influence the results. It was possible, for example, that a cracked section of the reinforced-concrete member might behave as a semi-plastic hinge or unit.

A study of *Fig. 2* made one wonder whether the frame would take the wind forces from right to left without the insertion of further hinges. If more hinges were required, would they not invalidate the analytical work? *Fig. 2* illustrate a novel method of construction and it seemed desirable to check the theoretical work by means of tests on model and full-scale structures.

The method of analysis was very complex and assumed that the hinges were correctly positioned before starting the calculations. The positioning of the plastic hinges in the case of mild-steel structures was considered a major problem by the Cambridge School. The necessity for the enormous labour in Tables 1 and 2 was apparently dictated by the requirement of limiting the plastic-hinge rotation, that difficulty being never considered in the collapse method of design for mild-steel structures. The virtual-work method was somewhat difficult to apply, as stated by the Author on p. 282, and it was doubtful whether the structural analyst could produce a "card" as illustrated in *Fig. 7* without considerable time and effort. In fact, it was questionable if *Fig. 4* (*b*) could be produced without a consideration of the slope-deflexion. Thus it would seem at first sight that $M_1 = 1$ at A would be divided between the beam and the column below. The difficulties of calculating the $\theta$ values made it necessary to have a simpler version of the present analysis before it could be introduced into the design office. Simplicity was always a primary consideration in design procedure.

On pp. 295–300 the problem of the behaviour of concrete in bending was reviewed and the complexity of the problem became apparent. That was particularly so when the collapse method was applied to cylindrical shells. There was no doubt that the Author had considered a most

difficult problem from a new approach and had deduced results which ultimately should prove of value to designers. It was hoped that the Author would endeavour to simplify his analysis and to follow that by a series of experiments on both models and full-scale structures.

**Dr E. H. Bateman** considered that in proposing to solve a framework of many members by shuffling a pack of cards, the Author had enhanced his reputation for resource and invention. That new card trick raised the question of whether the Paper had in fact reduced a complicated analytical problem to a game of "Snap," or whether the attention which should be focused on one-half of the problem was being distracted by flood lighting the other.

However much the distinction between vertical and horizontal loads on a framework of the type discussed in the Paper might be concealed by similarities in nomenclature and notation, the practical difference in the effects of those two types of loading was fundamental. Hardy Cross and all who had followed him had never seen any real difficulty in estimating the effects of vertical loads on a "building frame," but all had been conscious of the necessity for treating sidesway, in theory at least, with the greatest respect. Vertical loads were rapidly diffused into the structure and transmitted to the foundations as axial column forces with subsidiary bending at short distances from the loading points. On the other hand, however, a horizontal load at the top of a structure produced bending in every member. If there were inadequate stiffness of horizontal members, that bending might become excessive at ground level, as he (Dr Bateman) had pointed out 20 years previously.[21] The introduction of plastic hinges did not seem to him to make the computation of stresses due to lateral loads any simpler nor their practical effects less potentially serious.

In Great Britain it had so far been possible to ignore almost completely the effects of wind loads, when six or eight broadly based storeys were considered to constitute a tall building. The Author, however, to his credit, was well known for his advocacy of upward building, and in the Paper he had focused attention on a structure of twenty-five storeys. For such elevations, it might well be necessary to pay special attention to wind loads.

The fact that the Author and his associates could solve their problems with a pack of cards or a rudimentary abacus seemed to him (Dr Bateman) yet one more demonstration of the simplicity of the no-sidesway problem, and he did not believe that that was the true end of the search for analytical simplicity. For his own part he looked rather to the other end of the scale, to the work, for example, of Professor Louis Matheson and Mr R. K. Livesley of Manchester University who were now engaged in turning the power of the electronic computing machine to problems of structural analysis both elastic and plastic. Although he had so far seen only one Paper in print [22] on that new method of structural analysis he thought that there was a good prospect of electronic computing becoming available

before long for the rigorous testing of the simple formulae and criteria which the Author so rightly regarded as essential in an engineer's design office.

**The Author**, in reply to Dr Meyerhof, stated that insufficient tests had been carried out to show that plastic hinges at supports never weakened the ultimate strength of beam sections in shear, but the truss action assumed in designing shear reinforcement seemed to operate right up to failure. Even when no turned-up bars were provided, and the section was cracked at the top, diagonal thrust below the crack still resisted shear. Further tests, however, should be done, and if weakening in shear resistance was shown to occur for certain conditions, it would have to be allowed for. With regard to foundations which were liable to tilt or move horizontally, the rotations of the plastic hinges restraining such movement, if ultimate load was ever applied, would generally be increased by the movement, but the plastic restraining moment would continue to act so long as the rotation was not excessive. If the mode of failure was altered by such movement, the total rotation value of wrongly positioned hinges would be found to be negative, and appropriate adjustment to the assumed hinge-positions would be made. Such ground conditions, however, would generally require a raft or at least ties between the footings to prevent or minimize such movement.

Replying to Professor Marshall, the Author stated that he should have pointed out that hinges F and E, *Figs 24*, were exceptions to the general rule. There could only be two hinges at joint E and it was more convenient for adjustment to have one in the beam than at the top of the lower column. The hinge assumed at F was generally found to be well positioned since the moments at the bottoms of the columns restraining the bending of the end-span beams were usually too large to be seriously reduced by the sway moments. With regard to the shell, the segments had been reinforced, and the reinforcement, together with the uncracked concrete, resisted transverse bending. Frictional resistance along the transverse edges of the segments, which was present so long as the segments were pressed together, transmitted differences of shear from one segment to another. Such differences of shear, together with the load and the restraint of the edge beam, caused the transverse bending resisted by each segment, and which the deflexions indicated was present. The prestress which influenced the distribution of the longitudinal "beam" bending stress influenced the transverse bending, since the distribution of resultant longitudinal "beam" bending stress determined the distribution of shear stress along the edges of the segments, but the shape of the shell cross-section in relation to load distribution was a more important factor. The shape of the shell tested was suitable for the load distribution and hence transverse bending moments were small. The reference to "elastic theory simplified by neglecting torsion and longitudinal bending" was intended to mean elastic theory, using Schorer's simplifying assumptions.

With regard to the final point, uniformly distributed load had been assumed in the calculations. Weighed sandbags, in piles, having the same number of bags in each pile, were used. The shell surface was roughened with a thin layer of no-fines concrete, rather like pebble-dash, to prevent slipping. The piles of bags were kept apart by tie-rods placed across the bags at 27-inch vertical intervals.

Replying to Dr Gartner, the Author stated that he thought that an ultimate-load basis of design should be used to determine the cross-sections of frameworks. That need not mean calculating in terms of ultimate loads, but working loads could be used, as at present; permissible stresses and stress distributions could be adjusted where necessary and could be based on safe limiting values such as those given in Table 5, together with statistical analysis of specimen tests of concrete and steel. The diagrams in *Figs 8* were derived for assumed ultimate-load values. *Fig. 8* (*b*) indicated the distribution of bending moments on the assumption that no plasticity developed, and therefore represented the distribution, but not the values, of bending moments for working load. If the factor of safety was 2, the values of the loads assumed would be double the working-load values and have the same distribution. If working-load values were used to determine rotations of plastic hinges, for an assumed plastic condition which would develop for ultimate load, then the rotations thus found would need to be multiplied by the ratio of ultimate load to working load—that was to say, the factor of safety—in order to determine the rotations for the ultimate condition. In columns such as those supporting the lower floors of a high building, where direct stresses were large compared with bending stresses, it might be necessary in theory to assume that additional reinforcement was used between the plastic hinges as compared with the plastic-hinge sections, in order to ensure that plasticity did not extend much beyond the assumed length of the plastic hinge. The hinge section, too, might in some cases need to have close binding in order to increase the available rotation as a compressive type of plastic hinge (see *Fig. 11*). In practice, the same main reinforcement would for convenience be used throughout the column and be equal to the maximum area required. If the structure was tested, the plastic zones occurring before failure would probably be different from those assumed in the calculations. However, it would be certain that any structure used in practice would not be weakened, by strengthening certain sections, in order to provide uniformity. It might be possible to design a structure which could be weakened by strengthening a part, but such an unusual case, if it occurred in the course of normal practical work, would be evident to the designer, who would then take special steps to deal with it. Variations in the value of $E$ due to age, and the deformations taken up by lower parts of a structure before the upper were constructed, could cause actual stresses to be different from stresses calculated by elastic theory. Plastic-hinge theory, however, assumed that the structure was at least as strong as one which developed

the assumed plastic hinges and became statically determinate before failure. In the latter condition the above influences could be ignored in regard to their influence on stress values. Rotation values might be affected, but broad limits were used for $EI$-values in the framework members and permissible strain values at the hinges which covered such contingencies. In spite of that, economic distribution of bending moments were obtained without excessive rotations of the hinges. In the example *Fig. 6* (*i*) the moment diagram was drawn as shown in order to agree with the assumption made in the general case, *Fig. 9* (*l*). The bottom left-hand column-hinges would have tension on the left when wind blew from the left and was more influential than the vertical load. It might have been better to have drawn the diagram on the right-hand side for the general case, since vertical load generally had more influence, but it was interesting to note that when a diagram was drawn on the "wrong" side of a member, the trial-and-adjustment procedure revealed that the $\bar{X}$-value assumed for the hinge should be negative in order that $\theta$ might be positive. Use of the negative sign for the moment value gave the same result as plotting the diagram on the "right" side of the member.

The Author stated, in regard to Dr Horne's contribution, that he welcomed it as a good alternative approach to ultimate-load design. It probably had advantages for certain cases. However, it appeared to require the completion of an ordinary elastic-moment distribution first to determine the moments at a joint with which to make a start. In a building frame, fifteen to twenty storeys high, subject to sway and having stiff columns relative to the beams, the preliminary determination of those elastic moments might be difficult and onerous. The trial-and-adjustment method described by the Author was not difficult, when plastic-moment values based on the moment distributions shown in *Figs 3* were assumed and the adjustment rules given at the bottom of p. 283 of the Paper were followed. The same rules and influence coefficients could also be used to obtain fairly quickly an approximate elastic solution. Moreover, the practical designer should, with a little further development, be able to establish standard formulae, derived with the aid of the general elastic equations, such as $M = \frac{wl^2}{16}$ at mid-span and support sections of equi-span continuous beams when $\frac{l}{d} \not< 40$ (see reference 23). It should be possible also to establish that the ultimate distribution of bending moments may be safely assumed as shown in *Figs 3* for a wide range of stiffness ratios of beam to column. Similar simplifications might of course be obtained by the distribution method, but it appeared to be more appropriate for numerical examples. The Author hoped that Dr Horne would publish a further Paper and explain his very ingenious and promising method at greater length, and applied to a tall building frame subject to sway.

Turning to Dr Hajnal-Kónyi's comment the Author wished to re-state his own contention, which he thought perhaps had not been made clear.

Suppose two beams A and B were identical, except that mild-steel reinforcement was used for A and high-tensile steel for B. Also suppose the area of steel in beam A was just sufficient in a bending test to prevent the steel yielding before the concrete crushed, and the area of high-tensile steel in beam B was such that its total strength was equal to that of the steel in beam A, the yield strength of the high-tensile steel being about 3 to 4 times that of the mild steel. Beam B would fail by concrete crushing at a lower load than A, since the position of the neutral axis would be higher, because the strain in the high-tensile steel would be greater at the same stage of loading. The ratio $\frac{n}{d}$ would be about 0·5 for beam A and about 0·3 for beam B, corresponding to a linear distribution of strain, even if the high-tensile steel in beam B had greater bond strength. That was demonstrated by the *F* values which were close to unity, obtained by Gray [24] and Leung [25] for various steels, and which showed that bond strength, provided slip occurred only at the cracks, did not greatly influence the position of the neutral axis, although better bond reduced crack widths. The values of $\frac{n}{d}$ in Table 7 also confirmed the Author's contention, which was that the greater strains taken up by high-tensile steel, as compared with mild steel $\left(\frac{n}{d} = 0{\cdot}5\right)$, reduced the depth of the neutral axis. If that was accepted then the total concrete compression and concrete moment of resistance was reduced for identical grades of concrete. The latter result was not always evident from bending strengths obtained from tests, since although the concrete mixes employed might be identical the strengths might vary from 20 to 30 per cent. In a large number of tests the general trend, however, agreed with the Author's contention.

The Author was grateful to Mr Chan and Mr Yu for their contributions. The additional information regarding binders, and the improved rules for adjustment of the plastic-moment values which they had derived were valuable.

The Author hoped that practical engineers such as Mr Derrington would now develop simple bending-moment formulae for frames in common use which could be used within specified limits of relative stiffness of appropriate members. The continuous beam had been given as a simple example in reference 23. Concrete *E* values did vary, but even if broad limits were used to cover those variations, it would be found that the hinge-rotation values could still be satisfactorily taken up.

Dr Everard's proposals were of great interest, but more appropriate for research investigations. The practical designing engineer could calculate the ultimate strength of frameworks fairly accurately in terms of the

results of specimen tests on concrete and steel. Accuracy in regard to deformation values was not important as a rule, and rotations of hinges could be checked using safe limiting values of $EI$ which included the effects of creep and varying rates of application of load.

With regard to Mr Hun's comment, the Author had received confirmation from Mr Yu (who had prepared the Appendix) that he had not attempted to relate *Fig. 23* to the explanation given in the text. The photograph of the abacus had been taken at random, and the explanation written later.

The Author appreciated Professor Evan's contribution and in reply stated that, although certain sections of frameworks were regarded as potential hinges, standard systems of reinforcement could be used. A little additional binding might sometimes be necessary. The influence of cracking on deformation could be allowed for in the calculations by assuming throughout members a safe minimum $EI$-value equal to that which occurred at the cracked section of maximum concrete stress. The bulge in the plastic-stress distribution compensated the $EI$-value from being as low at that section as it might first appear. Moreover, the higher rotations of hinges obtained by assuming such safe minimum $EI$-values were generally not excessive. The plasticity of the hinge was adequate for them to be taken up, particularly when closed binders were used. With regard to frames such as the one shown in *Fig. 2*, in which sway forces could act from either side, the final frame would be designed so that each member was at least as strong as would be required for either of two imaginary frames designed for either case of loading. Under test such a frame might develop plastic hinges at quite different sections from those assumed, but it would not be weaker than the theoretically correct design for either case of loading. It would be possible to design a frame which was weakened by strengthening and hence stiffening certain sections, but such abnormal conditions were not likely to occur in practice and would be located as special cases if they did. The frame illustrated in *Fig. 2* need not be novel in construction. Plastic hinges developed in reinforced-concrete members conforming in design to standard practice, when either the concrete or steel commenced to yield. The only special steps required might be to provide additional binders at certain sections where strains in the concrete would be high if the structure was tested to destruction. In regard to *Fig. 4* (*b*), the moment applied at A to the frame below A would be resisted by the beam, since the remainder of the framework of the storey below A was hinged, so that it could not provide resistance to sway forces. Hence no bending moment occurred in the column. That particular part of the Paper was intended to give an alternative derivation of Müller-Breslau's well-known general elastic equations which were very commonly used by designers for elastic solutions. The tabulated trial-and-adjustment process was a method of solving those general equations which was direct and quick for the plastic case. It was also much quicker

than other methods for solving approximately the elastic case, which could otherwise be quite intractable in the case of high buildings many hundreds of times statically indeterminate. The Author, however, envisaged that it would be possible now to establish limits of stiffness and stiffness ratio, which would apply to many frames in common use, and for which the bending-moment values shown in *Figs 3* could be used without further, or perhaps only slight, adjustment.

Turning now to Dr Bateman, the Author stated that he had once heard an eminent engineer say that a design method was of no use to the practical man unless it could be done on the back of a post-card. The Author had failed to meet that requirement, but he had proposed a method which could be applied with the help of a pack of cards. That device, together with the model described by Mr Yu, had provided visual aid which was of great value in learning to use the trial-and-adjustment method. Having grasped the theory and applied it to a few cases it was anticipated that designers would soon establish and use standard formulae for ultimate-bending-moment values applicable to the common types of framework. Many twenty–thirty-story buildings had been designed for a distribution of sway moments as shown in *Fig. 3* (*c*). That, as Dr Bateman had shown, could be quite wrong for elastic conditions, particularly when the columns in the lower storeys were stiff relative to the beams, and almost fixed in direction at basement level. The plastic-hinge approach, however, could show that often the distribution of sway moments shown in *Fig. 3* (*c*) was correct for ultimate conditions. If assumed in the calculations, the resulting design under full working load would develop stresses greater than those normally permitted, but it would have an adequate factor of safety against failure, and would generally not be excessively strained unless over-loaded. The introduction of the plastic hinges made the computation of sway moments a practical operation. Elastic solutions were often based on assumptions which were not true, such as $EI$ values and non-settlement of supports and, as Dr Bateman had mentioned, required an electronic computing machine for their completion when the number of unknowns was high. Moreover, electronic machines could not determine the worst possible distributions of $EI$ values due to cracking, shrinkage, and creep. Whilst such analytical approaches were of great interest and value in research, there was no doubt that the soundest methods of design were those which could be carried out with simple slide-rule calculations, so that the influence on the answer of variations in the basic assumptions could be readily assessed.

The Author thanked Mr D. K. Doran for pointing out three errors in Table 1. The unit rotation values for hinge 2-2-*c* due to 1-1-*b* acting should have been $-2$; for hinge 2-2-*c* due to 2-1-*a* acting should have been $+2$; for hinge 1-2-*b* due to 2-2-*a* acting should have been $-1$. The results were not greatly affected. The adjustment made to 2-2-*c* was seen to have been unnecessary.

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